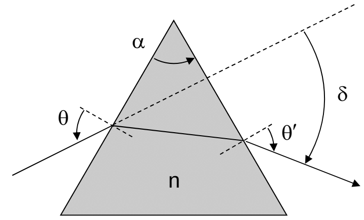
Section 18

Dispersing Prisms

OPTI-502 Optical Design and Instrumentation I
© Copyright 2019 John E. Greivenkamp

Dispersing Prism

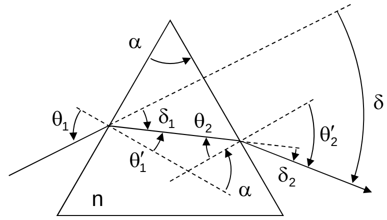


The net ray deviation δ is the sum of the deviations at the two surfaces.

The ray deviation as a function of the input angle θ:

$$\delta = \alpha - \sin^{-1}\left[\sqrt{n^2 - \sin^2\theta}\sin\alpha - \cos\alpha\sin\theta\right] - \theta$$

Prism Deviation - Derivation



Positive: $\theta_1, \theta_1', \alpha$

Negative: $\theta_2, \theta_2', \delta, \delta_1, \delta_2$

$$\delta = \delta_1 + \delta_2 \qquad \delta_1 = \theta_1' - \theta_1$$

$$\delta_2 = \theta_2' - \theta_2$$

$$\delta = \theta_1' - \theta_1 + \theta_2' - \theta_2$$

$$\theta_1' - \theta_2 + (180 - \alpha) = 180$$

$$\theta_1' - \theta_2 = \alpha$$

$$\delta = \alpha - \theta_1 + \theta_2'$$

Snell's Law: $\sin\theta_1' = \frac{1}{n}\sin\theta_1$

$$\sin\theta_2' = n\sin\theta_2$$

$$\theta_2 = \theta_1' - \alpha = \sin^{-1}\left(\frac{1}{n}\sin\theta_1\right) - \alpha$$

$$\sin\theta_2' = n\sin\theta_2 = n\sin\left[\sin^{-1}\left(\frac{1}{n}\sin\theta_1\right) - \alpha\right]$$

$$\sin\theta_2' = -n\sin\alpha\cos\left[\sin^{-1}\left(\frac{1}{n}\sin\theta_1\right)\right] + \cos\alpha\sin\theta_1$$

$$\cos\left(\sin^{-1}\beta\right) = \sqrt{1-\beta^2}$$

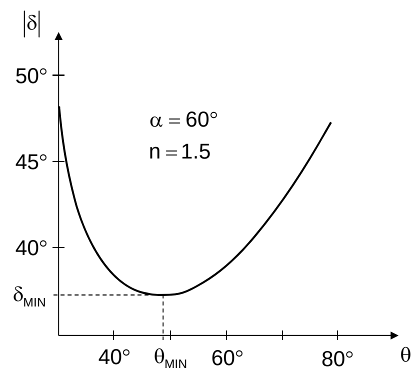
$$\sin\theta_2' = -\sin\alpha\sqrt{n^2 - \sin^2\theta_1} + \cos\alpha\sin\theta_1$$

$$\theta_2' = \sin^{-1}\left[-\sqrt{n^2 - \sin^2\theta_1}\,\sin\alpha + \cos\alpha\sin\theta_1\right]$$

$$\delta = \alpha - \theta_1 + \theta_2' \qquad \theta_1 = \theta$$

$$\delta = \alpha - \sin^{-1}\left[\sqrt{n^2 - \sin^2\theta}\,\sin\alpha - \cos\alpha\sin\theta\right] - \theta$$

Deviation versus Input Angle



There is some input angle θ_{MIN} for which the ray deviation is minimized.

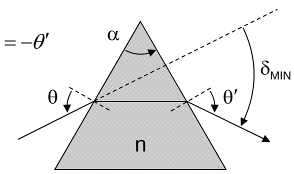
The corresponding deviation is the angle of minimum deviation δ_{MIN}.

Minimum Deviation

At minimum deviation, the ray path through a dispersing prism is symmetric . The ray is bent an equal amount at each surface. By sign convention, the deviation is negative for this prism orientation. The angle of minimum deviation is

$$\delta_{MIN} = \alpha - 2\sin^{-1}\left[n\sin\left(\alpha/2\right)\right]$$

$\theta = -\theta'$



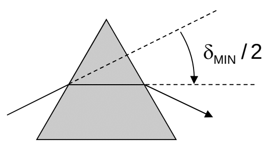
The minimum deviation condition is for the measurement of the index of refraction:

$$n = \frac{\sin\left[\left(\alpha - \delta_{MIN}\right)/2\right]}{\sin\left(\alpha/2\right)}$$

For $\alpha = 60°$

n	δ_{MIN}
1.3	-21.1°
1.4	-28.9°
1.5	-37.2°
1.6	-46.3°
1.7	-56.4°
1.8	-68.3°
2.0	-120°

At minimum deviation, 50% of the net deviation occurs at each surface.

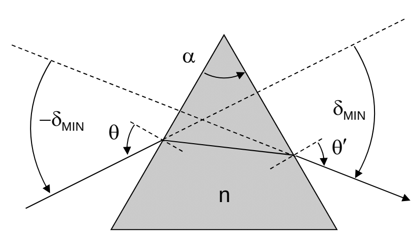


Minimum Deviation – Proof

At minimum deviation, assume path is not symmetric:

$$\left|\theta\right| \neq \left|\theta'\right|$$

But both the forward and reverse ray paths produce the same magnitude of deviation, assumed to be δ_{MIN}.



Both θ and θ′ produce minimum deviation, but they are not equal.

This is a contradiction as there cannot be two different input angles that produce the minimum.

Therefore, at δ_{MIN}: $\theta = -\theta'$

The ray path is symmetric through the prism.

Minimum Deviation – Derivation

Refer back to derivation of prism deviation.

$$\theta = -\theta'$$

$$\theta_1 = -\theta_2' \qquad \theta_1' = -\theta_2$$

$$\delta = \alpha - \theta_1 + \theta_2' \qquad \theta_1' - \theta_2 = \alpha$$

$$\delta_{MIN} = \alpha - 2\theta_1 \qquad \alpha = 2\theta_1'$$

$$\theta_1 = \left(\alpha - \delta_{MIN}\right)/2 \qquad \theta_1' = \alpha/2$$

$$\sin\theta_1 = n\sin\theta_1'$$

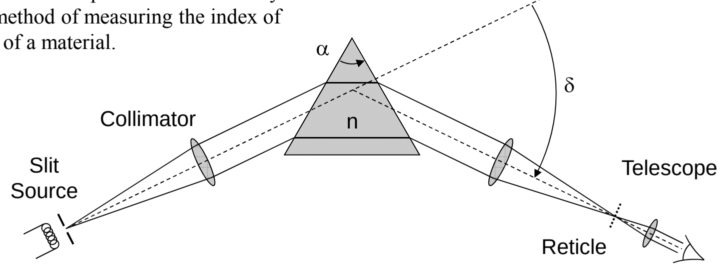
$$\sin\left[\left(\alpha - \delta_{MIN}\right)/2\right] = n\sin\left(\alpha/2\right)$$

$$\delta_{MIN} = \alpha - 2\sin^{-1}\left[n\sin\left(\alpha/2\right)\right]$$

$$n = \frac{\sin\left[\left(\alpha - \delta_{MIN}\right)/2\right]}{\sin\left(\alpha/2\right)}$$

Index Measurement – Prism Spectrometer

Minimum deviation provides an extremely accurate method of measuring the index of refraction of a material.



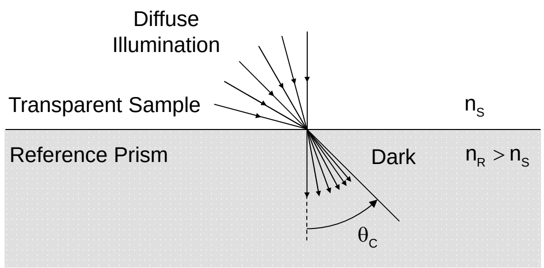
- The telescope and the collimator are mounted to a rotation stage.
- Use the telescope as an autocollimator to measure the prism apex angle α.
- Measure the straight through angle (no prism) I_1.
- Insert the prism and observe the angle of the refracted beam δ.
- Rotate the prism to obtain minimum deviation and measure this angle I_2.
- Subtract I_1 from I_2 to determine the angle of minimum deviation.

$$\delta_{\min} = I_2 - I_1 \qquad n = \frac{\sin\left[\left(\alpha - \delta_{MIN}\right)/2\right]}{\sin\left(\alpha/2\right)} \qquad I_1 > I_2 \rightarrow \delta_{min} < 0$$

Accuracies of one part in the sixth decimal place can be obtained.
Limited by the slit width and diffraction effects.

Index Measurement – Critical Angle Techniques

The critical angle for total internal refraction is often used for index measurement. The sample is placed in contact with a reference prism of higher index.



$$\theta_C = \sin^{-1}\left(\frac{n_S}{n_R}\right)$$

This instrument is called an Abbe Refractometer. Some instruments also measure the Abbe number of the material.

The typical accuracy for this method of index measurement is 0.0001.

Critical angle techniques can also be used in reflection to measure the index of opaque samples.

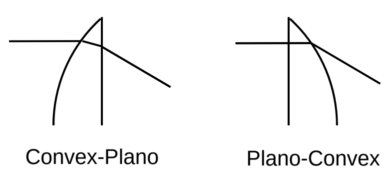
OPTI-502 Optical Design and Instrumentation I
© Copyright 2019 John E. Greivenkamp
College of Optical Sciences
The University of Arizona

Lens Orientation for Minimum Spherical Aberration

When focusing light with a plano-convex lens, it should be oriented with the curved side towards the long conjugate (the object side) – the "convex-plano" orientation. The minimum aberration occurs when the light is bent the same amount at each lens surface.

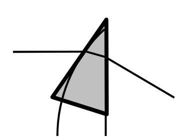
Object at Infinity:

$n \approx 1.5$



In the plano-convex configuration, all of the ray bending occurs at one surface. In the convex-plano configuration, the ray bending is split between the surfaces. This minimizes the angles of incidence at the surfaces and reduces the aberrations. Small angles make the situation as close to the assumptions of paraxial optics as possible.

Note that in the minimum aberration condition, the edge of the lens looks like a prism used at minimum deviation – equal ray bending at both surfaces.



The true minimum for n = 1.5 is a slightly biconvex lens. The plano surface has a long radius, but the difference is essentially insignificant.

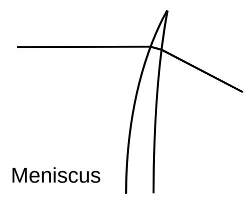
Lens Shape for Minimum Spherical Aberration

There is no bending that completely eliminates spherical aberration. It can only be minimized. Different object/image conjugates require different bendings to minimize spherical aberration.

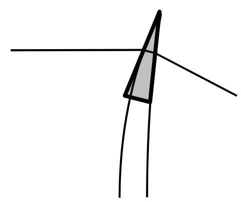
The optimum shape varies with index. At high index, as is often found in the IR, more bending occurs at the surface of the lens. For the same focal length as before, the front surface becomes flatter and the best shape is a meniscus.

Object at Infinity:

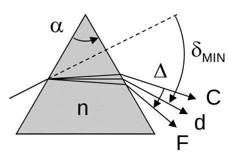
$n \approx 3$



Equal bending occurs at both surfaces of the lens and edge of the lens looks like a prism used at minimum deviation.



Dispersion of a Prism



The details of the prism dispersion depend on the geometry used and the index dispersion curve. However, assuming the prism is used at or near δ_{MIN}, the average prism dispersion over a wavelength band (F to C) can be estimated.

$$\text{Dispersion of a prism} = \frac{d\delta}{d\lambda}$$

$$\delta_{MIN} = \alpha - 2\sin^{-1}\left[n\sin\left(\alpha/2\right)\right]$$

$$\frac{d\delta}{d\lambda} = \frac{d\delta}{dn}\frac{dn}{d\lambda} \approx \frac{d\delta_{MIN}}{dn}\frac{dn}{d\lambda}$$

$$\frac{d\delta_{MIN}}{dn} = \frac{-2\sin\left(\alpha/2\right)}{\cos\left[\left(a-\delta_{MIN}\right)/2\right]} < 0$$

$$\frac{d\delta}{d\lambda} > 0 \qquad \frac{d|\delta|}{d\lambda} < 0$$

$$\frac{dn}{d\lambda} = \text{Glass Dispersion} < 0$$

Blue light is deviated more than red light.

As λ increases, δ increases, but δ is negative. The magnitude of the deviation decreases.

Dispersion of a Prism – Derivation

$$\delta_{MIN} = \alpha - 2\sin^{-1}\left[n\sin(\alpha/2)\right]$$

$$\frac{d\delta_{MIN}}{dn} = \frac{-2\sin(\alpha/2)}{\sqrt{1-n^2\sin^2(\alpha/2)}}$$

$$n = \frac{\sin\left[(\alpha-\delta_{MIN})/2\right]}{\sin(\alpha/2)}$$

$$n\sin(\alpha/2) = \sin\left[(\alpha-\delta_{MIN})/2\right]$$

$$1-n^2\sin^2(\alpha/2) = 1-\sin^2\left[(\alpha-\delta_{MIN})/2\right]$$

$$1-n^2\sin^2(\alpha/2) = \cos^2\left[(\alpha-\delta_{MIN})/2\right]$$

$$\frac{d\delta_{MIN}}{dn} = \frac{-2\sin(\alpha/2)}{\cos\left[(\alpha-\delta_{MIN})/2\right]}$$

Glass Dispersion – Example

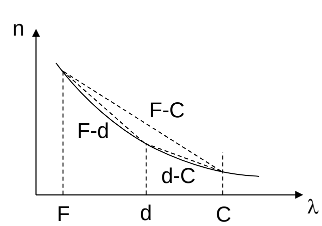
Since the refractive index is a complex function of wavelength, the glass dispersion can be approximated by finite differences:

$$\frac{dn}{d\lambda} \approx \frac{\Delta n}{\Delta\lambda} = \frac{n_2 - n_1}{\lambda_2 - \lambda_1}$$

	λ	Index (BK7)	Index (F2)
F	.4861 µm	1.52238	1.63208
d	.5876 µm	1.51680	1.62004
C	.6563 µm	1.51432	1.61503

Glass dispersion for different wavelength pairs:

$\Delta n/\Delta\lambda$	BK7	F2
F-C	-.0474/µm	-.1002/µm
F-d	-.0550/µm	-.1186/µm
d-C	-.0361/µm	-.0729/µm



Prism Dispersion – Example

$$\frac{d\delta}{d\lambda} = \frac{d\delta}{dn}\frac{dn}{d\lambda} \approx \frac{d\delta_{MIN}}{dn}\frac{\Delta n}{\Delta\lambda} \approx \frac{d\delta_{MIN}}{dn}\frac{(n_F - n_C)}{(\lambda_F - \lambda_C)}$$

$$\frac{d\delta_{MIN}}{dn} = \frac{-2\sin(\alpha/2)}{\cos\left[(a - \delta_{MIN})/2\right]} < 0$$

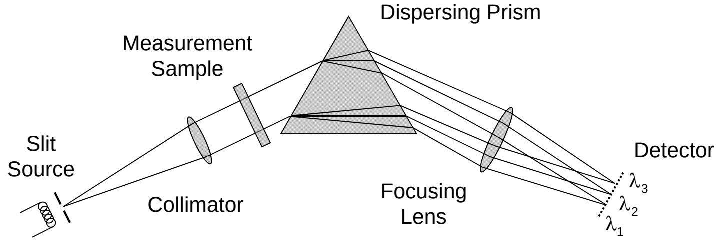
Example: 60° prism ($\alpha = 60°$)
d light
F-C glass dispersion

	BK7	F2
δ_{MIN}	-38.7° -.675 rad	-48.2° -.841 rad
$\frac{d\delta_{MIN}}{dn}$	-1.535	-1.705
$\frac{\Delta n}{\Delta\lambda}$	-.0474/µm	-.1002/µm
$\frac{d\delta}{d\lambda}$	0.073/µm 4.18°/µm .00418°/nm	0.177/µm 10.2°/µm .0102°/nm

OPTI-502 Optical Design and Instrumentation I
© Copyright 2019 John E. Greivenkamp
College of Optical Sciences
The University of Arizona

Prism Spectrometer

The dispersing power of a prism can be used to measure the spectral content of a light beam. This includes the spectrum of the source or the transmission/absorption spectrum of a material placed in the beam.



The source is focused on a slit, and the light is then collimated. After dispersion by the prism, a second lens images the slit onto the detector plane. The slit images for each wavelength are displaced producing a spectrum.

Each wavelength in the spectrum is spread over the width of the slit image. Each wavelength is further blurred by diffraction due to the lens diameter and prism size.

Resolving Power and Littrow Prisms

The Resolving Power of the spectrometer is defined as

$$R = \frac{\lambda}{\Delta\lambda}$$

R can be increased by using a narrow slit and large diameter beam. Large prisms are required for high-resolution spectroscopy.

Littrow Prism configurations use a double pass through the prism for increased dispersion: