

Optical Design and Instrumentation I

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MIL HDBK-141

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Schott Glass Technologies, Inc.



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THE COLLINEAR TRANSFORMATION

The collinear transformation is a geometrical mapping connecting two spaces, which we shall call the object space and the image space, although in the present context the names are merely labels for the spaces and have no other meaning.

In order for there to be a mapping between the two spaces, a necessary and sufficient condition is that points in either space must be in a one-to-one correspondence with the points in the other space; that is, for each object point there must be one and only one image point corresponding to it, and vice versa. If we use a cartesian coordinate system to locate points in each of the spaces, then a point in the object space is represented by its coordinates x, y, z and a point in the image space is represented by x', y', z' . Even though the locations of the origins of the coordinate systems and the orientation of the coordinate axes are arbitrary, the one-to-one correspondence can be represented by the implicit equations

$$\begin{aligned}x' &= x'(x, y, z) \\y' &= y'(x, y, z) \\ \text{and } z' &= z'(x, y, z).\end{aligned}\tag{1}$$

The mapping is collinear if, for every three collinear points in object space, the corresponding three points in image space are also collinear. It necessarily follows that the straight lines defined by the collinear sets are also in one-to-one correspondence with each other. Moreover, it also follows that planes in the two spaces are in one-to-one correspondence. Elements (points, lines, or planes) which are in one-to-one correspondence are called conjugate elements.

In order to obtain explicit relationships for equations (1) in a collinear transformation, it is easier to use the plane-to-plane correspondence rather than the line-to-line correspondence in the derivation.

An arbitrarily selected plane in object space and its conjugate in image space can be represented by the equations

$$\begin{aligned}Ax + By + Cz + D &= 0 \\ \text{and } A'x' + B'y' + C'z' + D' &= 0.\end{aligned}\tag{2}$$

For any set of coefficients (A, B, C, D) in object space there is a corresponding unique set (A', B', C', D') in image space.

Consider the functions

$$\begin{aligned} M &= A x + B y + C z + D \\ \text{and } M' &= A' x' + B' y' + C' z' + D'. \end{aligned} \quad (3)$$

Note that M' is implicitly a function of (x, y, z) as well.

As M and M' take on various values, equations (3) represent families of parallel planes in each space. However, only when M and M' are both zero do we necessarily have conjugate planes. Moreover, whenever M is zero then M' must simultaneously also be zero, and this must be true for all sets (A, B, C, D) with their corresponding sets (A', B', C', D') . In order for this to be true in general, M' must contain M as a factor; that is,

$$\begin{aligned} M' &= M(x, y, z) P(x, y, z) \\ \text{or } M'/P &= M, \end{aligned} \quad (4)$$

where P is yet to be determined.

The expression P can only vanish when M vanishes, for otherwise M' would be equal to zero with a nonzero value for M , which is not possible. Similarly, the reciprocal of P can only vanish when M' vanishes.

Writing out explicitly the second of equations (4), we obtain

$$A' x' / P + B' y' / P + C' z' / P + D' / P = A x + B y + C z + D. \quad (5)$$

Since the right side of the equation is linear in x , y , and z , then each term on the left, individually, must also be linear in x , y , and z . Thus we obtain

$$\begin{aligned} 1/P &= a_0 x + b_0 y + c_0 z + d_0 \\ x'/P &= a_1 x + b_1 y + c_1 z + d_1 \\ y'/P &= a_2 x + b_2 y + c_2 z + d_2 \\ \text{and } z'/P &= a_3 x + b_3 y + c_3 z + d_3. \end{aligned} \quad (6)$$

Solving equations (6) for x' , y' , and z' gives us our explicit forms for equations (1) in a collinear transformation. These are

$$\begin{aligned} x' &= (a_1 x + b_1 y + c_1 z + d_1) / (a_0 x + b_0 y + c_0 z + d_0) \\ y' &= (a_2 x + b_2 y + c_2 z + d_2) / (a_0 x + b_0 y + c_0 z + d_0) \\ z' &= (a_3 x + b_3 y + c_3 z + d_3) / (a_0 x + b_0 y + c_0 z + d_0). \end{aligned} \quad (7)$$

GENERAL MAPPING EQUATIONS

$$x' = \frac{a_1 x + b_1 y + c_1 z + d_1}{a_0 x + b_0 y + c_0 z + d_0} \quad y' = \frac{a_2 x + b_2 y + c_2 z + d_2}{a_0 x + b_0 y + c_0 z + d_0} \quad z' = \frac{a_3 x + b_3 y + c_3 z + d_3}{a_0 x + b_0 y + c_0 z + d_0}$$

① z' -AXIS CONJUGATE TO z -AXIS

$x'=0$ AND $y'=0$ WHEN BOTH x AND $y=0$ $c_1=d_1=c_2=d_2=0$

$$x' = \frac{a_1 x + b_1 y}{a_0 x + b_0 y + c_0 z + d_0} \quad y' = \frac{a_2 x + b_2 y}{a_0 x + b_0 y + c_0 z + d_0} \quad z' = \frac{a_3 x + b_3 y + c_3 z + d_3}{a_0 x + b_0 y + c_0 z + d_0}$$

② MERIDIONAL PLANES CONJUGATE

$x'=0$ WHEN $x=0$ AND $y'=0$ WHEN $y=0$ $a_2=b_2=0$

$$x' = \frac{a_1 x}{a_0 x + b_0 y + c_0 z + d_0} \quad y' = \frac{b_2 y}{a_0 x + b_0 y + c_0 z + d_0} \quad z' = \frac{a_3 x + b_3 y + c_3 z + d_3}{a_0 x + b_0 y + c_0 z + d_0}$$

③ ROTATIONAL SYMMETRY

$x'/x = y'/y$ $a_1 = b_2 = a$

FOR FIXED z , z' INDEPENDENT OF x AND y $a_3 = b_3 = a_0 = b_0 = 0$

$$x' = \frac{ax}{c_0 z + d_0} \quad y' = \frac{ay}{c_0 z + d_0} \quad z' = \frac{c_3 z + d_3}{c_0 z + d_0}$$

④ DIVIDE NUMERATORS & DENOMINATORS BY a

$$x' = \frac{x}{c_0 z + d_0} \quad y' = \frac{y}{c_0 z + d_0} \quad z' = \frac{c_3 z + d_3}{c_0 z + d_0}$$

⑤ DEFINE MAGNIFICATION $m \equiv \frac{x'}{x} = \frac{y'}{y}$

$$m = \frac{1}{c_0 z + d_0} \quad x' = mx \quad y' = my \quad z' = m(c_3 z + d_3)$$

(z AND z' ORIGINS ARE STILL ARBITRARY)

$$m = \frac{1}{c_3 z + d_0} \quad x' = mx \quad y' = my \quad z' = m(c_3 z + d_0) = \frac{c_3 z + d_0}{c_0 z + d_0}$$

ORIGINS AT FOCAL PLANES (NEWTONIAN EQUATIONS)

$$z' = 0 \text{ WHEN } z = \infty$$

$$z' = \frac{c_3 + d_0/z}{c_0 + d_0/z} \quad c_3 = 0$$

$$z = 0 \text{ WHEN } z' = \infty$$

$$\frac{1}{z'} = \frac{c_0 z + d_0}{c_3 z + d_3} \quad d_0 = 0$$

$$m = \frac{1}{c_0 z} \quad x' = mx \quad y' = my \quad z' = m d_3 = \frac{d_3}{c_0 z}$$

AT PRINCIPAL PLANES:

$$\text{FOR } m = 1 \quad z = -f_F \quad z' = -f'_R$$

$$1 = \frac{1}{c_0(-f_F)}$$

$$c_0 = -\frac{1}{f_F}$$

$$-f'_R = (1) d_3$$

$$d_3 = -f'_R$$

$$m = -\frac{1}{z/f_F} \quad x' = mx \quad y' = my \quad z' = \frac{f'_R}{z/f_F}$$

$$\frac{z}{f_F} = -\frac{1}{m}$$

$$\frac{z'}{f'_R} = -m$$

$$\left(\frac{z}{f_F}\right)\left(\frac{z'}{f'_R}\right) = 1 \quad \text{OR } z z' = f_F f'_R$$

$$m = \frac{1}{c_3 z + d_3} \quad x' = m x \quad y' = m y \quad z' = m (c_3 z + d_3) = \frac{c_3 z + d_3}{c_3 z + d_3}$$

ORIGINS AT PRINCIPAL PLANES (GAUSSIAN EQUATIONS)

$$z' = 0, z = 0 \text{ WHEN } m = 1$$

$$m = \frac{1}{c_3 z + d_3} \quad d_3 = 1$$

$$z' = m (c_3 z + d_3) \quad d_3 = 0$$

$$m = \frac{1}{1 + c_3 z} \quad x' = m x \quad y' = m y \quad z' = m (c_3 z)$$

AT FOCAL PLANES:

$$m = 0 \text{ WHEN } z = \infty, z' = f'_R$$

$$m = \infty \text{ WHEN } z = f_F, z' = 0$$

$$\frac{1}{m} = 1 + c_3 z \quad c_3 = -\frac{1}{f_F} \quad m = \frac{1}{1 - z/f_F}$$

$$z' = \frac{c_3 z}{1 - z/f_F} = \frac{c_3}{\frac{1}{z} - \frac{1}{f_F}} \quad c_3 = -\frac{f'_R}{f_F}$$

$$\frac{z}{f_F} = 1 - \frac{1}{m} \quad \frac{z}{f'_R} = 1 - m$$

$$\frac{z'}{z} = -\frac{f'_R}{f_F} m$$

$$\frac{f_F}{z} + \frac{f'_R}{z'} = 1$$

DISTANCES BETWEEN PAIRS OF CONJUGATE PLANES

$$\frac{\Delta z}{f_F} = \frac{z_2}{f_F} - \frac{z_1}{f_F} = 1 - \frac{1}{m_2} - \left(1 - \frac{1}{m_1}\right) = \frac{1}{m_1} - \frac{1}{m_2} = \frac{m_2 - m_1}{m_1 m_2}$$

$$\frac{\Delta z'}{f_R'} = \frac{z_2'}{f_R'} - \frac{z_1'}{f_R'} = 1 - m_2 - (1 - m_1) = -(m_2 - m_1)$$

$$\frac{\Delta z'/f_R'}{\Delta z/f_F} = \frac{-(m_2 - m_1)}{\frac{(m_2 - m_1)}{m_1 m_2}} = -m_1 m_2$$

$$\frac{\Delta z'}{\Delta z} = -\frac{f_R'}{f_F} m_1 m_2$$

$$m = \frac{1}{c_3 z + d_3} \quad x' = mx \quad y' = my \quad z' = m(c_3 z + d_3) = \frac{c_3 z + d_3}{c_3 z + d_3}$$

AFOCAL SYSTEM

$$z' = \infty \text{ WHEN } z = \infty$$

$$\frac{1}{z'} = \frac{c_3 z + d_3}{c_3 z + d_3} = \frac{c_3 + d_3/z}{c_3 + d_3/z} \quad c_3 = 0 \quad m = \frac{1}{d_3} = \text{CONST. FOR SYSTEM}$$

$$m = \frac{1}{d_3} \quad x' = mx \quad y' = my \quad z' = m(c_3 z + d_3) \quad \text{AFFINE}$$

ORIGINS AT CONJUGATE PLANES

$$z' = 0 \text{ WHEN } z = 0$$

$$d_3 = 0$$

$$z' = m c_3 z$$

DISTANCES BETWEEN PAIRS OF CONJUGATE PLANES

$$\Delta z = z_2 - z_1$$

$$\Delta z' = c_3 m \Delta z$$

$$\frac{\Delta z'}{\Delta z} = c_3 m$$

FOR FOCAL SYSTEM, LET $-\frac{f'_R}{f} = k$ (CONST.) WHILE $f'_R \rightarrow \infty$, $f_F \rightarrow \infty$

$$\frac{\Delta z'}{\Delta z} = k m_1 m_2 \rightarrow k m^2 \quad c_3 = k m$$

$$\frac{\Delta z'}{\Delta z} = k m^2$$

Standard Eye – Engineering Drawing – MIL HDBK-141

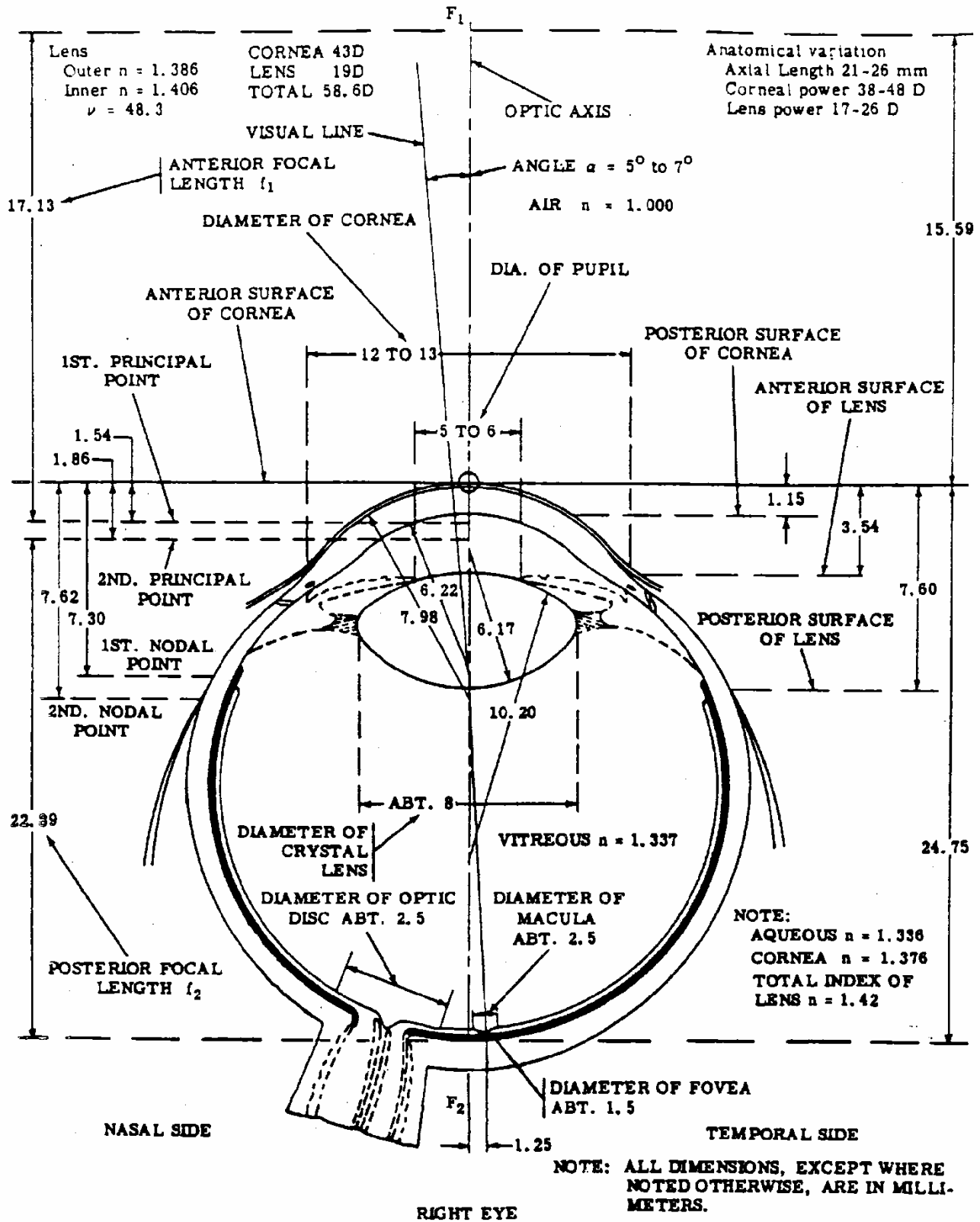
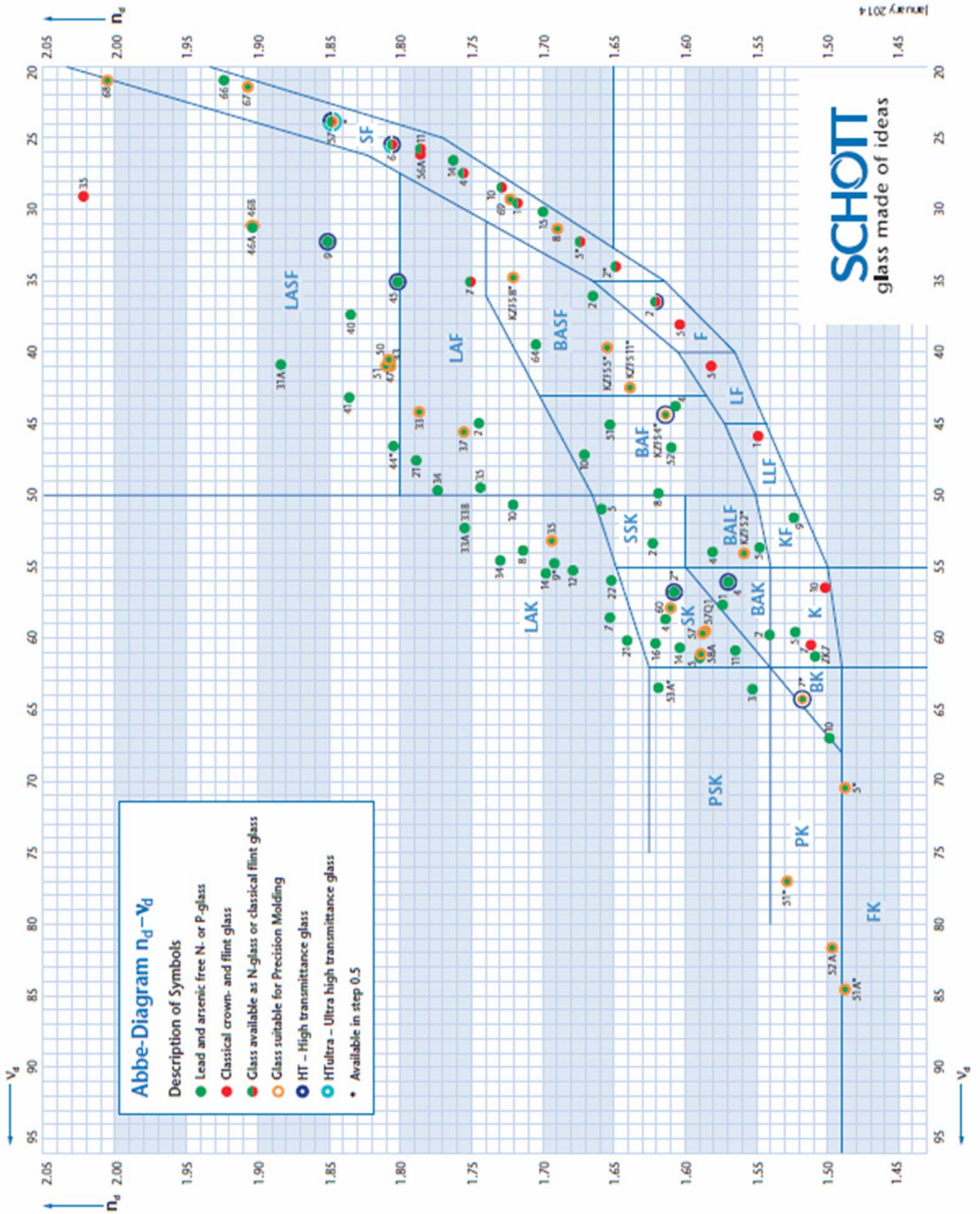


Figure 4.2 - Optical constants for a "standard eye."

Glass Map – Schott Glass Technologies, Inc.



Data Sheets

Refractive indices

The refractive indices n are listed for a maximum of 23 wavelengths in the range between 248.2 nm and 2325.4 nm.

Constants of the dispersion formula

From the Sellmeier dispersion formula

$$n^2(\lambda) - 1 = \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3}$$

the refractive indices for any wavelength within the range from the near UV to 2.3 μm can be calculated with the help of the constants $B_1, B_2, B_3,$ and $C_1, C_2, C_3.$

Constants of the formula dn/dT

The temperature dependence of the refractive index can be calculated using the following formula:

$$\frac{dn_{\text{abs}}(\lambda, T)}{dT} = \frac{n^2(\lambda, T_0) - 1}{2 n(\lambda, T_0)} \left(D_0 + 2 D_1 \Delta T + 3 D_2 \Delta T^2 + \frac{E_0 + 2 E_1 \Delta T}{\lambda^2 - \lambda_{\text{TK}}^2} \right)$$

The constants are valid for a temperature range from -100°C to $+140^\circ\text{C}$ and a wavelength range from 0.365 μm to 1.014 μm . The temperature coefficients in the data sheets are guideline values.

Temperature coefficient of refraction

$\Delta n_{\text{rel}} / \Delta T$ referring to air at normal pressure 1013.3 mbar

$\Delta n_{\text{abs}} / \Delta T$ referring to vacuum

Internal transmittance τ_i

The internal transmittance in the wavelength range between 250 nm and 2500 nm is listed for thickness of 10 and 25 mm. The internal transmittance and color code listed in the data sheet represent median values from several melts of one glass type. For HT and HTultra grade, the internal transmittance in the visible spectrum includes guaranteed minimum values.

Color code

The color code lists the wavelength λ_{80} and λ_5 at which the transmittance is 0.80 and 0.05 at 10 mm thickness. The values are rounded off to 10 nm and denoted by eliminating the first digit. For high index glass types with $nd > 1.83$, the data of the color codes (marked by *) refers to the transmittance values 0.70 and 0.05 (λ_{70} and λ_5).

Relative partial dispersion

The relative partial dispersions P_{xy} and P'_{xy} for the wavelengths x and y are derived from the equations.

$$P_{xy} = \frac{n_x - n_y}{n_F - n_C} \quad \text{und} \quad P'_{xy} = \frac{n_x - n_y}{n_F - n_C}$$

Deviation of the relative partial dispersion from the "normal line" ΔP

The term ΔP_{xy} quantitatively describes a deviation relation of the dispersion from the "normal glasses".

Other characteristics

$\alpha_{-30/+70}$	= The coefficient of thermal expansion in the temperature range between -30°C und $+70^{\circ}\text{C}$ in $10^{-6}/\text{K}$
$\alpha_{20/300}$	= The coefficient of linear thermal expansion in the temperature range between $+20^{\circ}\text{C}$ und $+300^{\circ}\text{C}$ in $10^{-6}/\text{K}$
Tg	= Transformation temperature in $^{\circ}\text{C}$
$T_{10^{13.0}}$	= Temperature of the glass in $^{\circ}\text{C}$ at a viscosity of 10^{13} dPa·s
$T_{10^{7.6}}$	= Temperature of the glass in $^{\circ}\text{C}$ at a viscosity of $10^{7.6}$ dPa·s
c_p	= average specific heat capacity in $\text{J}/(\text{g}\cdot\text{K})$
λ	= Thermal conductivity in $\text{W}/(\text{m}\cdot\text{K})$
AT*	= Yield point/sag temperature in $^{\circ}\text{C}$
ρ	= Density in g/cm^3
E	= Elasticity modulus in 10^3 N/mm ²
μ	= Poisson's ratio
K	= Stress optical coefficient in 10^{-6} mm ² /N
HK	= Knoop hardness
HG	= Grindability class (ISO 12844)
Abrasion Aa*	= Grindability according to JOGIS**
CR	= Climatic resistance Resistance to moisture in the air expressed in CR classes 1 (high) to 4 (low).
FR	= Stain resistance Resistance to stain formation expressed in FR classes 0 (high) to 5 (low).
SR	= Acid resistance Resistance to acid solutions expressed in SR classes 1 (high) to 4 (low) and 51 to 53 (very low).
AR	= Alkali resistance Resistance to alkaline solutions expressed in AR classes 1 (high) to 4 (low).
PR	= Phosphate resistance Resistance to alkaline phosphate containing solutions expressed in PR classes 1 (high) to 4 (low).

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$n_d = 1.51680$	$v_d = 64.17$	$n_F - n_C = 0.008054$
$n_e = 1.51872$	$v_e = 63.96$	$n_{F'} - n_{C'} = 0.008110$

Refractive Indices		
	λ [nm]	
$n_{2325.4}$	2325.4	1.48921
$n_{1970.1}$	1970.1	1.49495
$n_{1529.6}$	1529.6	1.50091
$n_{1060.0}$	1060.0	1.50669
n_t	1014.0	1.50731
n_s	852.1	1.50980
n_r	706.5	1.51289
n_C	656.3	1.51432
$n_{C'}$	643.8	1.51472
$n_{632.8}$	632.8	1.51509
n_D	589.3	1.51673
n_d	587.6	1.51680
n_e	546.1	1.51872
n_F	486.1	1.52238
$n_{F'}$	480.0	1.52283
n_g	435.8	1.52668
n_h	404.7	1.53024
n_i	365.0	1.53627
$n_{334.1}$	334.1	1.54272
$n_{312.6}$	312.6	1.54862
$n_{296.7}$	296.7	
$n_{280.4}$	280.4	
$n_{248.3}$	248.3	

Constants of Dispersion Formula	
B_1	1.03961212
B_2	0.231792344
B_3	1.01046945
C_1	0.00600069867
C_2	0.0200179144
C_3	103.560653

Constants of Dispersion dn/dT	
D_0	$1.86 \cdot 10^{-6}$
D_1	$1.31 \cdot 10^{-8}$
D_2	$-1.37 \cdot 10^{-11}$
E_0	$4.34 \cdot 10^{-7}$
E_1	$6.27 \cdot 10^{-10}$
λ_{TK} [μm]	0.17

Temperature Coefficients of Refractive Index						
	$\Delta n_{rel}/\Delta T [10^{-6}/K]$			$\Delta n_{abs}/\Delta T [10^{-6}/K]$		
[°C]	1060.0	e	g	1060.0	e	g
-40/ -20	2.4	2.9	3.3	0.3	0.8	1.2
+20/ +40	2.4	3.0	3.5	1.1	1.6	2.1
+60/ +80	2.5	3.1	3.7	1.5	2.1	2.7

Internal Transmittance τ_i		
λ [nm]	τ_i (10mm)	τ_i (25mm)
2500	0.665	0.360
2325	0.793	0.560
1970	0.933	0.840
1530	0.992	0.980
1060	0.999	0.997
700	0.998	0.996
660	0.998	0.994
620	0.998	0.994
580	0.998	0.995
546	0.998	0.996
500	0.998	0.994
460	0.997	0.993
436	0.997	0.992
420	0.997	0.993
405	0.997	0.993
400	0.997	0.992
390	0.996	0.989
380	0.993	0.983
370	0.991	0.977
365	0.988	0.971
350	0.967	0.920
334	0.905	0.780
320	0.770	0.520
310	0.574	0.250
300	0.292	0.050
290	0.063	
280		
270		
260		
250		

Color Code	
λ_{80}/λ_5	33/29
(*= λ_{70}/λ_5)	

Remarks
step 0.5 available

Relative Partial Dispersion	
$P_{s,t}$	0.3098
$P_{C,s}$	0.5612
$P_{d,C}$	0.3076
$P_{e,d}$	0.2386
$P_{g,F}$	0.5349
$P_{i,h}$	0.7483
$P'_{s,t}$	0.3076
$P'_{C,s}$	0.6062
$P'_{d,C'}$	0.2566
$P'_{e,d}$	0.2370
$P'_{g,F}$	0.4754
$P'_{i,h}$	0.7432

Deviation of Relative Partial Dispersions ΔP from the "Normal Line"	
$\Delta P_{C,t}$	0.0216
$\Delta P_{C,s}$	0.0087
$\Delta P_{F,e}$	-0.0009
$\Delta P_{g,F}$	-0.0009
$\Delta P_{i,g}$	0.0035

Other Properties	
$\alpha_{-30/+70^\circ C} [10^{-6}/K]$	7.1
$\alpha_{+20/+300^\circ C} [10^{-6}/K]$	8.3
T_g [°C]	557
$T_{10}^{13.0}$ [°C]	557
$T_{10}^{7.6}$ [°C]	719
c_p [J/(g·K)]	0.858
λ [W/(m·K)]	1.114
ρ [g/cm ³]	2.51
E [10 ³ N/mm ²]	82
μ	0.206
K [10 ⁻⁶ mm ² /N]	2.77
$HK_{0.1/20}$	610
HG	3
CR	1
FR	0
SR	1
AR	2.3
PR	2.3

