Cube - Gaussian Imagery

A 50 mm cube is to be imaged by three optical systems:

a) The center of the cube is positioned 500 mm from the front principal plane of an optical system with \( f_f = -200 \) mm and \( f'_r = 300 \) mm.

b) The center of the cube is positioned 500 mm from the front principal plane of an optical system with \( f_f = 300 \) mm and \( f'_r = -200 \) mm.

c) The cube is imaged with an afocal system in air having \( m = -1/2 \).

In each case, what are the dimensions of the image of the cube? For parts a and b, also determine the location of the cube image. You may assume that the cube is a wire grid so that obscuration, the index of the cube and transparency are not issues.

Solution: 50mm Cube

a) \( f_f = -200 \) mm \( f'_r = 300 \) mm (Positive Lens)

\[ z_o = -500 \text{ mm} \quad \text{← Center of Cube} \]

\[ z_L = -525 \text{ mm} \]

\[ z_R = -475 \text{ mm} \]

\[ l_o = l_L = l_R = 50 \text{ mm} \]

\[ m = -\frac{f_f}{z - f_f} \quad z' = (1 - m)f'_r \]
\[ m_o = -0.667 \quad z'_o = 500mm \quad l'_o = m_o l_o = 33.3mm \]
\[ m_L = -0.615 \quad z'_L = 485mm \quad l'_L = m_L l_L = 30.8mm \]
\[ m_R = -0.727 \quad z'_R = 518mm \quad l'_R = m_R l_R = 36.4mm \]

A cross-section through the cube image is a trapezoid:

Note that the use of the average magnification \( m_o \) and the longitudinal magnification \( \bar{m} = \left( -\frac{f'_R}{f'_F} \right) m_o^2 \) is only an approximation. It would not show the distortion of the cube.

b) \( f_F = 300mm \quad f'_R = -200mm \) (Negative Lens)

\[ z_o = -500mm \quad \text{← Center of Cube} \]
\[ z_L = -525mm \]
\[ z_R = -475mm \]
\[ l_o = l_L = l_R = 50mm \]
A virtual image is produced:

Note that in parts a) and b) the lines representing the top and bottom of the cube image converge to the rear focal point of the system.
c) Afocal system with $m = -1/2$

\[ m = -1/2 \]

\[ m = m^2 \]

\[ m^2 = 1/4 \]

The cross-section of the cube becomes a rectangle:

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25  12.5
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\[ z \]