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Section 10

Vignetting

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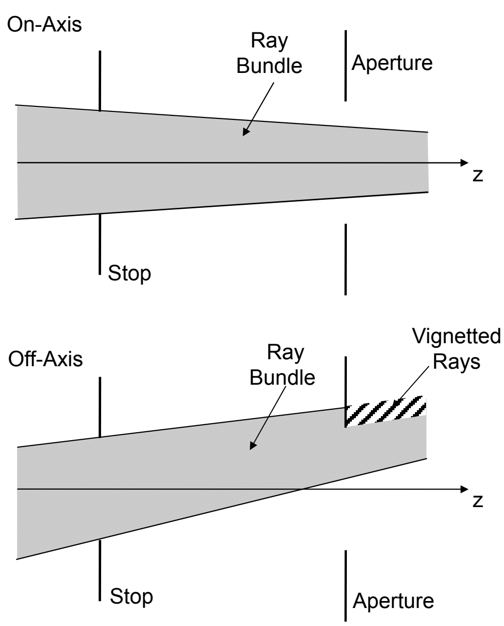
Vignetting

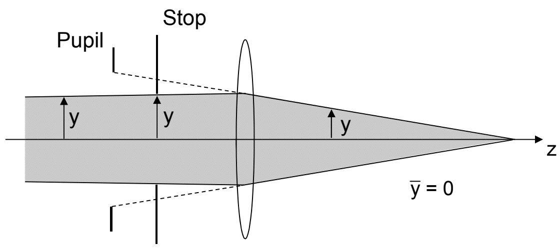
The stop determines the size of the bundle of rays that propagates through the system for an on-axis object.

As the object height increases, one of the other apertures in the system (such as a lens clear aperture) may limit part or all of the bundle of rays. This is known as vignetting.



Ray Bundle – On-Axis

The ray bundle for an on-axis object is a rotationally-symmetric spindle made up of sections of right circular cones. Each cone section is defined by the pupil and the object or image point in that optical space. The individual cone sections match up at the surfaces and elements.



At any *z*, the cross section of the bundle is circular, and the radius of the bundle is the marginal ray value. The ray bundle is centered on the optical axis.

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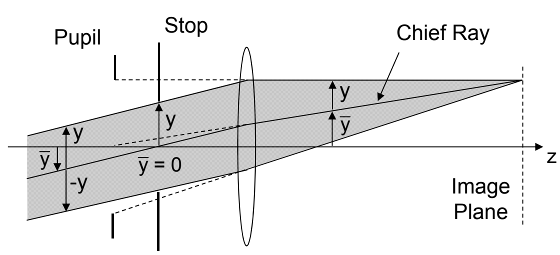
Ray Bundle – Off Axis

For an off-axis object point, the ray bundle skews, and is comprised of sections of skew circular cones which are still defined by the pupil and object or image point in that optical space.



Since the base of the cone is defined by the circular pupil, the cross section of the ray bundle at any *z* remains circular with a radius equal to the radius of the axial bundle. The off-axis bundle is centered about the chief ray height.

The maximum radial extent of the ray bundle at any *z* is

$$|y_{MAX}| = |y| + |\bar{y}|$$

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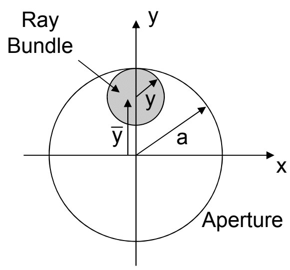
Unvignetted

While the stop alone defines the axial ray bundle, vignetting occurs when other apertures in the system, such as a lens clear aperture, block all or part of an off-axis ray bundle.

No vignetting occurs when all of the apertures pass the entire ray bundle from the object point. Each aperture radius *a* must equal or exceed the maximum height of the ray bundle at the aperture. Note that the required apertures size will change if the system FOV (chief ray) is changed.



Unvignetted:

$$a \geq |\bar{y}| + |y|$$

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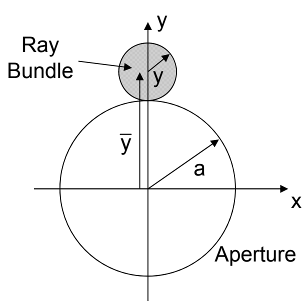
Fully Vignetted

The maximum FOV supported by the system occurs when an aperture completely blocks the ray bundle from the object point.



Fully Vignetted:

$$a \leq |\bar{y}| - |y|$$

and

$$a \geq |y|$$

The second part of this vignetting condition ensures that the aperture passes the marginal ray and is not the system stop. By definition, vignetting cannot occur at the aperture stop or at a pupil.

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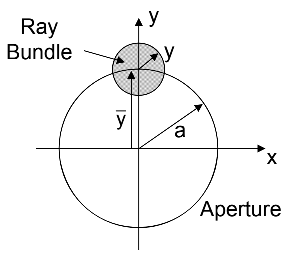
Half Vignetted

A third vignetting condition is defined when an aperture passes about half of the ray bundle from an object point.



Half Vignetted:

$$a = |\bar{y}|$$

and

$$a \geq |y|$$

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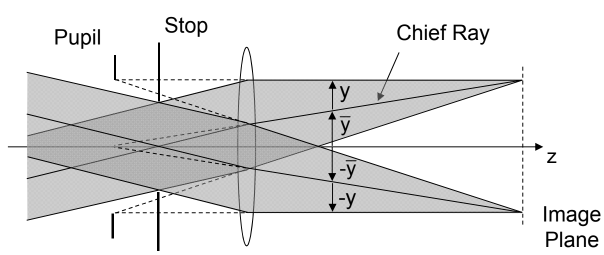
Ray Bundles and Aperture Diameter

Because the Field of View of an optical system is symmetric (i.e. ±h or $\pm\theta_{1/2}$), the vignetting conditions apply equally well to the upper and lower ray bundles.



The maximum radial extent of both ray bundles at any *z* is

$$|y_{MAX}| = |y| + |\bar{y}|$$

If a system is used with a rectangular detector, different amounts of vignetting can occur in the horizontal, vertical and diagonal directions.

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Vignetting Summary

The required clear aperture radius at a given z:

Unvignetted: $a \geq |\bar{y}| + |y|$

Fully Vignetted: $a \leq |\bar{y}| - |y|$ and $a \geq |y|$

Half Vignetted: $a = |\bar{y}|$ and $a \geq |y|$

The vignetting conditions are used in two different manners:
- For a given set of apertures, the FOV that the system will support with a prescribed amount of vignetting can be determined. A different chief ray defines each FOV.
- For a given FOV and vignetting condition, the required aperture diameters can be determined.

A system with vignetting will have an image that has full irradiance or brightness out to a radius corresponding to the unvignetted FOV limit. The irradiance will then begin to fall off, going to about half at the half-vignetted FOV, and decreasing to zero at the fully-vignetted FOV. This fully-vignetted FOV is the absolute maximum possible. This description ignores the obliquity factors of radiative transfer, such as the cosine fourth law.

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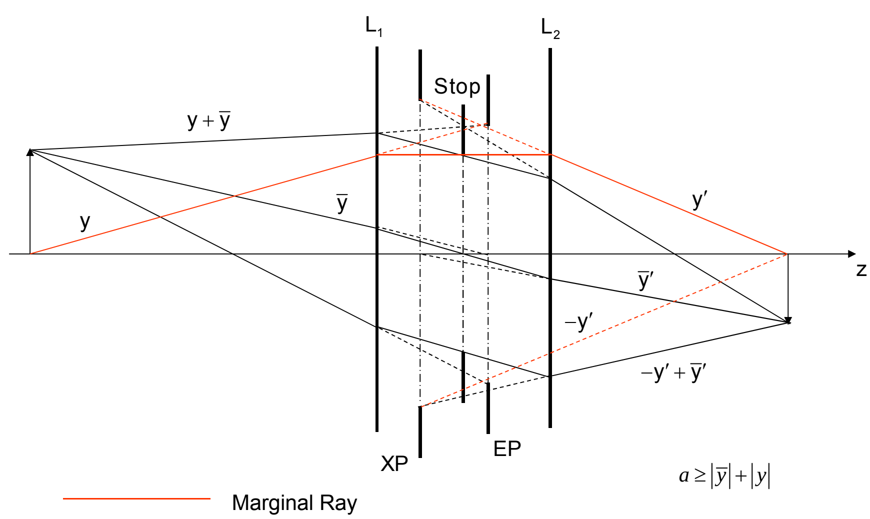
Example System – Ray Bundle Extent

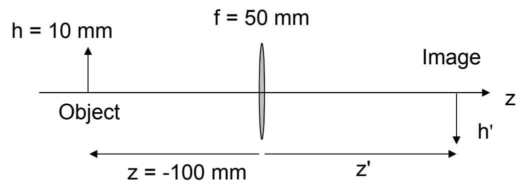


$a \geq |\bar{y}| + |y|$

Vignetting Example – Page 1

An object is located 100 mm to the left of a 50 mm focal length thin lens. The object has a height of 10 mm above the optical axis of the lens. The lens diameter is 20 mm, and the lens serves as the system stop. An aperture is placed 50 mm to the right of the lens. What is the required diameter of the aperture so that the system operates without vignetting?

First consider the imaging:



Stop at Lens

D_{STOP} = 20 mm

$$z = -100\,mm \qquad \frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \qquad z' = 100\,mm$$
$$f = 50\,mm$$

Lens is operating at 1:1 conjugates $\quad h' = -10\,mm$

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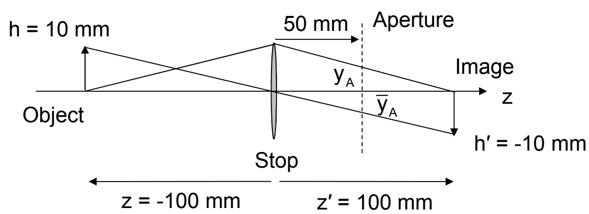
Vignetting Example – Page 2

Draw the Marginal and Chief Rays and evaluate ray heights at the aperture:



a_{STOP} = 10 mm

At the aperture:

$$y_A = a_{STOP} / 2 = 5\,mm$$

$$\bar{y}_A = h' / 2 = -5\,mm$$

For no vignetting:

$$a_{APERTURE} \geq |\bar{y}_A| + |y_A| = 10\,mm$$

$$D_{APERTURE} \geq 20\,mm$$

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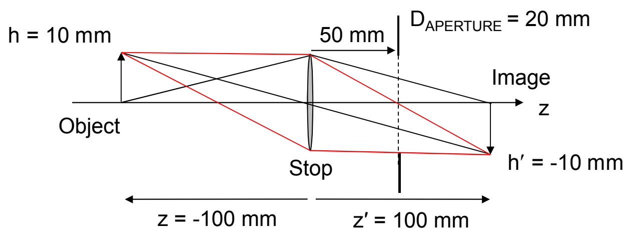
Vignetting Example – Page 3

The ray bundle with no vignetting:



The full ray bundle is passed by the aperture.

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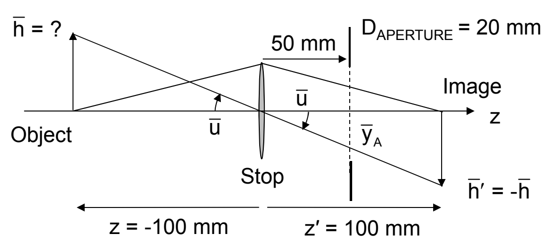
Vignetting Example – Page 4

Given this aperture diameter of 20 mm, what is the object height that will be imaged with half vignetting?

The marginal ray does not change, but the chief ray changes with object height. Start with an arbitrary object height. The chief ray goes through the center of the stop.



$\bar{u} = \frac{\bar{h}}{z} = \frac{\bar{h}'}{z'}$

At the aperture: $\bar{y}_A = (50\ mm)\bar{u} = (50\ mm)\frac{\bar{h}}{z}$

For half vignetting: $a_{APERTURE} = |\bar{y}_A| = 10\,mm$

Equating: $|\bar{y}_A| = \left|(50\ mm)\frac{\bar{h}}{z}\right| = \left|(50\ mm)\frac{\bar{h}}{-100mm}\right| = 10\,mm$

$\bar{h} = 20\,mm$ Half Vignetted Object Height

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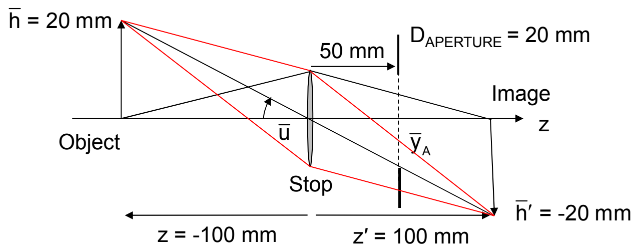
<u>Vignetting Example – Page 5</u>

The ray bundle at half vignetting



$$a_{APERTURE} = |\bar{y}_A| = 10\,mm$$

Half of the ray bundle is blocked by the aperture.

The chief ray goes through the edge of the aperture and the center of the stop.

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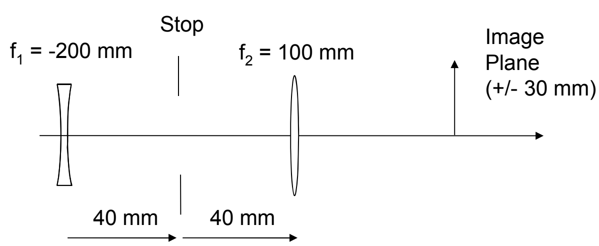
<u>Example System – Pupils and Vignetting – Page 1</u>

The following reverse telephoto objective is comprised of two thin lenses in air. The system stop is located between the two lenses.

The system operates at f/4.

The object is at infinity. The maximum image size is +/- 30 mm.



Determine the folowing:
- Entrance pupil and exit pupil locations and sizes.
- System focal length and back focal distance.
- Stop diameter.
- Angular field of view (in object space).
- Required diameters for the two lenses for the system to be unvignetted over the specified maximum image size.

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Example System – Pupils and Vignetting – Page 2

First set up the raytrace sheet.

Trace a potential Chief Ray starting at the center of the Stop. The Pupils are located where this ray crosses the axis in object space and image space.

	Object		EP		L_1		Stop		L_2		XP		Image
Surface	0		1		2		3		4		5		6
f					-200		-		100				
$-\phi$					0.005		-		-0.01				
t				-33.33		40		40		-66.67			
Potential Chief Ray:													
$\tilde{y}$			0		-4.00		0		4.00		0		
$\tilde{u}$				.12		0.1*		0.1*		0.06			

Entrance Pupil: Located 33.33 mm to the Right of L1
Exit Pupil: Located 66.67 mm to the Left of L2

Both Pupils are virtual.

* Arbitrary

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Example System – Pupils and Vignetting – Page 3

In order to determine the Focal Length and the Back Focal Distance, trace a potential Marginal Ray. Since the object is at infinity, this ray is parallel to the axis in object space. The Rear Focal Point is located where this ray crosses the axis in image space.

	Object		EP		L_1		Stop		L_2		XP		F' Image
Surface	0		1		2		3		4		5		6
f					-200		-		100				
$-\phi$					0.005		-		-0.01				
t		∞		-33.33		40		40		-66.67		222.22	
Potential Marginal Ray:													
y	1*		1		1		1.2		1.4		2.0		0
u		0		0		0.005		0.005		-0.009		-0.009	

The image or the rear focal point is located 222.22 mm to the Right of the XP.

$$BFD = \left(L2 \rightarrow XP\right) + \left(XP \rightarrow F'\right) = -66.67\ mm + 222.22\ mm$$
$$BFD = 155.56\ mm$$

System Power and Focal Length: $\phi = -\dfrac{\tilde{u}'}{\tilde{y}_1}$ $\quad \tilde{u}' = -0.009$ $\quad \tilde{y}_1 = 1.0\ mm$

$$\phi = 0.009 / mm \qquad f = \frac{1}{\phi} = 111.11\ mm$$

* Arbitrary

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Example System – Pupils and Vignetting – Page 4

To determine the Stop and Pupil sizes, make use of the fact that the system operates at f/4.

$$f/\# = \frac{f}{D_{EP}} = 4 \qquad f = 111.11\,mm \qquad D_{EP} = 27.78\,mm \qquad r_{EP} = 13.89\,mm$$

Scale the Potential Marginal Ray to obtain this ray height at the EP:

$$Scale\ Factor = \frac{r_{EP}}{\tilde{y}_{EP}} = \frac{13.89\,mm}{1\,mm} = 13.89$$

	Object	EP	L_1	Stop	L_2	XP	F' Image
Surface	0	1	2	3	4	5	6
f			-200	-	100		
-ϕ			0.005	-	-0.01		

	0–1	1–2	2–3	3–4	4–5	5–6
t	∞	-33.33	40	40	-66.67	222.22

Potential Marginal Ray:

Surface	0	1	2	3	4	5	6
ỹ	1	1	1	1.2	1.4	2.0	0

	0–1	1–2	2–3	3–4	4–5	5–6
ũ	0	0	0.005	0.005	-0.009	-0.009

Marginal Ray:

Surface	0	1	2	3	4	5	6
y	13.89	13.89	13.89	16.67	19.45	27.78	0

	0–1	1–2	2–3	3–4	4–5	5–6
u	0	0	0.0695	0.0695	-0.125	-0.125

$$r_{STOP} = y_{STOP} = 16.67\,mm \qquad D_{STOP} = 33.33\,mm$$
$$r_{XP} = y_{XP} = 27.78\,mm \qquad D_{XP} = 55.56\,mm$$

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Example System – Pupils and Vignetting – Page 5

For the Field of View calculation, the Potential Chief Ray is extended to the image plane.

The Potential Chief Ray is scaled to the required image height of 30 mm:

$$Scale\ Factor = \frac{\bar{y}_{IMAGE}}{\tilde{\bar{y}}_{IMAGE}} = \frac{30\,mm}{13.33\,mm} = 2.25$$

	Object	EP	L_1	Stop	L_2	XP	F' Image
Surface	0	1	2	3	4	5	6
f			-200	-	100		
-ϕ			0.005	-	-0.01		

	0–1	1–2	2–3	3–4	4–5	5–6
t	∞	-33.33	40	40	-66.67	222.22

Potential Chief Ray:

Surface	0	1	2	3	4	5	6
$\tilde{\bar{y}}$		0	-4.00	0	4.00	0	13.33

	0–1	1–2	2–3	3–4	4–5	5–6
$\tilde{\bar{u}}$	.12	.12	0.1	0.1	0.06	0.06

Chief Ray:

Surface	0	1	2	3	4	5	6
$\bar{y}$		0	-9.00	0	9.00	0	30.0

	0–1	1–2	2–3	3–4	4–5	5–6
$\bar{u}$	0.270	0.270	0.225	0.225	0.135	0.135

Object Space Chief Ray: $\bar{u}_0 = 0.270$

$$HFOV = \tan^{-1}(\bar{u}_0) = 15.1°$$
$$FOV = 30.2° \quad or \quad \pm 15.1°$$

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Example System – Pupils and Vignetting – Page 6

Summarizing the completed Marginal and Chief Rays:

	Object		EP		L_1		Stop		L_2		XP		F' Image
Surface	0		1		2		3		4		5		6
f					-200		-		100				
$-\phi$					0.005		-		-0.01				
t		∞		-33.33		40		40		-66.67		222.22	
Marginal Ray:													
y	13.89		13.89		13.89		16.67		19.45		27.78		0
u		0		0		0.0695		0.0695		-0.125		-0.125	
Chief Ray:													
$\bar{y}$			0		-9.00		0		9.00		0		30.0
$\bar{u}$		0.270		0.270		0.225		0.225		0.135		0.135	

For No Vignetting: $a \ge |y| + |\bar{y}|$

$L1:$ $y_1 = 13.89\ mm$ $a_1 \ge 22.89\ mm$
$\bar{y}_1 = -9.0\ mm$ $D_1 \ge 45.78\ mm$

$L2:$ $y_2 = 19.45\ mm$ $a_2 \ge 28.45\ mm$
$\bar{y}_2 = 9.0\ mm$ $D_2 \ge 56.9\ mm$

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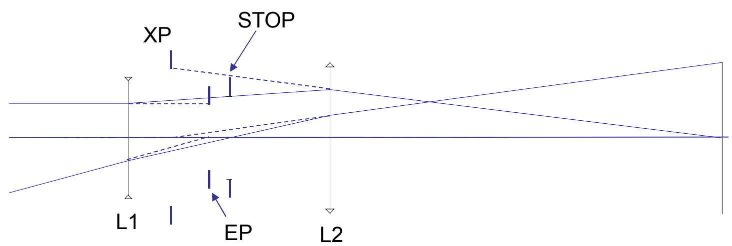
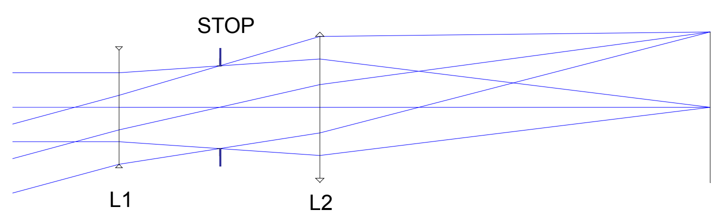
Example System – Pupils and Vignetting – Page 7

Marginal and Chief Rays:



Ray Bundles:



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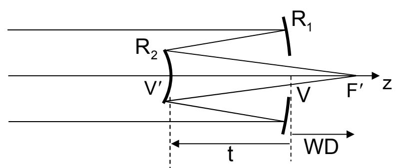
Dummy Surfaces

In a raytrace, zero-power surfaces can be inserted at any location (any z) in order to examine the ray properties (or image quality in the case of real rays). These are called "Dummy Surfaces."

An example of their use would be determining the required size of the hole in the primary mirror of a Cassegrain objective. It is nothing more than a vignetting problem with a dummy surface placed at the location of the hole at the primary mirror vertex.



$R_1 = -200\,mm$
$t = -80\,mm$
$R_2 = -50\,mm$
$n_1 = n = 1$
$n_2 = -1$
$n_3 = n' = 1$

Surface	0		Primary		Secondary		Hole		F′
R			-200		-50				
t		∞		-80		80		WD	
n		1.0		-1.0		1.0		1.0	
-ϕ			-.01		.04		0		
t/n		∞		80		80		WD	

The hole is located to the right of the secondary mirror.

Marginal and chief rays are traced, and the vignetting condition is applied to determine the hole size.

By using the dummy surface, the working distance (WD) is also directly computed on the raytrace sheet.

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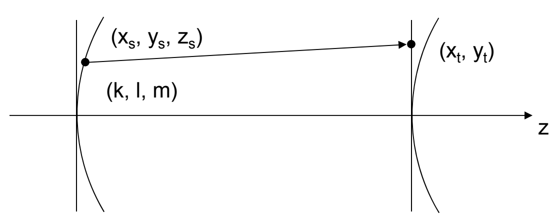
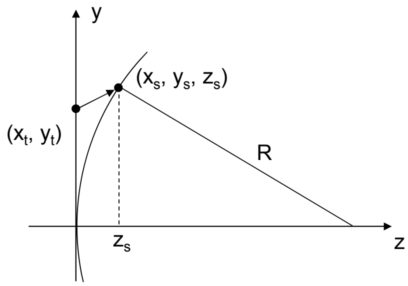
Real Raytrace

In addition to using actual angles instead of paraxial approximations, a real raytrace must use the actual surfaces instead of just the vertex planes. (It is a 3D problem).

1.) Start at a point on one surface and transfer to a target (vertex) plane for the next surface. The direction cosines of the initial ray are known.



Real Raytrace - Continued



$$(R - z)^2 + x^2 + y^2 = R^2$$

2.) Transfer from the target (vertex) plane to the actual surface. Find the intersection of the ray with the surface.
3.) Find the surface normal at the intersection point.
4.) Refract using Snell's law to determine the new direction cosines in the optical space after the surface.
5.) Transfer to the next surface. Repeat.

A real raytrace can also be done with aspheric surfaces. The expressions may get complicated, and iteration may be needed to determine the intersection point.

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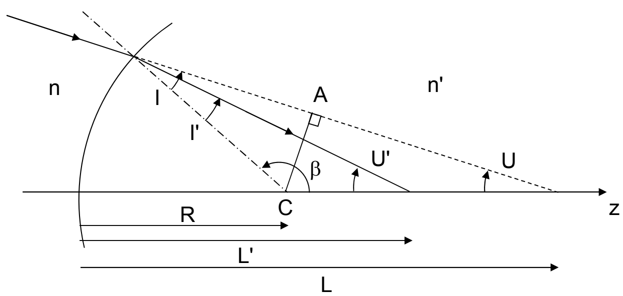
Trigonometric Raytrace

Used for manual raytracing. Rays restricted to the meridional plane.



$$\sin U = -\frac{\overline{CA}}{(L-R)} \qquad \sin I = \frac{\overline{CA}}{R} \qquad 180 = I' + \beta - U' = I + \beta - U$$

(3) $U' = I' - I + U$

(1) $\sin I = -\frac{(L-R)}{R}\sin U$

$$\sin I' = -\frac{(L'-R)}{R}\sin U'$$

(2) $n' \sin I' = n \sin I$

(4) $L' = R - \frac{R \sin I'}{\sin U'}$

Given R, L and U, these four equations allow L' and U' to be calculated. This defines the refracted ray.

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