Concave-Convex Lens with Embedded Stop

A plano-concave lens and a plano-convex lens have been glued together to create a thick lens. When the lenses were glued together, the system stop was placed between the two lenses. The lens is used in air.

\[ R_1 = -100 \text{ mm} \]
\[ R_2 = -50 \text{ mm} \]
\[ t_1 = 10 \text{ mm} \]
\[ t_2 = 15 \text{ mm} \]
\[ n = 1.5 \text{ (both lenses)} \]

Stop Diameter = 20 mm

a) Use Gaussian methods to determine the location and diameter of the Entrance Pupil of the system.

b) Use paraxial raytrace methods to determine the system Focal Length, the Back Focal Distance, the Exit Pupil location, and the Exit Pupil diameter. Note that the results from part (a) should not be used for this solution.

Solution

a) Entrance pupil:

Front surface:

\[ \phi = \frac{(1.5 - 1.0)}{R_1} = -0.005 \text{ / mm} \]

EP: Image the stop into object space.

The light is going from Right to Left for the EP imaging.

\[ \frac{n'}{z_{EP}'} = \frac{n}{z_{STOP}} + \phi \]
\[ z_{STOP} = t_1 = 10 \text{ mm} \]
\[ n = -1.5 \]
\[ n' = -1.0 \]
\[-1 = \frac{-1.5}{10} - 0.005\]

\[z'_{EP} = 6.45\text{mm} \text{ to the Right of } V_1\]

\[m_{EP} = \frac{z'_{EP}}{z'_{STOP}} \frac{n'}{n} = \frac{-6.45}{10 / -1.5} = 0.968\]

\[D_{EP} = m_{EP}D_{STOP} = 19.35\text{mm}\]

b) Focal Length and Exit Pupil

Trace a potential chief ray that starts at the center of the stop. The XP is located where this ray crossed the axis in image space.

\[z'_{XP} = -11.11\text{mm} \quad \text{(To the left of } V_2)\]

To determine the focal length and BFD, trace a potential marginal ray parallel to the axis in object space (\(\tilde{y}_1 = 1\)). The rear focal point is located where this ray crossed the axis.

\[z'_{XP \rightarrow F_r} = 196.8\text{mm}\]

\[BFD = z'_{XP \rightarrow F_r} + z'_{XP} = 196.8\text{mm} - 11.11\text{mm} = 185.7\text{mm}\]

\[\phi = -\frac{\tilde{u}'}{\tilde{y}_1} \quad \tilde{u}' = -0.005833 \quad \tilde{y}_1 = 1\]

\[\phi = 0.005833 / \text{mm}\]

\[f = \frac{1}{\phi} = 171.4\text{mm}\]
Scale the potential marginal ray to the Radius of the Stop:

\[ r_{STOP} = 10mm \]

Scale Factor: \[ \frac{r_{STOP}}{y_{STOP}} = \frac{10mm}{1.0333mm} = 9.678 \]

The height of the marginal ray at the XP is the radius of the XP:

\[ r_{XP} = y_{XP} = 11.11mm \]

\[ D_{XP} = 2r_{XP} = 22.22mm \]

Note that the marginal ray height in object space also gives the EP size as this ray is parallel to the axis. This matches the result from part a):

\[ r_{EP} = y_{EP} = 9.678mm \]

\[ D_{EP} = 2r_{EP} = 19.35mm \]
<table>
<thead>
<tr>
<th>Surface</th>
<th>Obj</th>
<th>R1</th>
<th>STOP</th>
<th>R2</th>
<th>XP</th>
<th>F'</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

| R       |       | -100 | -     | -50 |     |    |
| t       |       | ∞    | 10    | 15  | z'_{xp} | z'_{xp} \rightarrow F' |
| n       |       | 1.0  | 1.5   | 1.5 | 1.0  | 1.0 |

\[ \phi = -11.11 \]

| t/n     |       | ∞    | 6.667 | 10.0 | z'_{xp} | z'_{xp} \rightarrow F' |

**Potential Chief Ray – Zero at Stop and XP**

| \tilde{y} |     | 0    | 1.0   | 0    |     |    |
| n\tilde{u} |     | 0.1* | 0.1*  | 0.09 |     |    |
| u         |     |      |      |      |     |    |

**Potential Marginal Ray**

| \tilde{y} |     | 1    | 1     | 1.0333 | 1.0833 | 1.148 | 0    |
| n\tilde{u} |     | 0    | 0.005 | 0.005  | -0.005833 | -0.005833 | 0    |
| u         |     |      |      |      |      |      |      |

**Marginal Ray: Scale to Stop Radius**

\[ r_{\text{STOP}} / \tilde{y}_{\text{STOP}} = 10 / 1.0333 = 9.678 \]

| y       | 9.678 | 9.678 | 10.0 | 10.48 | 11.11 | 0    |
| nu      | 0     | 0.0484 | 0.0484 | -0.0565 | -0.0565 | 0    |
| u       |      |      |      |      |      |      |

*arbitrary*