Section 22

The Eye
The optical power of the human eye is about 60 D, of which the cornea provides 43 D. The base radius of curvature of the cornea is about 8 mm, and the overall length of the eye is about 25 mm. Since the vitreous ($n_v = 1.337$) fills the eye, the rear focal length differs from the focal length.

Optical variations between eyes can be as much as 25%.

The crystalline lens is a gradient index element; it has a higher index at its center. The relaxed power of the lens is about 19 D, and the eye focuses at infinity. To view near objects, the ciliary muscle contracts, causing the lens power to increase. The lens bulges and its radii of curvature become steeper. The range of accommodation varies with age, but can be as much as 15 D.

The iris is the stop of the eye. The pupil is the EP of the eye and has a typical diameter of about 4 mm, with a range of 2-8 mm.
Relaxed Eye (Distance Vision):

The relaxed ciliary muscle pulls the zonules taut and flattens the crystalline lens. In the relaxed eye, the crystalline lens is under tension.

The "natural" state of the isolated crystalline lens is actually that of the accommodated eye.

Accommodated Eye (Near Vision):

With accommodation, the ciliary muscle constricts and releases the tension on the zonules and the crystalline lens bulges. The power of the eye increases and near objects are brought into focus.

Cardinal Points of the Eye

The front and rear principal planes of the eye P and P' are located about 1.6 mm and 1.9 mm, respectively, behind the vertex of the cornea. The system nodal points N and N' are located near the anterior surface of the lens, 7.2 mm and 7.5 mm, respectively, from the corneal vertex.

The visual axis of the eye is defined by the macula and is displaced about 5° nasally from the optical axis.
Schematic Eye Models

Schematic eye models are simplified representations that allow the eye to be analyzed.
- Crystalline lens has a uniform index
- Crystalline lens sometimes ignored
- Posterior cornea sometimes ignored

The simplest is the reduced schematic eye: a single refractive surface which approximates the paraxial properties of the eye:

\[ R = 5.65 \text{ mm} \]
\[ n = 1.333 \]
\[ \text{length} = 22.6 \text{ mm} \]
\[ f = 17.0 \text{ mm} \]

The principal planes of this model are at the corneal surface, and the nodal points are at the center of curvature of the cornea.

More Complete Eye Model

A variety of more sophisticated eye models have been developed; some model the aberration content of the eye. The following schematic eye provides a more complete model of the paraxial properties of the eye (Y. Le Grand and S. G. El Hage, *Physiological Optics*, Springer Verlag, Berlin, 1980). The crystalline lens is assumed to have a uniform index.

<table>
<thead>
<tr>
<th>Surface</th>
<th>( R ) (mm)</th>
<th>( t ) (mm)</th>
<th>( n )</th>
<th>( \phi ) (D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anterior cornea</td>
<td>7.8</td>
<td>0.55</td>
<td>1.3771</td>
<td>48.35</td>
</tr>
<tr>
<td>Posterior cornea</td>
<td>6.5</td>
<td>3.05</td>
<td>1.3374</td>
<td>-6.11</td>
</tr>
<tr>
<td>Anterior lens</td>
<td>10.2</td>
<td>4.00</td>
<td>1.420</td>
<td>8.10</td>
</tr>
<tr>
<td>Posterior lens</td>
<td>-6.0</td>
<td>16.60</td>
<td>1.336</td>
<td>14.00</td>
</tr>
</tbody>
</table>

\[ \phi = 59.9D \]
\[ f = 16.9 \text{ mm} \]
\[ f'_n = 22.3 \text{ mm} \]
The retina covers the interior of the globe of the eye. The cones provide color vision at daylight illumination levels (photopic). The highest cone density is at the fovea in the center of the macula. The macula is about 3 mm in diameter (11° FOV), and the fovea has a diameter of about 1.5 mm (5° FOV). The rods are more uniformly distributed over the retina and are used for dark-adapted vision (scotopic). The peak wavelength of the photopic response is about 555 nm, and the peak wavelength of the scotopic response is about 510 nm.

The light sensitivity of the eye covers a dynamic range of $10^{10}$-10$^{14}$. Most of this range comes from dark adaptation of the retina as the variation in the pupil area is only a factor of 16. For comparison, film and most electronic sensors have a dynamic range of only about $10^{3}$-10$^{5}$.

Under bright illumination, the resolution of the eye is 1 arc min (1 mm at 3 m). This corresponds to about 100 lp/mm on the retina.

The vernier acuity of the eye (the ability to line up two line segments) is about 5 arc sec (0.1 mm at 3 m).

The eye will see a flicker up to a temporal frequency of about 10-30 Hz. Above this critical frequency, the image will appear continuous. This feature is exploited for movies and TV. The peripheral vision has a higher flicker frequency than the macula.

Retina—Photoreceptors

Retina has two types of photoreceptors:

- Cones and Rods

About 125 million rods:
  - Responding under dark conditions

About 6.4 million cones:
  - Responsible for daytime vision

Cones:Rods ~ 1:20
Rods and Cones

Fig. 11. Scanning electron micrograph of the rods and cones of the primate retina. Image adapted from one by Ralph C. Eagle/Photo Researchers, Inc.

Eye

Fig. 1.1. A drawing of a section through the human eye with a schematic enlargement of the retina.
Rod and Cone Distribution

Fig. 20. Graph to show rod and cone densities along the horizontal meridian.

1 degree of visual FOV corresponds to about 0.3 mm on the retina.
Macula: ~ 3-5 mm Diameter  Fovea: ~ 1-1.5 mm Diameter

Foveated System

Visual acuity decreases with the distance from the fovea due to the decrease in the density of cones.

When fixating at the center of the pattern, all of these letters are equally legible.

For high acuity, the eye must look directly at the object, placing the image at the fovea.
Luminous Sensitivity (Photopic)

Photopic condition: the rods are saturated and only the cones are producing a visual signal.

- Luminance \( > \sim 3 \text{ cd/m}^2 \)

For photopic vision, the spectral luminous efficiency of the eye, \( \sigma(\lambda) \), has a peak value located at a wavelength of 555 nm.

For photopic vision, one watt of radiant energy at the wavelength of 555 nm is equivalent to 683 lumens.

\[
\sigma(\lambda) = \frac{\text{Lumens}}{\text{Watt} \times \text{nm}}
\]

Luminous Sensitivity (Scotopic)

Scotopic condition: the light levels are too low to activate the cones, but the rods still respond.

- Luminance \( < \sim 0.03 \text{ cd/m}^2 \)

The scotopic luminous sensitivity, \( \sigma'(\lambda) \), has a peak value located at a wavelength of 507 nm.

For scotopic vision, one watt of radiant energy at the wavelength of 507 nm is equivalent to 1700 lumens.
The Photopic and Scotopic Luminous Sensitivity

CIE: Commission Internationale de l’Eclairage
International Commission on Illumination

<table>
<thead>
<tr>
<th>λ (nm)</th>
<th>Photopic Sensitivity σ(λ) Im/W</th>
<th>Scotopic Sensitivity σ(λ) Im/W</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.3</td>
<td>400</td>
</tr>
<tr>
<td>420</td>
<td>2.7</td>
<td>420</td>
</tr>
<tr>
<td>440</td>
<td>15.7</td>
<td>440</td>
</tr>
<tr>
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<tr>
<td>480</td>
<td>95.0</td>
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<td>11.6</td>
<td>680</td>
</tr>
<tr>
<td>700</td>
<td>2.8</td>
<td>700</td>
</tr>
<tr>
<td>720</td>
<td>0.7</td>
<td>720</td>
</tr>
</tbody>
</table>

Tabulated values may be found at http://www.cvrl.org
and other locations.

Refraction Terms

Emmetropia: Distant objects are imaged correctly onto the retina; normal vision.

Myopia or nearsightedness: the eye is too powerful for its axial length. Images of distant objects are in front of the retina; corrected with a negative spectacle lens.

Hyperopia or farsightedness: the eye is too weak for its axial length. Images of distant objects are behind the retina; corrected with a positive spectacle lens. Accommodation can cause distant objects to be in focus.

Far point: the object distance that is in focus without accommodation. The far point is virtual with hyperopia.

Near point: the object distance that is in focus with maximum accommodation.
Eyeglasses or Spectacles

The rear focal point of the correcting eyeglasses or spectacle lens should be placed at the far point of the relaxed eye. This allows distant objects to be seen without accommodation. In addition, eyeglasses or spectacle lenses are often located at the front focal point of the eye.

\[ t = f_{\text{EYE}} = \frac{1}{\phi_{\text{EYE}}} \]

\[ \phi = \phi_{\text{GLASSES}} + \phi_{\text{EYE}} - \phi_{\text{GLASSES}} \phi_{\text{EYE}} t = \phi_{\text{GLASSES}} + \phi_{\text{EYE}} - \phi_{\text{GLASSES}} \phi_{\text{EYE}} \frac{1}{\phi_{\text{EYE}}} \]

Eyeglasses do not change the system power.

\[ \delta' = \frac{d'}{n'} = -\frac{\phi_{\text{GLASSES}}}{\phi_{\text{EYE}}} \quad n' = n_{\text{EYE}} = 1.336 \quad n = a' = 1 \]

\[ d' = -n_{\text{EYE}} \frac{\phi_{\text{GLASSES}}}{\phi_{\text{EYE}}} \]

Eyeglasses shift the rear principal plane of the system. Since the power of the eye is not changed, the rear focal point of the system shifts by the same amount to place the rear focal point of the system onto the retina. There is no apparent visual magnification change.

Myopia:

Myopia (nearsightedness) – the eye is too powerful for its axial length. The rear focal point must shift away from the cornea:

\[ d' > 0 = -n_{\text{EYE}} \frac{\phi_{\text{GLASSES}}}{\phi_{\text{EYE}}} \quad \phi_{\text{GLASSES}} < 0 \quad \text{Negative Spectacle Lens} \]

Hyperopia (farsightedness) – the eye is too weak for its axial length. The rear focal point must shift towards the cornea:

\[ d' < 0 = -n_{\text{EYE}} \frac{\phi_{\text{GLASSES}}}{\phi_{\text{EYE}}} \quad \phi_{\text{GLASSES}} > 0 \quad \text{Positive Spectacle Lens} \]

If the spectacle lens is not placed at the front focal point of the eye, the focal length and power of the eye will change, as will the rear principal plane location.

\[ t \neq f_{\text{EYE}} \implies f \neq f_{\text{EYE}} \phi \neq \phi_{\text{EYE}} \quad d' = -n_{\text{EYE}} \frac{\phi_{\text{GLASSES}}}{\phi_{\text{EYE}}} t \]

The system is set up so that the rear focal point of the system is located at the retina to form an in-focus image of distant objects. However, since the focal length of the system has changed, an apparent magnification or size change of the image on the retina results.
Contact Lenses

Contact lenses sit in contact with the anterior corneal surface, and effectively change the radius of curvature of the cornea. There is an approximate index match between the contact lens and the cornea.

The radius of curvature at the air interface is changed and the majority of the refractive power of the eye occurs at the air interface.

The power and focal length of the eye system is changed to place the rear focal point of the system on the retina. An in-focus image of distant objects is obtained. However, since the system focal length has changed, an apparent magnification or size change of the image on the retina results.

For both spectacle lenses and contact lenses, near objects are brought into focus on the retina by an increase in the power of the crystalline lens due to accommodation.

More Ophthalmic Terms

Presbyopia: the loss of accommodative response due to a stiffening of the crystalline lens with age. Occurs after age 40 and is compensated by additional positive spectacle power (as with reading glasses, bifocals or progressive lenses).

Visual astigmatism: a variation of the power of the eye with meridional cross section due to a non-rotationally symmetric cornea or lens. Linearly blurred images result. Because there is no field dependence, this effect is different from aberrational astigmatism $W_{22}$. Visual astigmatism is characterized by a wavefront aberration coefficient $W_{02}$.

Stiles-Crawford effect: the reduction in effectiveness of light rays entering the edge of the pupil due to the shape and orientation of the cones. The light efficiency as a function of pupil radius is approximately: 1 mm – 90%; 2 mm – 70%; 3 mm – 40% and 4 mm – 20%. There is no Styles-Crawford effect for the rods and night vision.

Intra-ocular lens IOL: with age, the crystalline lens becomes opaque. The lens can be surgically removed and replaced with an artificial lens or IOL.
Reading Glasses

With presbyopia, there is insufficient accommodation for the eye to focus on near objects. Reading glasses are positive spectacle lenses used to image the near object to the far point of the eye (at infinity for an emmetropic eye).

The correct focal length for the spectacle lens is just the object distance, and the required dioptric power is the reciprocal of this focal length (in meters):

\[
\text{Power (D)} = \frac{1}{\text{Object Distance (m)}}
\]

The powers of reading glasses typically run from $+1.00 \, \text{D}$ to $+3.00 \, \text{D}$ with higher powers available.

For myopic and hyperopic eyes, the power in the reading glasses is added to the required distance correction. Bifocals (invented by Benjamin Franklin) and trifocals have zones of different power. The top zone is corrected for distance and the lower zone is used for near objects.

Progressive spectacle lenses are designed to have a continuous variation in the available correction as the eye looks through different regions of the lens.

Another Way of Thinking About Reading Glasses

Imaging the near object to infinity is the simple way to think about the use of reading glasses. However, considering the reading glasses and the eye as a system allows the application of the principles of geometrical optics.

Reading glasses are spectacle lenses placed at or near the front focal point of the eye. As with spectacle lenses, they will not change the power of the eye:

\[
t = f_{\text{eye}} \quad \phi_{\text{GLASSES,EYE}} = \phi_{\text{EYE}}
\]

Also the Front Principal Plane of the system remains located at the Front Principal Plane of the eye (as with a field lens).

Reading glasses work by shifting the rear principal plane of the eyeglass/eye system without changing the focal length of the eye.

\[
d' = \frac{n_{\text{eye}}}{z}\phi_{\text{GLASSES}} - \phi_{\text{EYE}}
\]

The Rear Principal Plane of the system $P'$ is shifted so that the retina is at the proper image distance $z'$ for the near object.

\[
\frac{n_{\text{eye}}}{z} - \frac{1}{z} + \phi_{\text{EYE}} = \frac{1}{z} + \phi_{\text{GLASSES,EYE}}
\]

The image is brought into focus. The image size does not change and there is no visual magnification change.
Visual Acuity

Snellen visual acuity VA: a single number measure of the resolution of the visual system based upon the ability of the subject to identify characters or symbols. The value 20/XX implies that the subject can identify a letter at 20 feet that a standard observer can just identify at XX feet. The 20/20 line of characters on the VA chart subtends 5 arc min. The letters on the 20/40 line subtend 10 arc min. Note that a 20/20 letter can be broken down into 5 segments of size 1 arc min. The human retina is capable of supporting a VA of better than 20/10. Metric VA is based upon distances in meters and reads as 6/6, etc.

A 20/20 E corresponds to about 100 lp/mm on the retina. Note that visual acuity is really an identification task, and not a true resolution measurement.

Refractive Surgery

A variety of techniques have been developed to correct the refractive error of the eye:

- RK – Radial keratotomy: A series of non-penetrating incisions are made in the periphery of the cornea to relax the cornea and change its shape.

- PRK – Photorefractive keratectomy: the outer layer (epithelium) of the cornea is removed to expose the body of the cornea (stroma). An excimer laser (193 nm) is used to ablate the stroma to change the corneal shape and power. The healing process must regrow the epithelium.

- LASIK – Laser in situ keratomileusis: a variation on PRK where a flap is shaved into the cornea to reveal the stroma and save the epithelium. The flap is replaced after ablation.

- Phakic IOL: a small addition lens surgically implanted in front of the natural lens to correct the power of the eye.
Visual System Design

Both diffraction and the resolution of the eye (1 arc min) limit the performance of a visual optical instrument. The best that can be done is to match the visual resolution to the diffraction resolution.

The maximum useful magnifying power of a telescope is related to the diameter of its objective or entrance pupil:

\[ [MP] \leq 0.43D_{ep} \quad D_{ep} \text{ in mm} \]

For a microscope, the maximum useful visual magnification is

\[ m_v \leq 230NA \quad \lambda = 0.55\mu m \]

Any magnification beyond these levels is termed “Empty Magnification” as the image appears larger, but does not contain any additional information. Empty magnification can be useful as the operator is no longer working at the limit of their visual resolution. This reduces fatigue and improves operator accuracy. Addition magnifications of 2-3X (and perhaps even 5-10X) are often built into systems.

The purpose of the optical instrument is to “improve” the native resolution of the eye. For example, a 10X telescope allows features of size about 6 arc sec to be viewed.

\[ 1 \text{ arc min}/10 = 6 \text{ arc sec} \]