Section 4

Imaging and Paraxial Optics

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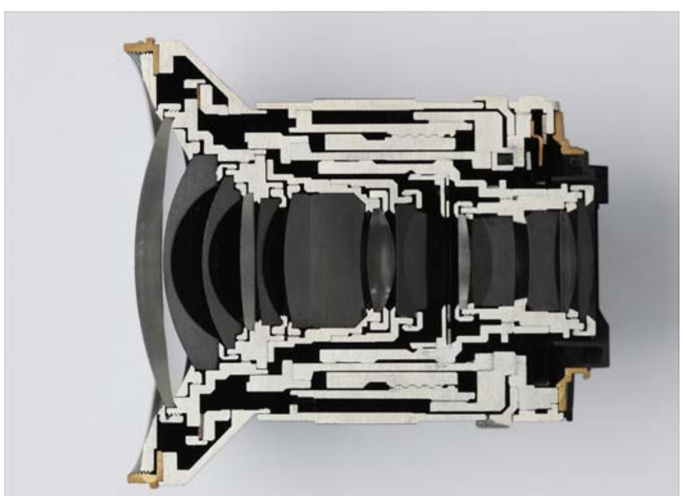
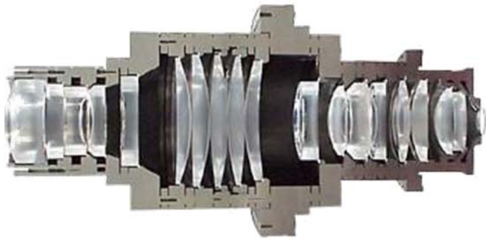
Optical Systems

An optical system is a collection of optical elements (lenses and mirrors). While the optical system can contain multiple optical elements, the first order properties of the system are characterized by a single focal length or magnification.



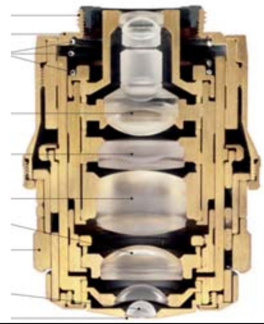
lenses.zeiss.com

Optical Systems



tf.uni-kiel.de

base24.com



oddstuffmagazine.com

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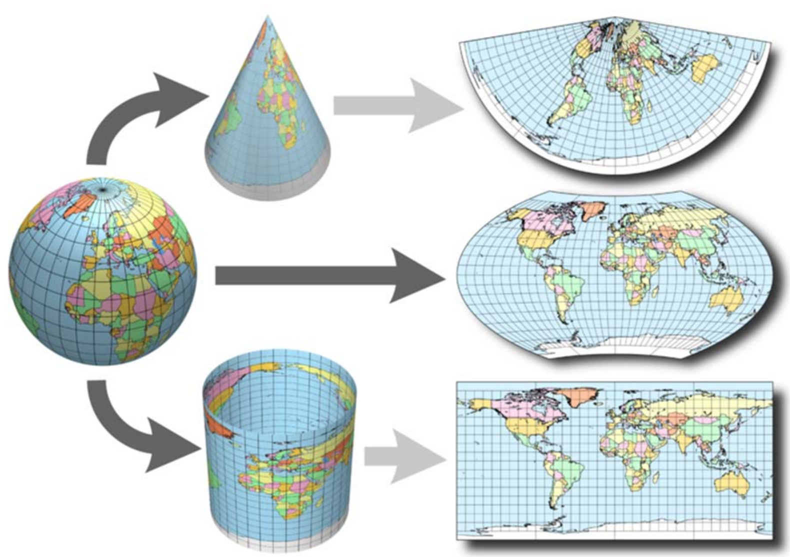
Mappings

A mapping is a conversion or transformation from one representation to another representation.



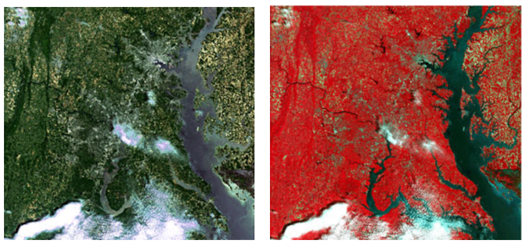
progonos.com

Color Mappings

Color to Grayscale:

stackoverflow.com

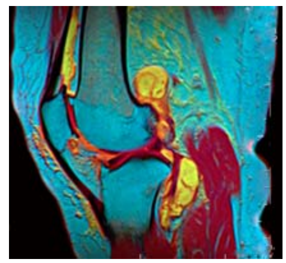
False-color:



IR mapped to a visible color

Wikipedia

Pseudo-color:



Different tissues are displayed as different colors.

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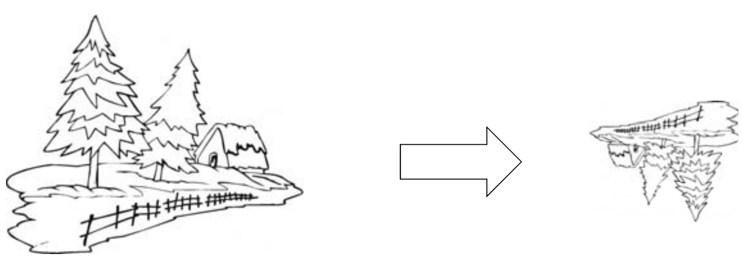
Imaging as a Mapping

First-order optics is the optics of perfect imaging systems. Aberrations are ignored.

The object is mapped or transformed to its image. The process can be analyzed without knowing the details of the optical system.

A small number of system properties will completely define and determine the mapping or first-order imaging properties. These are known as the *cardinal points* of the imaging system.



This is a mapping from object space to image space.

Image from: supercoloring.com

Imaging

Goal: Determine the size, orientation and location of an image for a given object. This is the optics of perfect imaging systems; aberrations are ignored.

Collinear Transformation – A generalized mapping from one space into another in which points map to points, lines map to lines, and planes map to planes.

Gaussian Imagery – A specific collinear transformation using assumptions that are appropriate for optical systems. The cardinal points result.

First-Order Optics – The actual ray paths through a system can be expanded in a power series of heights and angles. An axially symmetric system will have only odd power terms, and the first-order terms give the position and size of the image. First-order optics is the optics of perfect optical systems. The deviations from this perfection are the system aberrations.

Paraxial Optics – A method of determining the first-order properties of an optical system by tracing rays using the slopes of the rays instead of the ray angles. The angles of incidence and refraction or reflection at surfaces are assumed to be small. The sag* of the refracting or reflecting surface is ignored or is considered to be negligible compared to other distances.

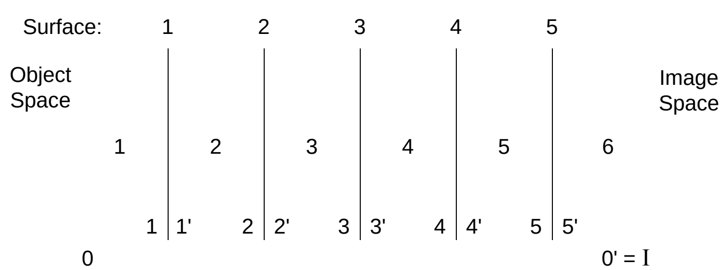
Paraxial analysis is also useful for relating the physical properties of a refracting or reflecting surface (curvature and index) to its Gaussian properties (focal length and cardinal points).

Fortunately, all of these theories are consistent and give the same results.

*The sag of a surface is the separation of the surface from a plane tangent to the surface vertex.

Optical Spaces

Each time a refracting or reflecting surface is encountered, a new optical space is entered. Each space extends from $-\infty$ to $+\infty$ and has an associated index. If a system has N surfaces, there will be N+1 optical spaces. The first space is usually called object space and the last is called image space.

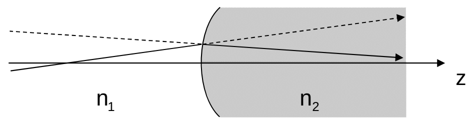


A single object or image exists in each space. Each surface has an object space and image space of its own. These intermediate optical spaces serve as the image space for the preceding surface and the object space for the following surface. Each surface will form an image of the object in the preceding space.

There are real and virtual segments of each optical space. The real segment of an optical space is the volume between surfaces defining entry and exit into that space.

Real and Virtual

Rays can be traced from optical space to optical space. Within any optical space, a ray is straight and extends from $-\infty$ to $+\infty$ with real and virtual segments. Rays from adjoining spaces meet at the common optical surface.



A real object is to the left of the surface; a virtual object is to the right of the surface. A real image is to the right of the surface; a virtual image is to the left of the surface.

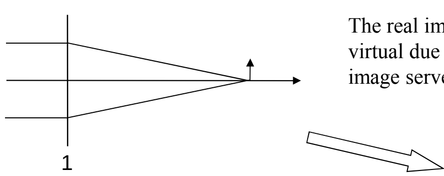
In an optical space with a negative index (light propagates from right to left), left and right are reversed in these descriptions of real and virtual.

Alternate description: If an object is upstream of the surface or system, it is considered to be real. A downstream object is virtual. Images downstream of the surface or system are real; images upstream are virtual.

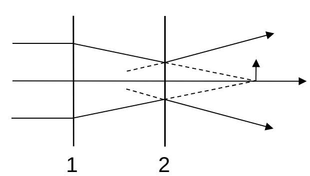
It is also common to combine multiple optical surfaces into a single system element and only consider the object and image spaces of the element; the intermediate spaces within the element are then ignored.

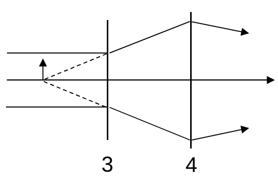
Real and Virtual Confusion

The distinctions between real and virtual may become confused when discussing intermediate optical spaces. Real and virtual can be discussed either from the perspective of a single surface or from the perspective of the system. For example, it is common for the real image created by one surface to serve as a virtual object for the next surface.



The real image formed by Surface 1 becomes virtual due to the presence of Surface 2, and this image serves as the virtual object for Surface 2.

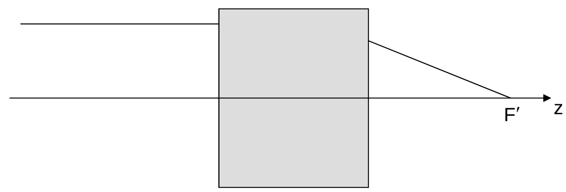




In a similar manner, the virtual image produced by Surface 3 can be considered to be a real object for Surface 4.

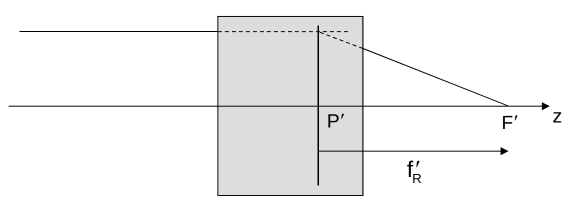
General System

Consider a “black-box” model of an optical system. An optical system is any collection of optical elements (lenses, mirrors, etc.) comprising a rotationally symmetric optical system. A ray from an object at infinity will emerge from the system and go through the Rear Focal Point of the system F':



Planes of Effective Refraction

By extending the image space ray back to the height of the object space ray, the plane of effective refraction into image space for the system is found. All of the refractions at the individual elements within the system are combined into a single effective refraction.

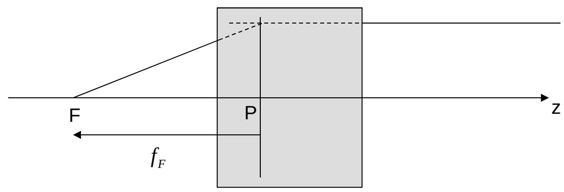


This plane of effective refraction into image space is called the Rear Principal Plane P'.

The distance from the Rear Principal Plane to the Rear Focal Point is the *Rear Focal Length* of the system f_R' .

Planes of Effective Refraction

In a similar manner, the plane of effective refraction out of object space can be found by using a ray starting at the Front Focal Point of the system F. The image ray is parallel to the axis.

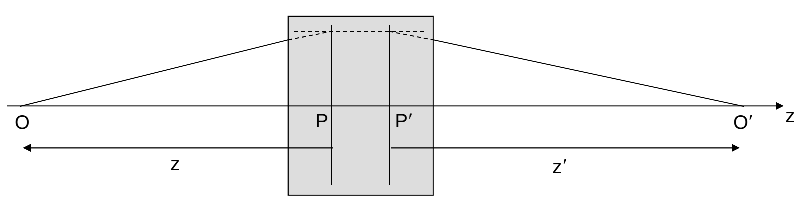


This plane of effective refraction out of object space is called the Front Principal Plane P.

The distance from the Front Principal Plane to the Front Focal Point is the *Front Focal Length* of the system f_F .

Planes of Effective Refraction

For the system used at finite conjugates, the ray from the object point will appear to refract at the Front Principle Plane and emerge from the Rear Principal Plane at the same height. The image point is located where this ray crosses the axis.



The object and image distances z and z' are measured from the respective Principal Planes.

This treatment allows the system to be considered to be just like a thin lens except the refraction occurs at separated planes of effective refraction.

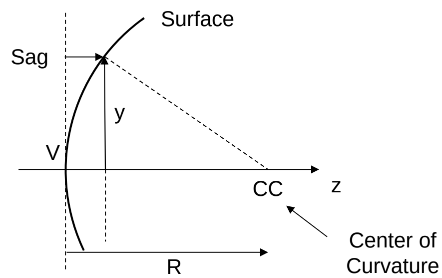
Paraxial Optics

In order to relate the physical parameters of an optical system (radii of curvature of surfaces, spacings and thicknesses) to its imaging properties, rays must be traced through the system using Snell's law or the law of reflection.

While exact raytracing can be used, the first-order or imaging properties of the system can be found using the approximate ray paths computed by paraxial optics. Rays are traced using the slopes of the rays instead of the ray angles. The amount a ray is bent at the refracting or reflecting surface is assumed to be small. The sag of the refracting or reflecting surface is ignored or is considered to be negligible compared to other distances.

The radius of curvature *R* of a surface is defined to be the distance from its vertex to its center of curvature CC.

The sag of a surface is measured relative to a plane tangent to the surface vertex V. The sag will vary with the radial position on the surface y.

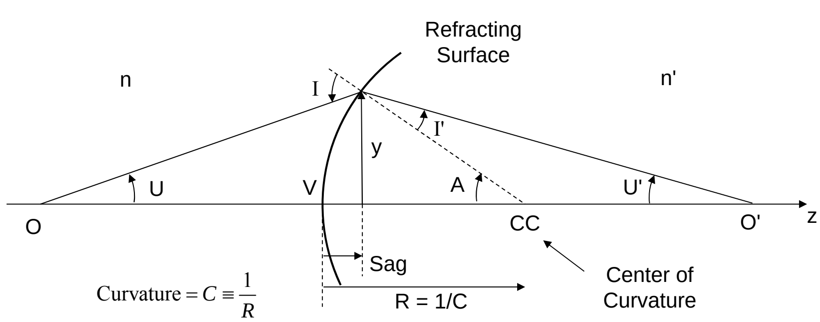


Single Refracting Surface

Consider a single refracting surface with a radius of curvature of R that separates an index of refraction n from and index of refraction n'.

The angles of incidence and refraction (I and I') are measured with respect to the surface normal.

The ray angles U and U', as well as the elevation angle A of the surface normal at the ray intersection, are measured with respect to the optical axis. The usual sign conventions apply.



$$\text{Curvature} = C \equiv \frac{1}{R}$$

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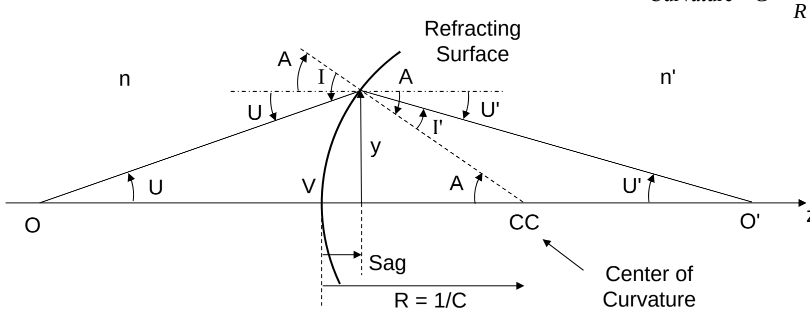
Single Refracting Surface

Curvature $= C \equiv \frac{1}{R}$



Relating the angles at the ray intersection with the surface:

$$A = U' - I'$$

$$I = U - A \qquad I' = U' - A$$

Apply Snell's Law:

$$n \sin I = n' \sin I'$$

$$n \sin(U - A) = n' \sin(U' - A)$$

Single Refracting Surface and Snell's Law

$$n \sin I = n' \sin I'$$

$$n \sin(U - A) = n' \sin(U' - A)$$

$$n[\sin U \cos A - \cos U \sin A] = n'[\sin U' \cos A - \cos U' \sin A]$$

$$n\left[\sin U - \cos U \frac{\sin A}{\cos A}\right] = n'\left[\sin U' - \cos U' \frac{\sin A}{\cos A}\right]$$

$$n[\sin U - \cos U \tan A] = n'[\sin U' - \cos U' \tan A]$$

Approximation #1: $\cos U \approx \cos U'$

Approximation #1 implies that the ray bending at the surface is small.

$$n\left[\frac{\sin U}{\cos U} - \tan A\right] = n'\left[\frac{\sin U'}{\cos U'} - \tan A\right]$$

$$n[\tan U - \tan A] = n'[\tan U' - \tan A]$$

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Paraxial Angles

$$n\left[\tan U - \tan A\right] = n'\left[\tan U' - \tan A\right]$$

Define the paraxial angles:

$$u \equiv \tan U \qquad u' \equiv \tan U' \qquad \alpha \equiv \tan A$$

$$n\left(u - \alpha\right) = n'\left(u' - \alpha\right)$$

$$n'u' = nu + \left(n' - n\right)\alpha$$

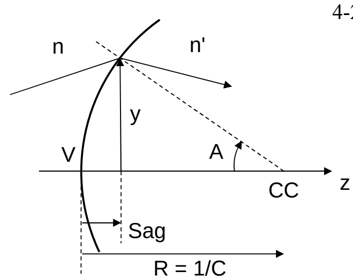
Paraxial angles are the tangents of the real ray angles and are the slopes of the rays.

Paraxial angles are not angles and are incorrectly called angles only because of tradition.

Paraxial angles are ray slopes.

Paraxial angles are unitless.

Power of a Surface and Paraxial Raytrace Equation



$$n'u' = nu + \left(n' - n\right)\alpha$$

$$\alpha = \tan A = -\frac{y}{\left(R - Sag\right)}$$

Approximation #2: $|\text{Sag}| << |\text{R}|$

$$\alpha \approx -\frac{y}{R}$$

$$n'u' = nu - \left(n' - n\right)\frac{y}{R} = nu - \left(n' - n\right)yC$$

Approximation #2 implies that the sag of the surface at the ray intersection is much less than the radius of curvature of the surface.

Define the Power ϕ of the surface:

$$\phi = \left(n' - n\right)C = \frac{\left(n' - n\right)}{R}$$

$$n'u' = nu - y\phi$$

This is the Paraxial Raytrace Equation.

Note that the power of the surface depends only on the construction parameters (R, n, n') of the surface. It is ray independent.

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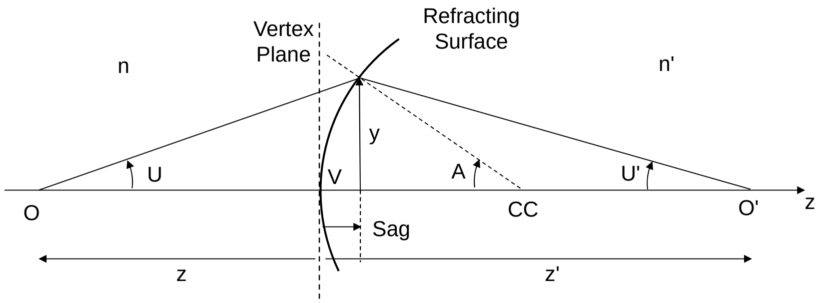
Object and Image distances for a Single Refracting Surface

The object and image distances (z and z') are also both measured from the surface vertex.



Approximation #3: The object and image distances are much greater than the sag of the surface at the ray intersection.

$$|Sag| << |z| \qquad |Sag| << |z'|$$

$$u = \tan U = -\frac{y}{(z - Sag)} \approx -\frac{y}{z}$$

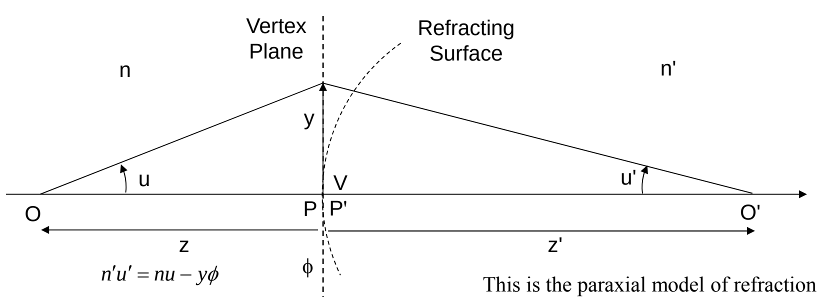
$$u' = \tan U' = -\frac{y}{(z' - Sag)} \approx -\frac{y}{z'}$$

Surface Vertex Plane and Principal Planes

Approximations #2 and #3 state that the sag is considered to be small with respect to the radius of curvature of the surface and with respect to the object and image distances. Since these conditions are only truly met for small ray heights y on the surface, this method of analysis is called paraxial optics or paraxial raytracing. Paraxial means "near axis".

By ignoring the surface sag in paraxial optics, the planes of effective refraction for the single refracting surface are located at the surface vertex plane V. The Front and Rear Principal Planes (P and P') of the surface are both located at the surface.

Here, the paraxial "angles" u and u' are shown with the refraction at the vertex plane. The paraxial rays shown approximate the actual rays as the surface sag is ignored. Refraction is considered to occur at the surface vertex.



$$n'u' = nu - y\phi$$

This is the paraxial model of refraction at a surface with power ϕ.

Imaging and Focal Lengths of a Single Refracting Surface

$$n'u' = nu - y\phi$$

$$u = \frac{y}{-z} \qquad u' = -\frac{y}{z'}$$

$$n'\left(-\frac{y}{z'}\right) = n\left(-\frac{y}{z}\right) - y\phi$$

z and z' are measured from the surface vertex or the Principal Planes.

$$\frac{n'}{z'} = \frac{n}{z} + \phi$$

Object at Infinity
Image at the Rear Focal Point

$$z = \infty \qquad z' = f'_R$$

$$\frac{n'}{f'_R} = \phi \qquad f'_R = \frac{n'}{\phi}$$

Image at Infinity
Object at the Front Focal Point

$$z' = \infty \qquad z = f_F$$

$$\frac{-n}{f_F} = \phi \qquad f_F = -\frac{n}{\phi}$$

$$\phi = \frac{n'}{f'_R} = -\frac{n}{f_F} \qquad \frac{f'_R}{f_F} = -\frac{n'}{n}$$

Object-Image Relationship and the Focal length

$$\frac{n'}{z'} = \frac{n}{z} + \phi \qquad \phi = \frac{n'}{f'_R} = -\frac{n}{f_F}$$

Define the "THE" Focal Length, also called the Effective (or Equivalent) Focal Length EFL:

$$f = f_E \equiv \frac{1}{\phi} = -\frac{f_F}{n} = \frac{f'_R}{n'}$$

Note that the focal length *f* is the reduced front and rear focal lengths.

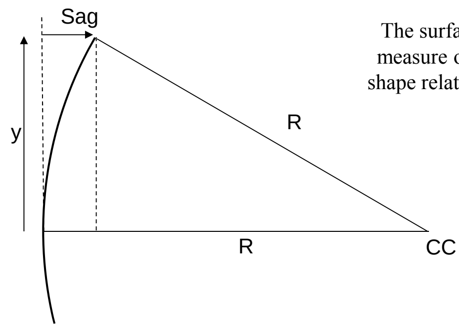
$$\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E} \qquad \boxed{\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f}}$$

The "effective" or "equivalent" in EFL is actually unnecessary. There is a single focal length *f*.

The front and rear focal lengths are related to the focal length through the indices of refraction:

$$f_F = -\frac{n}{\phi} = -nf_E = -nf \qquad f'_R = \frac{n'}{\phi} = n'f_E = n'f$$

Surface Sag



The surface sag is the measure of the surface shape relative to a plane.

Circle (or Sphere):

$$y^2 + (R - Sag)^2 = R^2$$

$$y^2 + R^2 - 2R\,Sag + Sag^2 = R^2$$

$$Sag^2 << y^2$$

$$y^2 - 2R\,Sag \approx 0$$

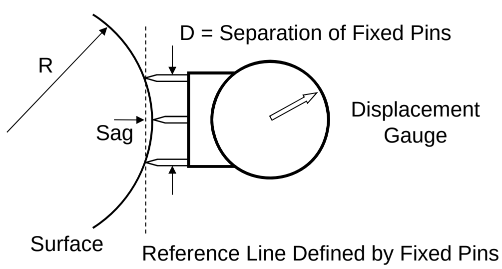
$$Sag \approx \frac{y^2}{2R}$$

This is the parabolic approximation for a circle or sphere.

Radius of Curvature Measurement

The surface sag over a fixed baseline is a common method of measuring radius of curvature in the optics shop.

The instrument is known as a Lens Clock or a Geneva Gauge. It consists of three pins that contact the surface. The outside two pins are fixed, separated by a distance D, and define a reference line. The middle pin is spring loaded and connected to a displacement gauge.



$$Sag \approx \frac{D^2}{8R}$$

$$R \approx \frac{D^2}{8\,Sag}$$

The gauge often reads in Diopters and assumes a specific index of refraction. A convex surface reads a positive power and a concave surface reads a negative power.

A spherometer is a more precise instrument for measuring Radius of Curvature. The surface is contacted with three fixed pins or a large ring. A micrometer in the center reads the sag of the surface relative to the plane defined by the three pins or the ring.

Approximation Summary for Paraxial Optics

Approximation #1: Cosine Condition

$$\cos U \approx \cos U' \qquad \text{or} \qquad \frac{\cos U'}{\cos U} \approx 1$$

This condition is met if the ray bending at a surface is small. This will occur when the incident ray is approximately perpendicular to the surface at the ray intersection. Note that this approximation does not require that the ray angles U and U' are small.

Approximation #2: $|Sag| << |R|$

This condition requires that the sag of the surface at the ray intersection is much less than the radius of curvature of the surface.

Approximation #3: $|Sag| << |z| \qquad |Sag| << |z'|$

The object and image distances are much greater than the sag of the surface at the ray intersection.

> The surface sag is ignored and paraxial refraction occurs at the surface vertex.
>
> The ray bending at each surface is small.

Approximation Summary for Paraxial Optics

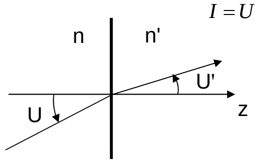
Approximation #1: Cosine Condition

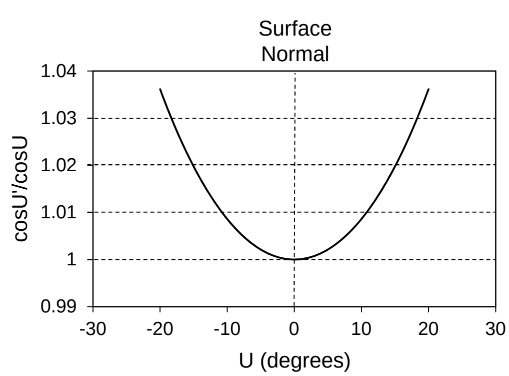
$$\cos U \approx \cos U' \qquad \text{or} \qquad \frac{\cos U'}{\cos U} \approx 1$$

Consider a surface perpendicular to the optical axis:

$n = 1.0 \quad n' = 1.5 \qquad A = 0$

$I = U$





For $|U| < 10$ deg

The error is about 1%

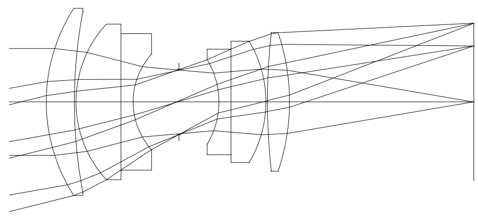
For $|U| < 20$ deg

The error is about 3.5%

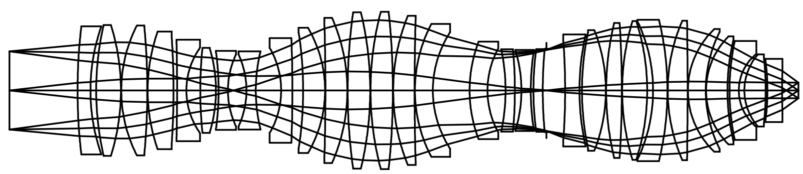
Approximation Summary for Paraxial Optics

Cosine Condition: The situation of small ray bending at each surface is common in well-design systems. The rays are gently guided through the system.

Double Gauss photographic objective with 6 elements



Lithography lens with 30 elements

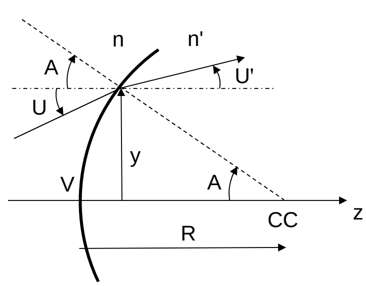


US Patent 5,835,285 – Nikon Corporation

Cosine Condition – Curved Surfaces

With curved surfaces, this approximation is more difficult to interpret as it relates to the ray angle with respect to the optical axis, not with respect to the surface normal.

Consider a surface with a radius of curvature R = 100 mm with a ray height of 10 mm. The surface normal at the ray intersection is tilted at about -5.7 degrees.



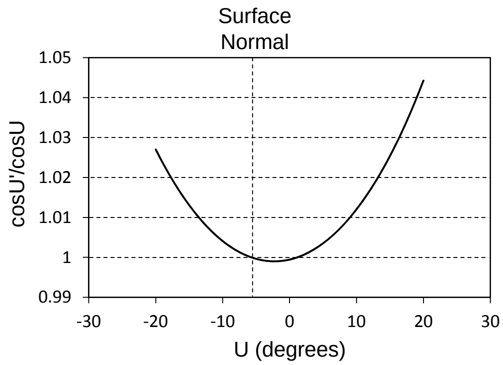
$I = U - A \qquad I' = U' - A$

A ray will be incident perpendicular to the surface when $U = A$.

Cosine Condition – Curved Surfaces

$$\cos U \approx \cos U' \qquad \text{or} \qquad \frac{\cos U'}{\cos U} \approx 1$$

Consider: $R = 100\ mm$ $\quad y = 10\ mm$ $\quad A = -5.7\ deg$ $\quad n = 1.0$ $\quad n' = 1.5$



The approximation error skews, but is not symmetric about the surface normal:

For rays incident within about 10 deg of the surface normal, the error is about 1%.

The approximation is in terms of the ray angle with respect to the optical axis, not the angle of incidence at the surface.

Cosine Condition – Curved Surfaces

$$\cos U \approx \cos U' \qquad \text{or} \qquad \frac{\cos U'}{\cos U} \approx 1$$

The curious aspect of this plot is that there are two conditions where there is no approximation error:

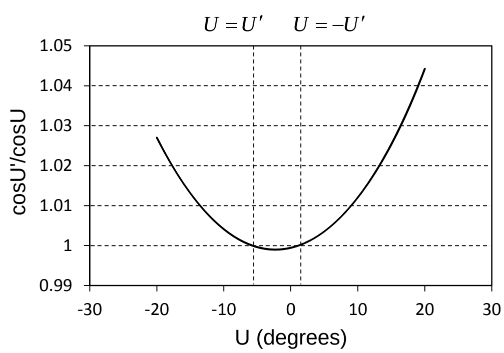
1) The ray is perpendicular to the surface (as expected).

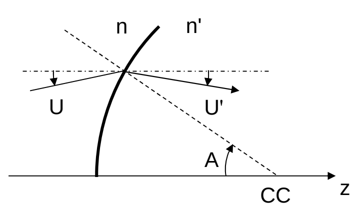
   $$U \approx U' \quad or \quad I = I' = 0$$

2) The ray angle with respect to the optical axis changes sign.

   $$U \approx -U'$$

   The cosine is an even function.





The ray is bent towards the surface normal to obtain the anti-symmetric condition with respect to the optical axis.

Approximation Summary for Paraxial Optics

Approximation #2 states that the sag of the surface at the ray intersection is much less than the radius of curvature of the surface.

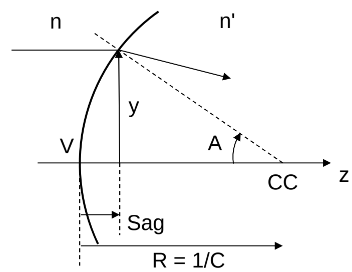
$$|Sag| << |R|$$

Approximation #3 states that the object and image distances are much greater than the sag of the surface at the ray intersection. The object and image distances for a ray are the distances to the ray crossings at the optical axis.

$$|Sag| << |z| \qquad |Sag| << |z'|$$

Consider: $R = 100\ mm$ $\quad y = 10\ mm$ $\quad n = 1.0$ $\quad n' = 1.5$

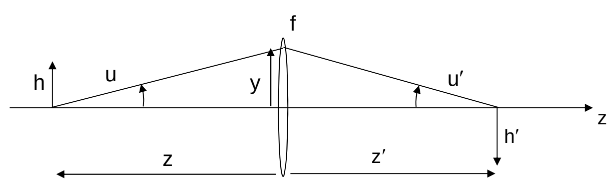
$$Sag \approx \frac{y^2}{2R} = 0.5\ mm << R$$



For an incident ray parallel to the axis:

$$z' \approx f'_R = \frac{n'}{\phi} = \frac{n'R}{(n'-n)} = 300\ mm >> Sag$$

Paraxial Raytrace – Thin Lens



$$u = \frac{y}{-z} \qquad u' = -\frac{y}{z'}$$

$$\frac{1}{z} = -\frac{u}{y} \qquad \frac{1}{z'} = -\frac{u'}{y}$$

Apply the imaging equation for a thin lens:

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$-\frac{u'}{y} = -\frac{u}{y} + \frac{1}{f}$$

$$u' = u - \frac{y}{f}$$

Let $f = f_E \equiv \frac{1}{\phi}$

$$u' = u - y\phi$$

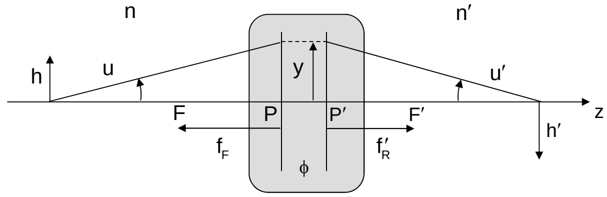
This is the paraxial raytrace equation of a refracting surface where n = n′ = 1 $\quad n'u' = nu - y\phi$

General System

The refractive properties of a general system can be used to define its system focal length f or system power ϕ.

For an arbitrary optical system, the system power ϕ or system focal length f is determined so that the system obeys the paraxial refraction equation.

The system is treated as if it were a single refracting surface where the principal planes of the system are the planes of effective refraction.

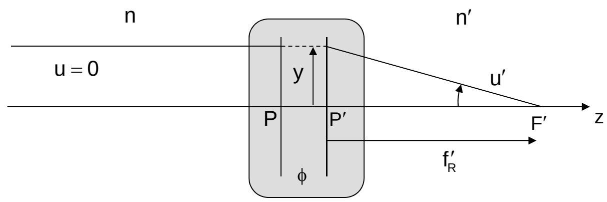


$$n'u' = nu - y\phi \qquad f = f_E \equiv \frac{1}{\phi}$$

The paraxial raytrace equation becomes the definition of the system power and focal length

Rear Focal Length of a General System

Trace a ray parallel to the axis. This corresponds to an object at infinity located on the optical axis. The conjugate ray crosses the axis at the rear focal point.



Apply the paraxial refraction equation to the system:

$$n'u' = nu - y\phi$$
$$nu = 0$$
$$n'u' = -y\phi$$
$$-\frac{y}{u'} = \frac{n'}{\phi}$$

$$u' = -\frac{y}{f'_R}$$
$$-\frac{y}{u'} = f'_R$$

Equate:

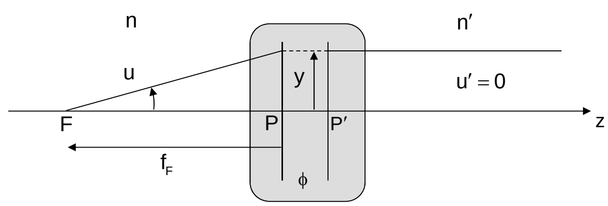
$$f'_R = \frac{n'}{\phi}$$

$$f = f_E \equiv \frac{1}{\phi} = \frac{f'_R}{n'}$$

This is exactly the same relationship as for a single refracting surface.

Front Focal Length of a General System

Trace a ray from an object at the front focal point. The conjugate ray is parallel to the optical axis in image space. This corresponds to an image at infinity located on the optical axis.



Apply the paraxial refraction equation to the system:

$$n'u' = nu - y\phi$$
$$n'u' = 0$$
$$nu = y\phi$$
$$\frac{y}{u} = \frac{n}{\phi}$$

$$u = -\frac{y}{f_F}$$
$$-\frac{y}{u} = f_F$$

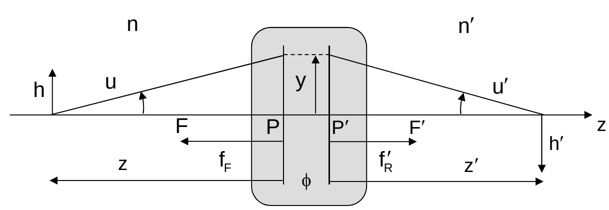
Equate:

$$f_F = -\frac{n}{\phi}$$

$$f = f_E \equiv \frac{1}{\phi} = -\frac{f_F}{n}$$

This is also exactly the same relationship as for a single refracting surface.

Paraxial Raytrace and Imaging for a General System



$$u = \frac{y}{-z} \qquad u' = -\frac{y}{z'}$$

Apply the paraxial refraction equation that defines the system power::

$$n'u' = nu - y\phi$$

$$-\frac{n'y}{z'} = -\frac{ny}{z} - y\phi$$

$$\frac{n'}{z'} = \frac{n}{z} + \phi$$

$$\boxed{\frac{n'}{z'} = \frac{n}{z} + \frac{1}{f_E}}$$

$$f = f_E \equiv \frac{1}{\phi}$$

This is the same relationship as was determined for a single refracting surface.

Focal Lengths of a General System

Repeating the results for the front and rear focal lengths:

$$f'_R = \frac{n'}{\phi} = n'f \qquad f_F = -\frac{n}{\phi} = -nf$$

$$f \equiv \frac{1}{\phi}$$

$$f = f_E \equiv \frac{1}{\phi} = \frac{f'_R}{n'} = -\frac{f_F}{n} \qquad \frac{f'_R}{f_F} = -\frac{n'}{n}$$

The focal length is the *reduced* rear focal length and minus the *reduced* front focal length. In general, the focal length is not a physical distance.

The front and rear focal lengths are physical distances. They are the directed or signed distances from the Principal Planes to the respective Focal Points.

These are exactly the same relationships between power and focal length as found for a single refracting surface, and they can be applied to any optical system.

The principal planes are the effective planes of refraction in object space and image space.

For a refractive system in air: $f \equiv \frac{1}{\phi} = -f_F = f'_R \qquad n = n' = 1$

Comments on Paraxial Optics

There is a one-to-one correspondence between object and image points in paraxial optics. This is just a way of saying that there are no aberrations in paraxial or first-order optics – it is the optics of perfect imaging systems.

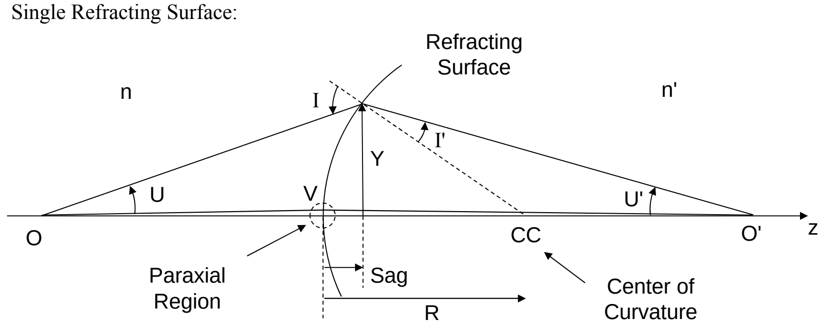
A paraxial raytrace is linear with respect to ray angles and heights since all paraxial angles are defined to be the tangent of the actual ray angle – they are actually the slope of the ray. In a paraxial analysis, these approximations (and the resulting linearity) are maintained even for large object and image heights and large ray angles.

Traditional Derivation of the Paraxial Raytrace Equation

A narrow bundle of rays in the vicinity of the optical axis is used. All of the ray heights and angles are assumed to be infinitesimal. The surface sag is ignored or is negligible. Angles of incidence and refraction are small. Small-angle Snell's law is used.

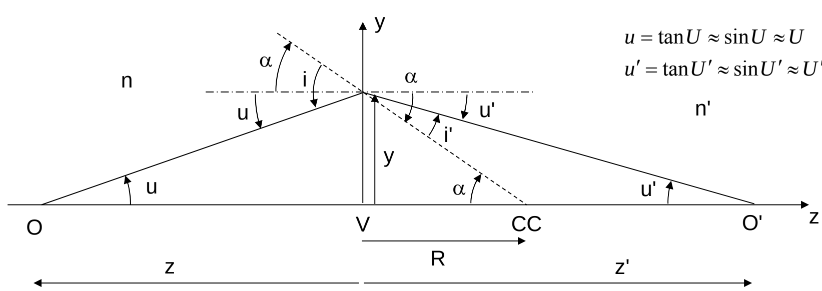
$n \sin I = n' \sin I'$ becomes $ni = n'i'$ where $i = \tan I \approx \sin I \approx I$ and $i' = \tan I' \approx \sin I' \approx I'$

Single Refracting Surface:



Refraction at a Surface

Greatly expand the vertical scale to examine the region in the vicinity of the optical axis. The scales in the horizontal and vertical directions are greatly different (anamorphic), and the sag of the surface is approximately zero and not visible. Because of the anamorphic representation, the surface normal does not appear to be perpendicular to the surface.



$u = \tan U \approx \sin U \approx U$

$u' = \tan U' \approx \sin U' \approx U'$

Paraxial Angles: $u = \dfrac{y}{-z}$ $\quad u' = -\dfrac{y}{z'}$ $\quad$ Curvature $= C = \dfrac{1}{R}$

$$\alpha = -\frac{y}{R} = -y\,C$$

$$i = u - \alpha \qquad i' = u' - \alpha$$

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Paraxial Refraction

Small Angle Snell's Law:

$$n'i' = ni$$

$$i = u - \alpha \qquad i' = u' - \alpha$$

$$n'(u' - \alpha) = n(u - \alpha)$$

$$n'u' = nu + (n' - n)\alpha$$

$$\alpha = -yC$$

$$n'u' = nu - (n' - n)yC$$

Define the Power of Surface: $\phi = (n' - n)C = \dfrac{(n' - n)}{R}$

$$n'u' = nu - y\phi$$

This is the Paraxial Refraction Equation.

The two derivations of the paraxial raytrace equation produce the same results, but the traditional derivation obfuscates and overstates the inherent approximations. In spite of the approximations, the paraxial raytrace equation is used for large ray heights and angles.