October 24, 2018 Lecture 19 Name________SOLUTIONS________

Closed book; closed notes. Time limit: 75 minutes.
An equation sheet is attached and can be removed. A spare raytrace sheet is also attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, that method must be used.
Raytraces must be done on the raytrace form. Be sure to indicate the initial conditions for your rays.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Provide your answers in a neat and orderly fashion. No credit if it can't be read/followed.
Use a ruler or straight edge!
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.
Note: On some quantities, only the magnitude of the quantity is provided. The proper sign conventions and reference definitions must be applied.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

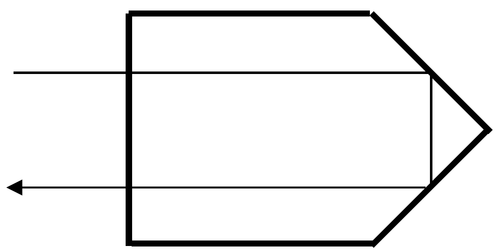
1) (10 points) An object is located 10 m to the left of a 50 mm focal length thin lens. The detector size is 5 x 5 mm. What is the approximate maximum object size that can be imaged?

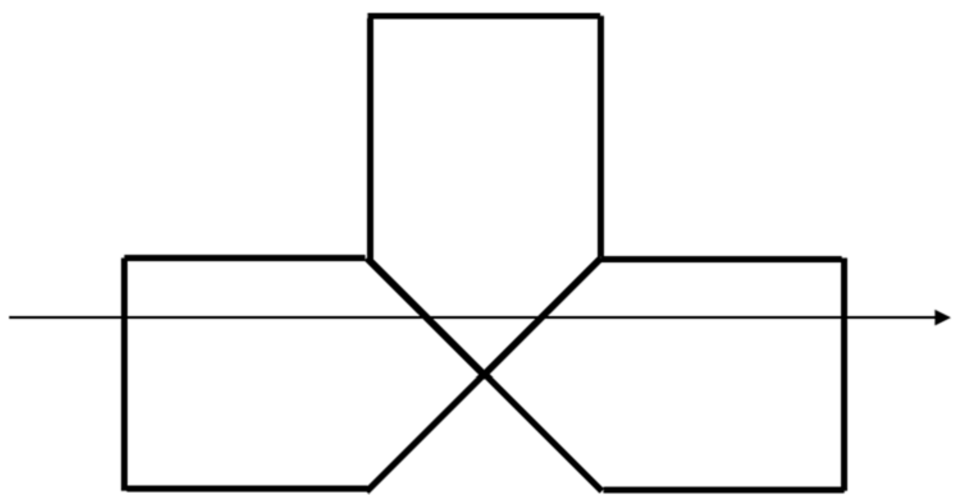
$$m = \frac{z'}{z} \approx \frac{f}{z} = \frac{50mm}{-10m} = \frac{50mm}{-10,000mm} = -0.005 \qquad h = \frac{h'}{m} = \frac{5mm}{-0.005} = -1000mm = -1.0m$$

$$200:1 \qquad h > h' \qquad h = 200h' = 200(5mm) = 1000mm = 1.0m \qquad \textit{Inverted}$$

Object Size ≈ __1.0__ m x __1.0__ m

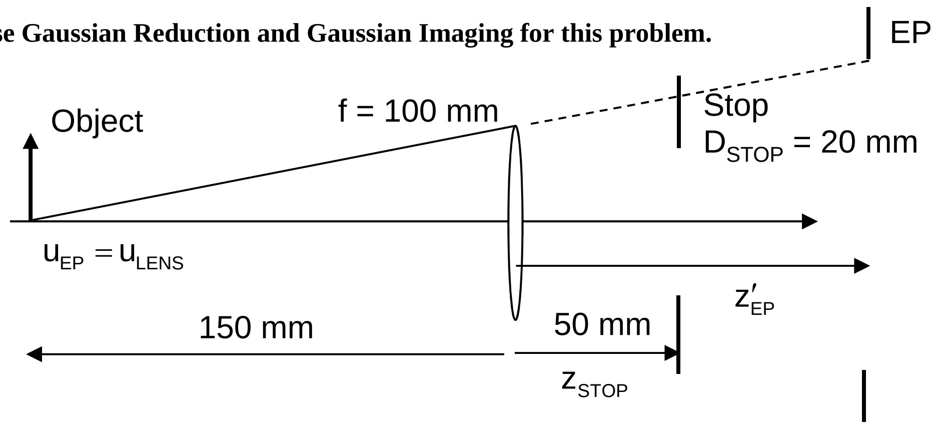
2) (10 points) Draw the tunnel diagram for this prism and the ray path shown.





3) (15 points) An object is located 150 mm to the left of a 100 mm thin lines in air. The system aperture stop is 50 mm to the right of the lens. The stop diameter is 20 mm.

NOTE: Use Gaussian Reduction and Gaussian Imaging for this problem.



a) Determine the location and size of the Entrance Pupil.

Light from the stop travels R to L: $n = n' = -1$

$$z_{STOP} = 50mm \qquad f = 100mm$$

$$\frac{n'}{z'_{EP}} = \frac{n}{z_{STOP}} + \frac{1}{f} \qquad \frac{-1}{z'_{EP}} = \frac{-1}{50mm} + \frac{1}{100} \qquad z'_{EP} = 100mm \qquad \text{Right of the Lens}$$

$$m_{EP} = \frac{z'_{EP}/n'}{z_{STOP}/n} = \frac{100mm}{50mm} = 2.0 \qquad D_{EP} = m_{EP} D_{STOP} = 2.0(20.0mm) = 40.0mm$$

EP: __100__ mm to the ___R__ of the lens; D_{EP} = __40_ mm

b) What is the required minimum lens diameter so that the lens does not serve as the system stop?

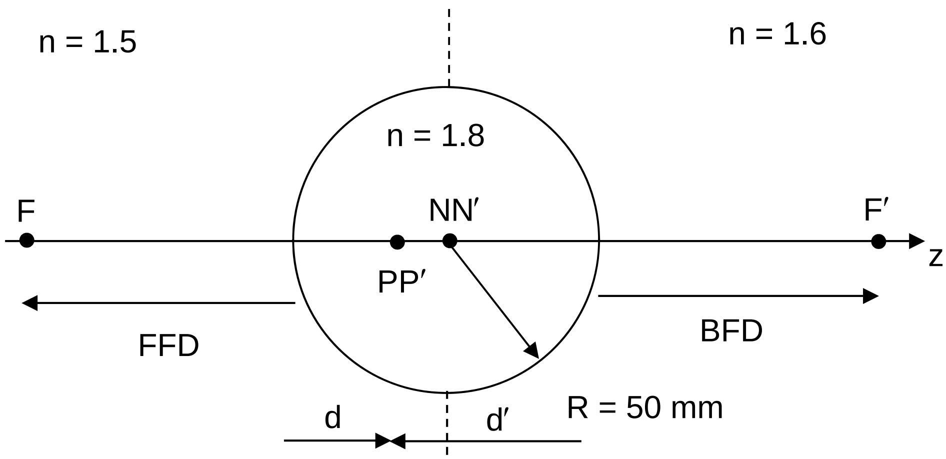
The angular size of the lens as viewed from the object must equal the angular size of the EP as viewed from the object (actually it needs to be greater than this angular size).

$$z = -150 \qquad r_{EP} = 20mm \qquad u_{EP} = \frac{r_{EP}}{-z + z'_{EP}} = \frac{20mm}{250mm} = 0.08$$

$$u_{LENS} = \frac{r_{LENS}}{-z} = \frac{r_{LENS}}{150mm} = u_{EP} = 0.08 \qquad r_{LENS} = 12mm \qquad D_{LENS} = 24mm$$

Minimum Lens Diameter = ___24__mm

4) (25 points) A spherical ball of index 1.8 is mounted between two media with indices of refraction of 1.5 and 1.6. The radius of the ball is 50 mm.



An object of size ±10 mm is placed 500 mm to the left of the front vertex of the sphere.

Determine: System Focal Length
Locations of the Principal Planes relative to the respective vertices
Locations of the Nodal Points relative to the respective vertices
Front Focal Distance
Back Focal Distance
Image Location relative to the rear vertex of the sphere
Image Size

Sketch the approximate locations of F, F', P, P', N, N' on the above figure.

NOTE: Use Gaussian Reduction and Gaussian Imaging for this problem. Cascaded imaging may not be used (you may not image through one lens and then use this image as an object for the other lens).

$R_1 = 50mm \qquad C_1 = 0.02/mm \qquad\qquad R_2 = -500mm \qquad C_2 = -0.02/mm$

$\phi_1 = (n_2 - n_1)C_1 = (1.8 - 1.5)C_1 \qquad\qquad \phi_2 = (n_3 - n_2)C_2 = (1.6 - 1.8)C_2$

$\phi_1 = 0.006/mm \qquad\qquad \phi_2 = 0.004/mm$

Reduce the two surfaces of the sphere:

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau \qquad \tau = \frac{t}{n_2} = \frac{2R}{n_2} = \frac{100mm}{1.8} = 55.56mm$$

$$\phi = 0.00867/mm$$

$$f = \frac{1}{\phi} = 115.4mm$$

$$f_F = -n_1 f = -1.5 f = -173.1mm \qquad f'_R = n_3 f = 1.6 f = 184.6mm$$

Now, the Principal Plane locations and the BFD and FFD:

$$\delta = \frac{d}{n_1} = \frac{\phi_2}{\phi}\tau = 25.67mm \qquad \delta' = \frac{d'}{n_3} = -\frac{\phi_1}{\phi}\tau = -38.43mm$$

$$d = 38.5mm \quad n_1 = 1.5 \qquad d' = -61.5mm \quad n_3 = 1.6$$

$$FFD = d + f_F = -134.6mm \qquad BFD = d' + f'_R = 123.1mm$$

$$\overline{PP'} = t + d + d' = 0$$ The Principal Planes are physical coincident

Nodal Point Locations:

$$\overline{PN} = \overline{P'N'} = f_F + f'_R = -173.1mm + 184.6mm = 11.5mm$$

The Nodal Points are located 11.5 mm to the right of the Principal Planes

N: Location from the front Vertex = $d + 11.5mm = 50.0mm$ (to the right)

N′: Location from the rear Vertex = $d' + 11.5mm = -50.0mm$ (to the left)

The Nodal Points are physically coincident at the center of curvature of the sphere. This result could be obtained by inspection. The Principal Planes must therefore also be coincident since $\overline{PN} = \overline{P'N'}$.

The object position relative to the Front Principal Plane P:

$$s = -500mm$$

$$z = s - d = -538.5mm$$

Image!

$$\frac{n_3}{z'} = \frac{n_1}{z} + \phi \qquad \frac{1.6}{z'} = \frac{1.5}{-538.5} + 0.00867$$

$$\frac{z'}{1.6} = 169.9mm$$

$$z' = 271.9mm$$

Convert to vertex distance:

$$s' = z' + d' = 266.7mm - 61.5mm$$

$$s' = 210.4mm$$

Real image to the right of the rear vertex.

Image magnification and size:

$$m = \frac{z'/n_3}{z/n_1} = \frac{271.9/1.6}{-538.5mm/1.5}$$

$$m = 0.473$$

$$h' = mh \qquad h = \pm 10mm$$

$$h' = \mp 4.73mm$$

Inverted image.

Focal Length = ___115.4____ mm

P: Located ___*38.5*__mm to the ___R____ of the front vertex.

P': Located ___61.5__mm to the ____L___ of the rear vertex.

N: Located ___50.0___mm to the ___R____ of the front vertex.

N': Located ___50.0__mm to the ____L___ of the rear vertex.

FFD = ___-134.6____ mm BFD = ____123.1___ mm

Image': Located __210.4__mm to the __R____ of the rear vertex.

Image Size = ± ___4.73___mm Inverted

5) (10 points) An afocal system in air is constructed with two positive thin lenses. The system is 150 mm long and has a longitudinal magnification $\bar{m} = 25$.

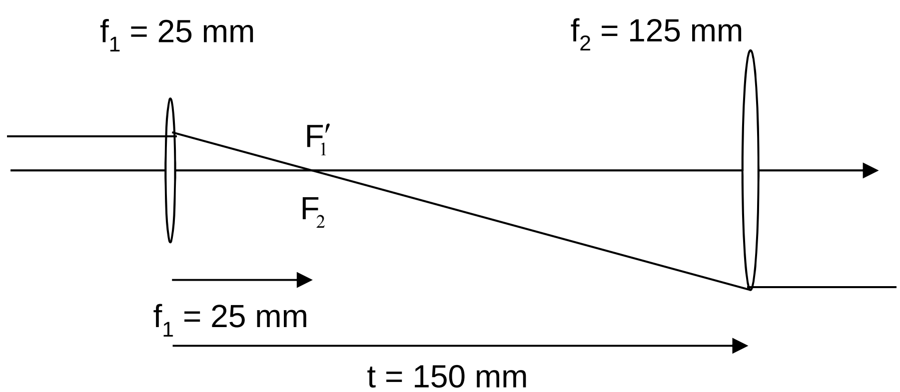
Sketch the system and provide the focal lengths of the two lenses.

An afocal system constructed out of two positive lenses must have a negative lateral magnification m.

$$\bar{m} = m^2 = 25 \qquad m = -5 = -\frac{f_2}{f_1} \qquad f_2 = 5 f_1$$

$$t = f_1 + f_2 = f_1 + 5 f_1 = 6 f_1 = 150mm$$

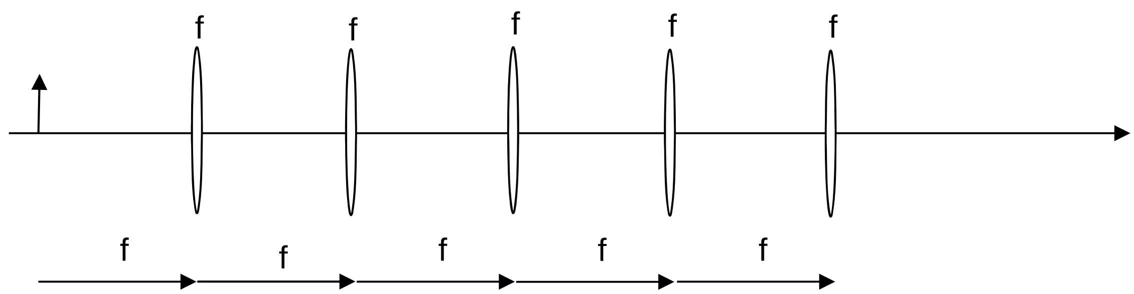
$$f_1 = 25mm \qquad f_2 = 125mm$$



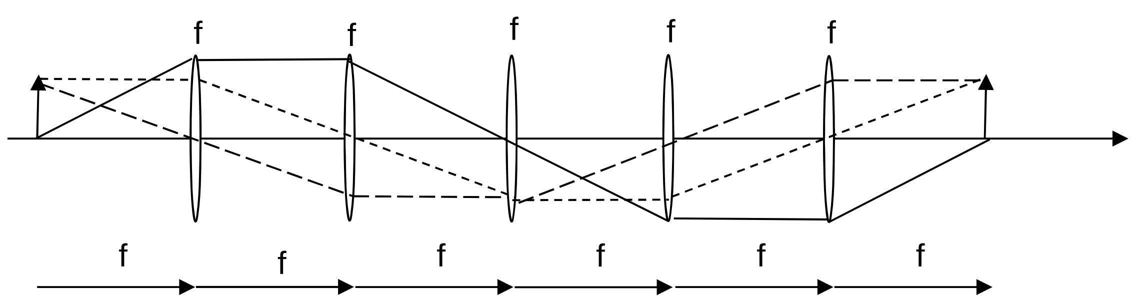
f_1 = ___25_____mm f_2 = ___125____mm

6) (10 points) Consider the following optical system comprised of five identical thin lenses of focal length f that are each separated by this same distance f.

An object is located at the front focal point of the first lens element. Determine the image location and size by sketching rays. Please use a straightedge. No calculations are required or permitted.



Sketch rays starting at the axial object location and from the top of the object. Make use of the properties of focal points. There are two possible rays from the top of the object.

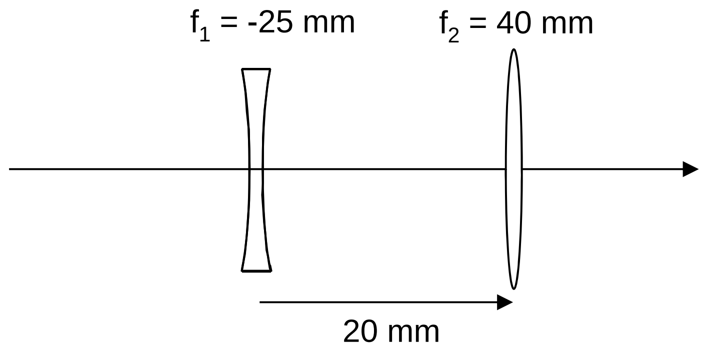


Two of the three rays shown must be used.

The image is at the rear focal point of the final lens element.

The system operates at a true 1:1 magnification. The image is at the rear focal point of the final lens. The rear focal point of the system is located at the final lens element.

7) (20 points) An optical system in air is comprised of two thin lenses:



An object is placed 300 mm to the left of the first lens. The object size is ± 10 mm. Use paraxial raytrace methods to determine the system focal length and the location and size of the image.

Determine:
- System Focal Length
- Back Focal Distance
- Front Focal Distance
- Image Location and Size

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the image size must be determined from a separate raytrace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Calculations may NOT be done in the margins of the raytrace sheet. Gaussian imaging methods may not be used for any portion of this problem.

System Focal Length = ____200____ mm

Back Focal Distance = _____360___ mm

Front Focal Distance = ____-100_____ mm

Image Location = ___560_____ mm to the __R___ of the second lens

Image Size = +/- ____10_____ mm (Inverted)

	F or Object		L1		L2		F' or Image						
Surface	0		1		2		3		4		5		6
f			-25		40								
$-\phi$			0.040		-0.025								
t				20									

Forward Focal Ray 360

	0		1		2		3		4		5		6
y	1		1		1.8		0						
u		0		0.040		-0.005							

Reverse Focal Ray 100

	0		1		2		3		4		5		6
y	0		0.5		1		1						
u		0.005		0.025		0							

Image Location Ray 300 560

	0		1		2		3		4		5		6
y	0		30		56.0		0						
u		0.1*		1.3		-0.1							

Image Size Ray

	0		1		2		3		4		5		6
y	10		10		18.0		-10.0						
u		0*		0.4		-.05							

y													
u													

y													
u													

y													
u													

* Arbitrary

Continues…

Method of Solution:

Method of Solution:

From the forward focal ray – crosses the axis at F':

$$\overline{f_2F'} = BFD = 360mm$$

$$u' = -0.005 \qquad \phi = -\frac{u'}{y_0} = 0.005/mm \qquad y_0 = 1$$

$$f = \frac{1}{\phi} = 200mm$$

From the reverse focal ray – crosses the axis at F:

$$\overline{Ff_1} = -FFD = 100mm \qquad FFD = -100mm$$

$$u = 0.005 \qquad \phi = \frac{y'}{u} = 0.005/mm \qquad y' = 1$$

$$f = \frac{1}{\phi} = 200mm$$

Using the rays defining the object location and object height. The axial ray crosses the axis at the image location (s'). The ray from the top of the object determines the image height (h').

The image is 560 *mm* to the right of the second lens.

The image height is -10 mm.

The image is inverted and the system is operating at 1:1 conjugates.

Note that the image height ray that was traced is a scaled version of the forward focal ray extended to the image plane.