Focal Length Measurement

The following two methods determine the focal length of a positive lens by using two pairs conjugate locations. In both cases, real objects and images are required (the object to image distance must be larger than 4f).

a) Bessel's Method uses the two reciprocal magnification positions for a fixed object-to-image distance D. The lens is translated between a fixed object and a fixed viewing screen. Two positions of the lens will form an image on the viewing screen (reciprocal magnifications). The separation between these two lens positions is L. Derive the following expression for the focal length in terms of D and L:

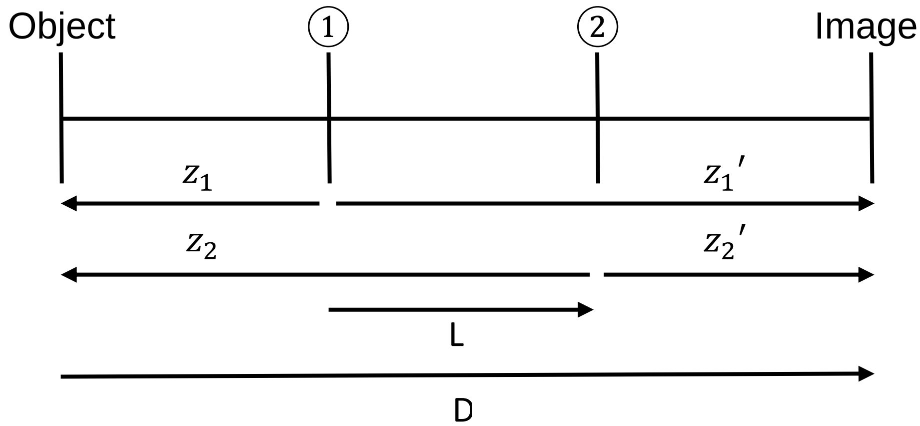
$$f = \frac{D^2 - L^2}{4D}$$

b) In Abbe's method, the image of an object is formed on a viewing screen. The object position z_1 and image magnification m_1 are measured. The object is then moved and the new object position and image magnification z_2 and m_2 are measured. Derive this expression for the focal length as a function of z_1, z_2, m_1 and m_2:

$$f = \frac{z_1 - z_2}{1/m_1 - 1/m_2}$$

Solution

Bessel's Method: Reciprocal Magnification



$$D - L = z_2' - z_1$$

$$D = z_1' - z_1$$

$$D + L = z_1' - z_2$$

Reciprocal Magnification:

$$z_2' = -z_1$$

$$z_1' = -z_2$$

$$D - L = -2z_1$$

$$D + L = -2z_2$$

$$\frac{1}{z_1'} = \frac{1}{z_1} + \frac{1}{f}$$

$$f = \frac{z_1' z_1}{z_1 - z_1'} = \frac{z_1 z_2}{D}$$

$$f = \frac{(D-L)(D+L)}{4D}$$

$$\boxed{f = \frac{D^2 - L^2}{4D}}$$

Abbe’s Method:

$$\frac{1}{z_1'} = \frac{1}{z_1} + \frac{1}{f} \qquad \frac{1}{z_2'} = \frac{1}{z_2} + \frac{1}{f}$$

$$z_1' = \frac{fz_1}{f + z_1} \qquad z_2' = \frac{fz_2}{f + z_2}$$

$$\frac{1}{m_1} = \frac{z_1}{z_1'} = \frac{f + z_1}{f} \qquad \frac{1}{m_2} = \frac{z_2}{z_2'} = \frac{f + z_2}{f}$$

$$\frac{1}{m_1} - \frac{1}{m_2} = \frac{f + z_1}{f} - \frac{f + z_2}{f} = \frac{z_1 - z_2}{f}$$

$$\boxed{f = \frac{z_1 - z_2}{1/m_1 - 1/m_2}}$$

Alternate Solution for Abbe's Method:

$$\frac{1}{z_1'} = \frac{1}{z_1} + \frac{1}{f} \qquad \frac{1}{z_2'} = \frac{1}{z_2} + \frac{1}{f}$$

$$1 = \frac{z_1'}{z_1} + \frac{z_1'}{f} \qquad 1 = \frac{z_2'}{z_2} + \frac{z_2'}{f}$$

$$1 = m_1 + \frac{z_1'}{f} \qquad 1 = m_2 + \frac{z_2'}{f}$$

$$m_1 + \frac{z_1'}{f} = m_2 + \frac{z_2'}{f}$$

$$\frac{z_1' - z_2'}{f} = m_2 - m_1$$

$$f = \frac{z_1' - z_2'}{m_2 - m_1}$$

$$z_1' - z_2' = m_1 z_1 - m_2 z_2 \qquad (\textit{also used below})$$

$$f = \frac{m_1 z_1 - m_2 z_2}{m_2 - m_1} \frac{1/m_1 m_2}{1/m_1 m_2}$$

$$f = \frac{z_1/m_2 - z_2/m_1}{1/m_1 - 1/m_2}$$

This equally valid solution for Abbe's Method differs significantly from the standard answer given above. Comparing the two results implies that

$$z_1/m_2 - z_2/m_1 = z_1 - z_2$$

This certainly doesn't look correct, but can it be? In order to prove it, we will need to jump ahead and use the equation for Thickness Magnification that relates the longitudinal object separation to the conjugate image separation. This is found in Chapter 5 of the class notes.

$$\frac{z_1' - z_2'}{z_1 - z_2} = m_1 m_2 \qquad \frac{z_1' - z_2'}{m_1 m_2} = z_1 - z_2$$

Returning to the quantity in question:

$$z_1 / m_2 - z_2 / m_1 = \frac{m_1 z_1 - m_1 z_2}{m_1 m_2}$$

From above:

$$z_1' - z_2' = m_1 z_1 - m_2 z_2$$

$$z_1 / m_2 - z_2 / m_1 = \frac{z_1' - z_2'}{m_1 m_2}$$

Using the Thickness Magnification relationship proves the equivalence of the results:

$$z_1 / m_2 - z_2 / m_1 = z_1 - z_2$$

$$f = \frac{z_1 - z_2}{1 / m_1 - 1 / m_2} = \frac{z_1 / m_2 - z_2 / m_1}{1 / m_1 - 1 / m_2}$$