Reverse Telephoto with FOV – Raytrace

An f/4 reverse telephoto objective is comprised of two thin lenses in air. The system stop is located between the two lenses.

The system is now to be used with a finite conjugate object that is located 150 mm to the left of the first lens. The maximum image size is +/- 20 mm.

Determine the following:
- System focal length and back focal distance.
- Stop size; Entrance pupil and exit pupil locations and sizes.
- Image location.
- Object size corresponding to the image size.
- Required diameters for the two lenses for the system to be unvignetted over the specified maximum image size and conjugate location.

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the object size must be determined from the chief ray. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. Gaussian imaging methods may not be used for any portion of this problem.
Solution:

Summary:

System Focal Length = 50 mm  Back Focal Distance = 80 mm
Entrance Pupil: 11.54 mm to the R of the first lens.  \( D_{EP} = 12.5 \) mm
Exit Pupil: 24.0 mm to the L of the second lens.  \( D_{XP} = 26.0 \) mm
\( D_{STOP} = 16.25 \) mm  Object Size = \(+/-\) 55.0 mm
Image Location: 98.2 mm to the R of the second lens.
Lens 1 Diameter = 19.46 mm  Lens 2 Diameter = 28.75 mm
<table>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>f</td>
<td>-</td>
<td>-50</td>
<td>-</td>
<td>40</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-t</td>
<td>0.02</td>
<td>-0.015</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Values for Finite Object

Potential Chief Ray

\[
\begin{align*}
\Delta \xi & = 141.54 \\
\Delta \eta & = -11.54 \\
\Delta \xi' & = 15 \\
\Delta \eta' & = 15 \\
\Delta \xi'' & = -24.0 \\
\Delta \eta'' & = 122.2
\end{align*}
\]

Potential Infinite Marginal Ray

\[
\begin{align*}
\Delta \xi & = 1 \\
\Delta \eta & = 1 \\
\Delta \xi' & = 1.30 \\
\Delta \eta' & = 1.60 \\
\Delta \xi'' & = 2.08 \\
\Delta \eta'' & = 0
\end{align*}
\]

\[
\begin{align*}
\Delta \xi & = 0 \\
\Delta \eta & = 0.02 \\
\Delta \xi' & = 0.02 \\
\Delta \eta' & = -0.02 \\
\Delta \xi'' & = -0.02 \\
\Delta \eta'' & = 0
\end{align*}
\]

Infinite Marginal Ray: Scale 6.25

\[
\begin{align*}
\Delta \xi & = 6.25 \\
\Delta \eta & = 6.25 \\
\Delta \xi' & = 6.25 \\
\Delta \xi'' & = 8.125 \\
\Delta \eta'' & = 10.0 \\
\Delta \xi''' & = 13.0 \\
\Delta \xi''' & = 0
\end{align*}
\]

\[
\begin{align*}
\Delta \xi & = 0 \\
\Delta \eta & = 0.125 \\
\Delta \xi' & = 0.125 \\
\Delta \xi'' & = -0.125 \\
\Delta \xi''' & = -0.125 \\
\Delta \xi''' & = 0
\end{align*}
\]

Marginal Ray

\[
\begin{align*}
\Delta \xi & = 0.0357 \\
\Delta \eta & = 0.0357 \\
\Delta \xi' & = 0.1548 \\
\Delta \xi'' & = 0.1548 \\
\Delta \xi''' & = -0.1064 \\
\Delta \xi''' & = -0.1064 \\
\Delta \xi''' & = 0
\end{align*}
\]

Finite

Potential Chief Ray - Extended

\[
\begin{align*}
\Delta \xi & = -21.0 \\
\Delta \eta & = 0 \\
\Delta \xi' & = -1.5 \\
\Delta \xi'' & = 0.15 \\
\Delta \xi''' & = 0 \\
\Delta \xi''' & = 7.625
\end{align*}
\]

\[
\begin{align*}
\Delta \xi & = 0.13 \\
\Delta \xi' & = 0.13 \\
\Delta \xi'' & = 0.1 \\
\Delta \xi''' & = 0.1 \\
\Delta \xi''' & = 0.0625 \\
\Delta \xi''' & = 0.0625
\end{align*}
\]

Chief Ray - Scale 2.618

\[
\begin{align*}
\Delta \xi & = -55.0 \\
\Delta \eta & = 0 \\
\Delta \xi' & = -3.927 \\
\Delta \xi'' & = 3.927 \\
\Delta \xi''' & = 0 \\
\Delta \xi''' & = 20.0
\end{align*}
\]

\[
\begin{align*}
\Delta \xi & = 0.340 \\
\Delta \xi' & = 0.340 \\
\Delta \xi'' & = 0.262 \\
\Delta \xi''' & = 0.262 \\
\Delta \xi''' & = 0.164 \\
\Delta \xi''' & = 0.164
\end{align*}
\]

\[
\begin{align*}
\Delta \xi & = 0 \\
\Delta \xi' & = 0 \\
\Delta \xi'' & = 0 \\
\Delta \xi''' & = 0 \\
\Delta \xi''' & = 0 \\
\Delta \xi''' & = 0
\end{align*}
\]

\[
\begin{align*}
\Delta \xi & = 0 \\
\Delta \xi' & = 0 \\
\Delta \xi'' & = 0 \\
\Delta \xi''' & = 0 \\
\Delta \xi''' & = 0 \\
\Delta \xi''' & = 0
\end{align*}
\]

* arbitrary
Method of Solution:

-Trace a potential chief ray – height of zero at stop.

-Pupil Positions: Located where the chief ray crosses the axis in object and image space.

\[ EP: \quad EP \rightarrow L_1 = -11.54 \text{ mm} \]
EP 11.54 mm to the right of L1

\[ XP: \quad L_2 \rightarrow XP = -24.0 \text{ mm} \]
XP 24.0 mm to the left of L2.

Note that the potential chief ray and pupil positions (and sizes) are independent at conjugates.

-To determine the focal length and BFD, trace a potential infinite marginal ray (parallel to the axis).

\[ \ddot{u}' = -0.02 \quad \ddot{y}_0 = 1 \]

\[ \phi = \frac{\ddot{u}'}{\ddot{y}_0} = 0.02 \]

\[ f = 50 \text{ mm} \]

\[ BFD = (L_2 \rightarrow XP) + (XP \rightarrow F') = -24.0 \text{ mm} + 104.0 \text{ mm} = 80.0 \text{ mm} \]

-The system operates at f/4:

\[ \frac{f}{D_{EP}} = 4 \quad D_{EP} = 12.5 \text{ mm} \quad r_{EP} = 6.25 \text{ mm} \]

Scale Factor = 6.25
-Scale the potential infinite marginal ray to determine the stop and XP sizes.

\[
\begin{align*}
  r_{\text{STOP}} &= 8.125 mm \\
  D_{\text{STOP}} &= 16.25 mm \\
  r_{\text{XP}} &= 13.0 mm \\
  D_{\text{XP}} &= 26.0 mm
\end{align*}
\]

-Now trace a marginal ray for the finite conjugate object.

\[
\begin{align*}
  \text{Object} \rightarrow L_1 &= 150 mm - (\text{Object} \rightarrow EP) + (EP \rightarrow L_1) \\
  EP \rightarrow L_1 &= -11.54 mm \\
  \text{Object} \rightarrow EP &= 161.54 mm
\end{align*}
\]

\[
u_0 = \frac{r_{EP}}{\text{Object} \rightarrow EP} = \frac{6.25 mm}{161.54 mm} = 0.0387
\]

Image Location: 122.2 mm to the right of XP

\[
\begin{align*}
  L_2 \rightarrow \text{Image} &= (L_2 \rightarrow XP) + (XP \rightarrow \text{Image}) = -24.0 mm + 122.2 mm = 98.2 mm
\end{align*}
\]

The image is located 98.2 mm to the right of L2

-Extend the potential chief ray to the object and image locations.

-Scale the potential chief ray to the given image height.

Image Height = 20mm

Scale Factor = 20.0 mm/7.638 mm = 2.618

Object Size = ±55.0 mm
-Vignetting:

L1:

\[ y_1 = 5.803\text{mm} \quad \bar{y}_1 = -3.927\text{mm} \]

\[ a_1 = |y_1| + |\bar{y}_1| = 9.73\text{mm} \]

\[ D_1 = 19.46\text{mm} \]

L2:

\[ y_2 = 10.446\text{mm} \quad \bar{y}_2 = 3.927\text{mm} \]

\[ a_2 = |y_2| + |\bar{y}_2| = 14.37\text{mm} \]

\[ D_2 = 28.75\text{mm} \]