

Section 2

Mirrors and Prism Systems

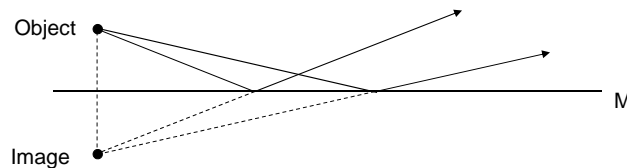


Plane Mirrors

Plane mirrors are used to:

- Produce a deviation
- Fold the optical path
- Change the image parity

Each ray from the object point obeys the law of reflection at the mirror surface. A virtual image of the object point is produced.



The rules of plane mirrors:

- The line connecting an object point and its image is perpendicular to the mirror and is bisected by the mirror.
- Any point on the mirror surface is equidistant from a given object point and its image point.
- The image parity is changed on reflection.



Parity Change on Reflection

2-3

In addition to bending or folding the light path, reflection from a plane mirror introduces a parity change in the image.

R R R R
Right Handed Parity (RH)

R R R R
Left Handed Parity (LH)

Each ray from an object obeys the law of reflection at a plane mirror surface, and a virtual image of the object is produced.

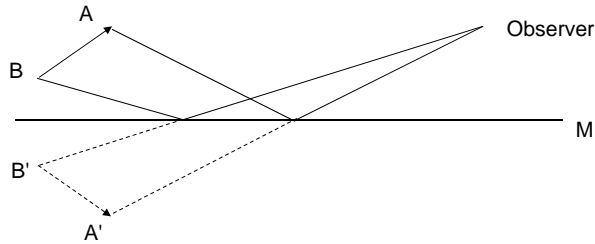


Image Parity and Orientation

2-4

Terminology:

Image Rotation – the image is rotated about a line normal to the image (optical axis). An image rotation does not produce a parity change.

Image Parity Changes

Invert – Parity change about horizontal line; vertical flip.

Revert – Parity change about a vertical line; horizontal flip.

An inversion and a reversion is equivalent to a 180° rotation.

Inversion:

R → R

Reversion:

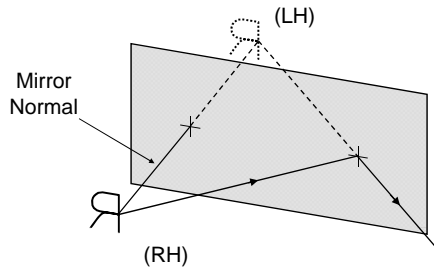
R → R

180° Rotation:

R → R

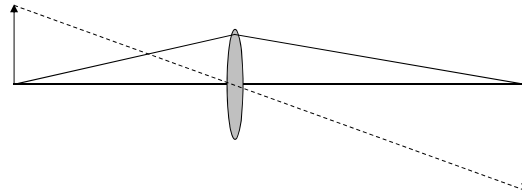
An image seen by an even number of reflections maintains its parity. An odd number of reflections changes the parity.

Parity is determined by looking back against the propagation direction towards the object or image in that optical space; let the light from the object or image come to you.



Parity and Imaging

The object and the image formed by a lens are
 flipped top and bottom
 flipped left and right



This constitutes an inversion and a reversion.

The image is rotated 180° about the optical axis.

There is no parity change.



Systems of Plane Mirrors

The rules of plane mirrors are used sequentially at each mirror in a system of plane mirrors.

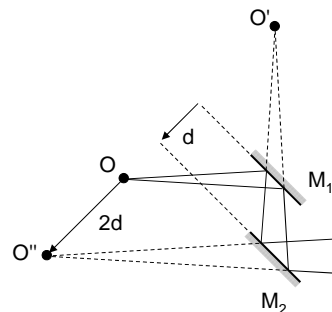
Two Parallel Mirrors

Two parallel plane mirrors act as a periscope and displace the line of sight.

There is no parity change.

All image rays are parallel to the corresponding object rays.

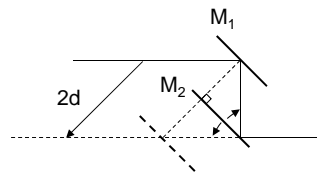
The image is displaced by twice the perpendicular separation of the mirrors d .



Proof:

Use a single ray.

Two identical triangles are formed.



Two Non-Parallel Plane Mirrors

2-7

The dihedral line is the line of intersection of two non-parallel plane mirrors. A principal section is a plane perpendicular to the dihedral line.

In a plane perpendicular to the dihedral line (a principal section), the projected ray path is deviated by twice the angle between the mirrors (the dihedral angle α).

$$\gamma = 2\alpha$$

This deviation is independent of the input angle.

$\alpha < 90^\circ$: The input and output rays cross.

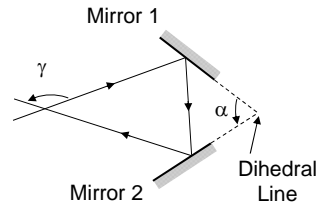
$\alpha > 90^\circ$: The input and output rays diverge.

(virtual rays cross behind the mirrors)

$\alpha = 90^\circ$: The input and output rays are anti-parallel.

When the dihedral angle is 90° , the input and output rays are anti-parallel in the principal section. This combination of two mirrors is called a roof mirror.

The projection of the ray paths into a plane containing the dihedral line shows a simple reflection at the dihedral line.



Two Non-Parallel Plane Mirrors - Derivation

2-8

Negative quantities:

$$\theta_1 \theta_1'' \theta_2 \text{ and } \delta$$

$$\theta_1 = -\theta_1'$$

$$\theta_2 = -\theta_2'$$

$$90 = \theta_1' - \theta_1''$$

$$90 = \theta_2'' - \theta_2$$

$$\theta_1'' = \theta_1' - 90$$

$$\theta_2'' = 90 + \theta_2$$

Triangle 1: $\alpha + \theta_2'' - \theta_1'' = 180$

$$\alpha + (90 + \theta_2) - (\theta_1' - 90) = 180$$

$$\theta_2 = \theta_1' - \alpha = -\theta_1 - \alpha$$

Triangle 2: $-\delta - 2\theta_1 - 2\theta_2 = 180$

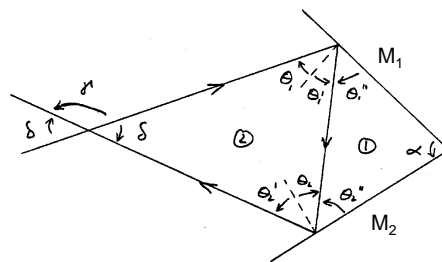
$$-\delta - 2\theta_1 - 2(-\theta_1 - \alpha) = 180$$

$$-\delta + 2\alpha = 180$$

$$-\delta = 180 - 2\alpha$$

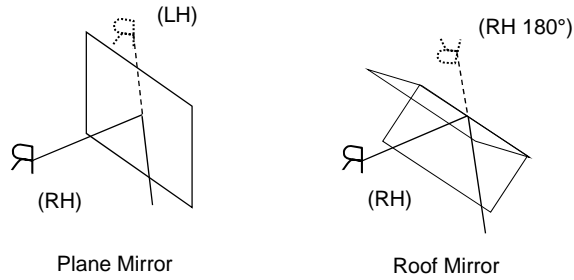
$$\gamma = 180 + \delta$$

$$\gamma = 2\alpha$$

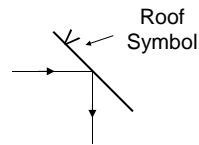


Roof Mirror

A roof mirror is two plane mirrors with a dihedral angle of 90° , and the input and output rays are anti-parallel. This roof mirror can replace any flat mirror to insert an additional reflection or parity change. It preserves parity on reflection.

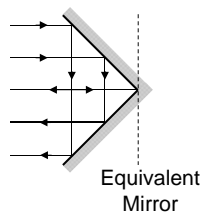


The dihedral line is often in the plane of the drawing, and the presence of a roof mirror is indicated by a "V" at the equivalent mirror or dihedral line.

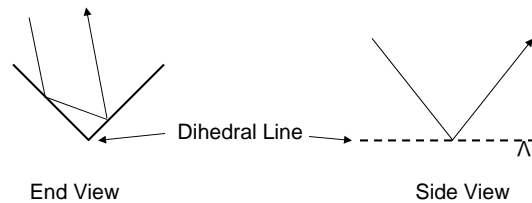


Roof Mirror

The addition of a roof mirror does not add any extra optical path. An equivalent plane mirror is formed at the dihedral line.



The projection of the ray paths into a plane containing the dihedral line shows a simple reflection at the dihedral line.



Prism Systems

2-11

Prism systems can be considered systems of plane mirrors. An excellent reference for prisms is MIL_HDBK-141 Section 13.10

If the angles of incidence allow, the reflection is due to TIR. Prisms fold the optical path and change or correct the image parity. Surfaces where TIR fails must have a reflective coating.

A tunnel diagram unfolds the optical path at each reflection through the prism and shows the total length of the path through the prism. The prism is represented as a block of glass of the same thickness. A ray propagates straight through the diagram. The tunnel diagram aids in determining Field of View (FOV), clear aperture, and vignetting. The addition of a roof mirror to the prism does not change the tunnel diagram.



Prisms are classified by the overall ray deviation angle and the number of reflections (# of R's).

90° Deviation Prisms

2-12

Right angle prism (1 R) – the actual deviation depends on the input angle and prism orientation. Image is inverted or reverted depending on prism orientation.



Amici or Roof prism (2 R) – a right angle prism with a roof mirror. The image is rotated 180°. No parity change.

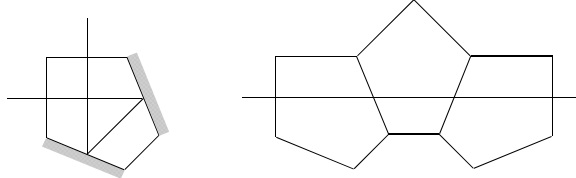


The tunnel diagram is identical to the right angle prism.

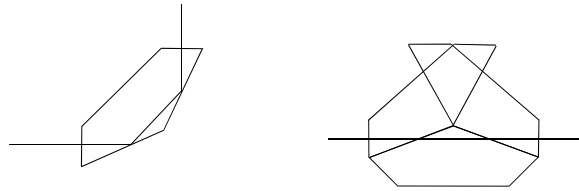
More 90° Deviation Prisms

2-13

Pentaprism (2 R) – two surfaces at 45° produce a 90° deviation independent of the input angle. It is the standard optical metrology tool for defining a right angle. The two reflecting surfaces must be coated. No parity change.



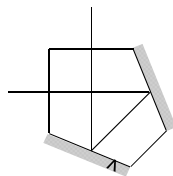
Wollaston Prism (2 R) – two surfaces at 135° produce a deviation of 270 ° (or 90°).



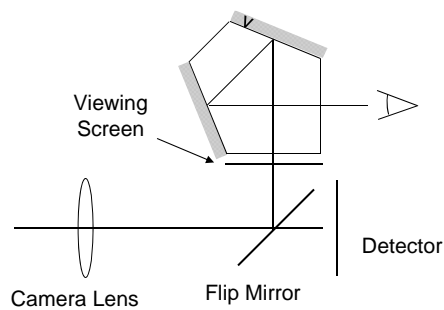
More 90° Deviation Prisms – Reflex Prism

2-14

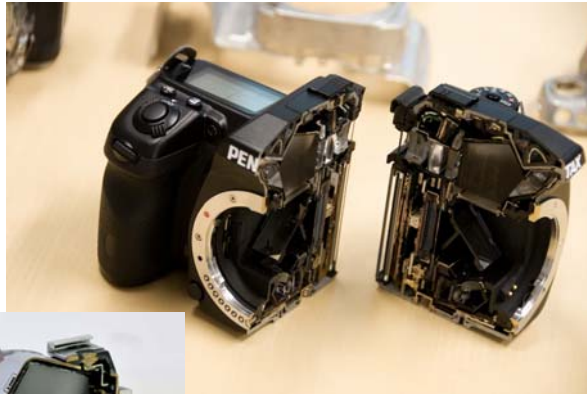
Reflex prism (3 R) – a pentaprism with an added roof mirror. Used in single lens reflex (SLR) camera viewfinders to provide an erect image of the proper parity. The roof surfaces must also be coated.



SLR Camera – the flip mirror rotates up during the exposure. The film and viewing screen planes are optically coincident.



SLR Camera



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Wikipedia

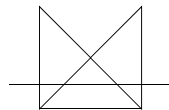
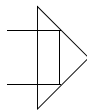


LoneTreeImages

180° Deviation Prisms

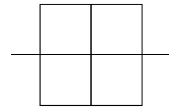
2-16

Porro prism (2 R) – a right angle prism using the hypotenuse as the entrance face. It controls the deviation in only one dimension.



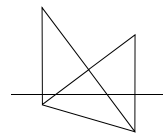
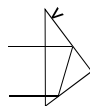
Dihedral line perpendicular to the plane of the paper

or



Dihedral line in the plane of the paper. Note that both representations of the tunnel diagram have the same length.

Corner cube (3 R) – three surfaces at 90°. The output ray of this retroreflector is truly anti-parallel to the input ray. The deviation is controlled in both directions, and light entering the corner cube returns to its source.



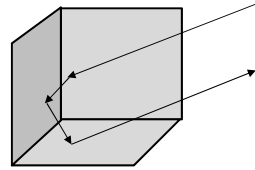
These figures appear skewed due to the compound angles needed to represent a prism face and a roof edge when all three prism faces have equal angles with the optical axis.

Retroreflectors

2-17



Three reflections are made and the light returns to the source.



Taillights and Bicycle Reflectors:



bannereengineering.com

Applications of Corner Cubes – Time-of-Flight Measurements

2-18

Surveying
Prism



Lunar Laser Ranging Retroreflector Arrays
Apollo 11 and 14: 100 prisms Apollo 15: 300 prisms



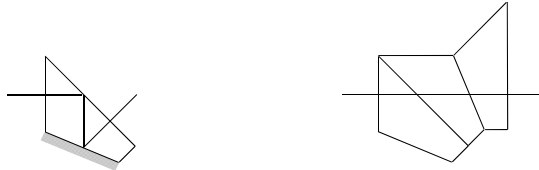
Laser Geodynamics Satellite (Orbit at 5,900 km):

The LAGEOS mission goals:

- Provide an accurate measurement of the satellite's position with respect to Earth,
- Determine the planet's shape (geoid), and
- Determine tectonic plate movements associated with continental drift.

45° Deviation Prisms

45° Prism (2 R) – half a pentaprism.



Schmidt prism (4 R) – a 3 R version without a roof also exists. No coated surfaces

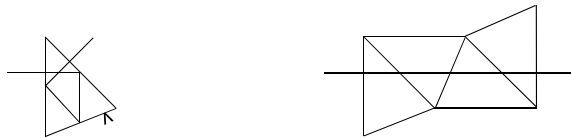


Image Rotation Prisms

Image Rotation Prisms – as the prism is rotated by θ about the optical axis, the image rotates by twice that amount (2θ). In all of these prisms, the input and output rays are co-linear (the light appears to go straight through) and there are an odd number of reflections (and therefore a parity change).

Symmetry and the parity change explains the image rotation. Each prism has an inversion direction and flip axis associated with it.

As the prism rotates relative to the object, this flip axis rotates, and the object is inverted about this axis. By symmetry, the object must rotate twice as fast (the prism at 0° and 180° must give the same output).

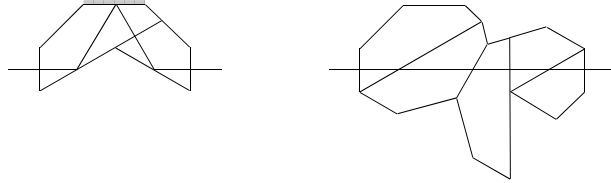
Object with Flip Axis:						
Prism with Inversion Direction:						
Rotated Image:						
Prism Angle:	0°	45°	90°	135°	180°	225°
Image Rotation Angle:	0°	90°	180°	270°	0°	90°

Image Rotation Prisms

Dove prism (1 R) – because of the tilted entrance and exit faces of the prism, it must be used in collimated light. Lateral chromatic aberration is introduced.



Reversion or K-prism (3 R) – the upper face must be coated.



Pechan prism (5 R) – a small air gap provides a TIR surface inside the prism. This compact prism supports a wide FOV. The two exterior surfaces must be coated. The Pechan prism is a combination of a 45° prism (2R) and a 45° equivalent of a non-roof Schmidt prism (3 R).

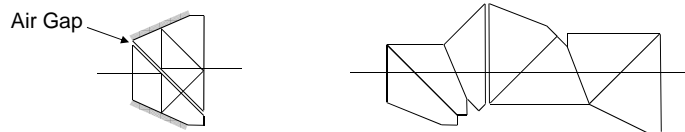
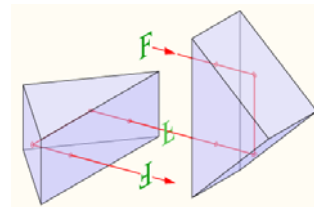
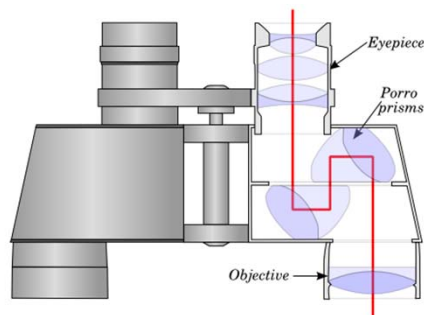


Image Erection Prisms

Image erection prisms are inserted in an optical system to provide a fixed 180° image rotation. They are commonly used in telescopes and binoculars to provide an upright image orientation. No parity change.

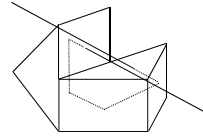


Porro Prism System

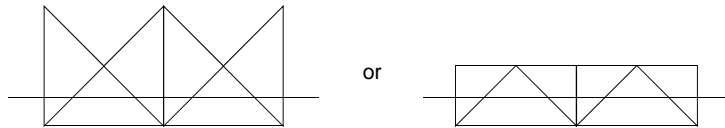
Image Erection Prisms

2-23

Porro system (4 R) – two Porro prisms. The first prism flips the image in one plane and the second prism flips the image in the other plane. This prism accounts for the displacement between the objective lenses and the eyepieces in binoculars. Invented in 1854 by Ignatio Porro – the first practical implementation was by Zeiss in 1894.



The tunnel diagram shows two Porro prisms. In the second representation, only the available entrance face is shown.



Porro-Abbe system (4 R) – a variation of the Porro system where the sequence of reflections is changed.

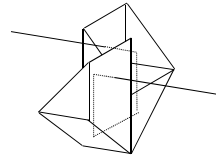
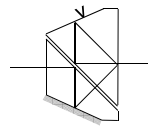


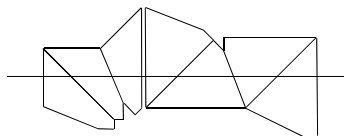
Image Erection Prisms

2-24

Pechan-roof prism (6 R) – a roof is added to a Pechan prism. This prism is used in compact binoculars and provides a straight-through line of sight. It is a combination of a 45° prism and a Schmidt prism. Note that the roof surface does not need to be coated. This prism system is also known as a Schmidt-Pechan prism as it is the combination of a 45° prism (2 R) and a Schmidt prism. It appears that this prism was first used in 1964.



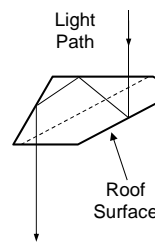
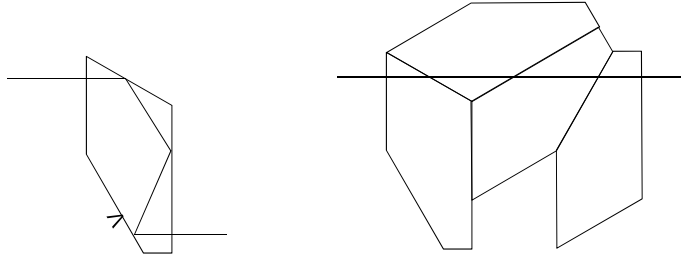
The tunnel diagram is the same as that of the Pechan prism.



This prism system is used in what are marketed as “roof-prism” binoculars.

Image Erection Prisms

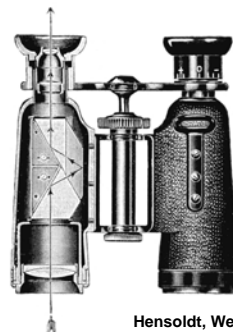
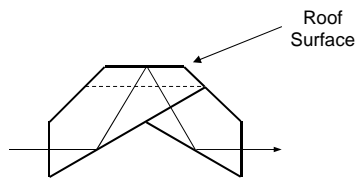
Lehman prism or Sprenger-Lehman prism (4 R) – the usable aperture is only a small portion of the entrance and exit faces.



Theatis 3½X ; J.D. Möller, Wedel, Germany, late 1920s

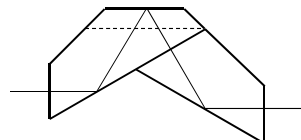
Image Erecting Prisms

Abbe-König erecting prism (4 R) – this prism system is a reversion or K-prism with a roof surface. This prism system appears in the early 1900s, and image erection is obtained without a displacement of the optical axis.



Hensoldt, Wetzlar 1905

Some asymmetry can be built into the Abbe-König prism to provide an offset of the optical axis. This is useful for large diameter objective lenses.



Abbe-König Prism Binoculars

2-27



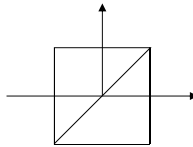
Jagd-Dialyt 7x44 S/N 13486
Hensoldt, Wetzlar

Cube Beamsplitter

2-28

A beamsplitter splits one input beam into two output beams.

The cube beamsplitter is a combination of two right angle prisms. A partially reflecting coating is applied to one hypotenuse before the two prisms are glued together. There is no TIR at the interface.



The beamsplitter usually provides a 50/50 split of the beam into the resulting two beams. The density of the coating can be varied to produce different ratios (such as 70/30).

The coating can be metallic or a dielectric (thin film) interference coating.

Coatings can also be applied so that the output beams are polarized. One beam would be vertically polarized and the other is horizontally polarized. This device is called a polarizing beamsplitter (PBS).

Prism Comments

Prisms with entrance and exit faces normal to the optical axis can be used in converging or diverging light. They will, however, introduce the same aberrations as an equivalent thickness plane parallel plate. Spherical aberration and longitudinal chromatic aberration are introduced into an on-axis beam.

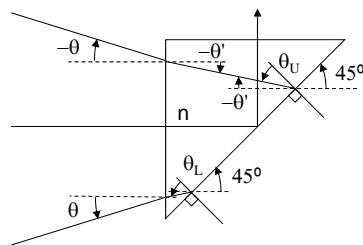
TIR often fails when prisms are used with converging or diverging beams. In laser or polarized light applications, TIR at the prism surfaces will change the polarization state of the light. In both of these situations, silvered or coated prisms must be used.

This polarization situation occurs frequently with corner cubes used with laser-based distance measuring interferometers.

TIR Limit in Prism Systems

When using a prism with a converging beam, the angle of incidence on the hypotenuse surface will vary, and rays below the critical angle can occur. TIR is then lost. In these cases, the surface must be coated with a reflective coating.

Consider a right angle prism in air used with a beam that has a convergence of $\pm\theta$:



$$\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = 41.8^\circ \quad n = 1.5$$

$$\theta_U + \theta' = 45^\circ$$

$$\theta_U = 45^\circ - \theta' < 45^\circ$$

$$\theta_L = 45^\circ + \theta' > 45^\circ$$

$$\sin \theta = n \sin \theta'$$

$$\theta > 0 \quad \theta' > 0$$

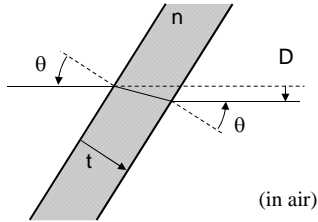
The lower ray is at an angle greater than the critical angle, and it will TIR.

As θ and θ' increase, the angle of the upper ray with respect to the surface normal of the hypotenuse will decrease. At some value of θ , the ray will lose TIR.

Plane Parallel Plate

2-31

A ray passing through a plane parallel plate is displaced but not deviated; the input and output rays are parallel.

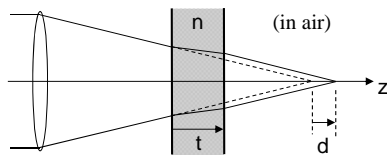


$$D = -t \sin \theta \left(1 - \sqrt{\frac{1 - \sin^2 \theta}{n^2 - \sin^2 \theta}} \right)$$

Small angle approximations:

$$D \approx -t \theta \left(\frac{n-1}{n} \right)$$

An image formed through a plane parallel plate is longitudinally displaced, but its magnification or size is unchanged.



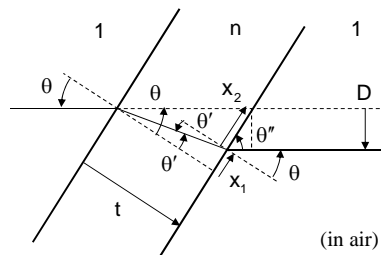
$$d = \left(\frac{n-1}{n} \right) t$$

$$d \approx \frac{t}{3} \text{ for } n = 1.5$$

Ray Displacement Derivation

2-32

$$\begin{aligned} \tan \theta'' &= \frac{x_1}{t} \\ \tan \theta &= \frac{(x_1 + x_2)}{t} \\ x_2 &= t (\tan \theta - \tan \theta') \quad \theta'' = 90 - \theta \\ \sin \theta'' &= \sin (90 - \theta) = \cos \theta = \frac{-D}{x_2} \\ D &= -x_2 \cos \theta = -t \cos \theta (\tan \theta - \tan \theta') \\ D &= -t \left(\sin \theta - \frac{\cos \theta \sin \theta'}{\cos \theta'} \right) \\ \sin \theta &= n \sin \theta' \\ D &= -t \sin \theta \left(1 - \frac{\cos \theta}{n \cos \theta'} \right) \\ \cos \theta' &= \sqrt{1 - \sin^2 \theta'} \\ \cos \theta' &= \sqrt{1 - \frac{1}{n^2} \sin^2 \theta} \\ n \cos \theta' &= \sqrt{n^2 - \sin^2 \theta} \end{aligned}$$

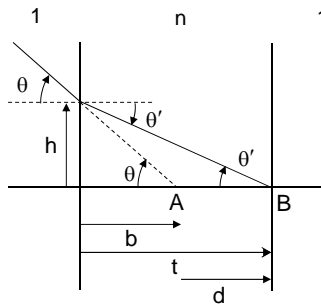


$$D = -t \sin \theta \left(1 - \sqrt{\frac{1 - \sin^2 \theta}{n^2 - \sin^2 \theta}} \right)$$

Small angle approximations:

$$D \approx -t \theta \left(1 - \frac{1}{n} \right) = -t \theta \left(\frac{n-1}{n} \right)$$

Image Displacement Derivation



A: The original image location (no plate)

B: The displaced image position

Set the problem up so that B is located at the second surface of the plate (i.e. translate the plate to put the image or ray just at its rear surface).

Use small angle approximations.

A: $\tan \theta = \frac{h}{-b} \approx \theta$

B: $\tan \theta' = \frac{h}{-t} \approx \theta'$

Refraction: $\theta \approx n\theta'$

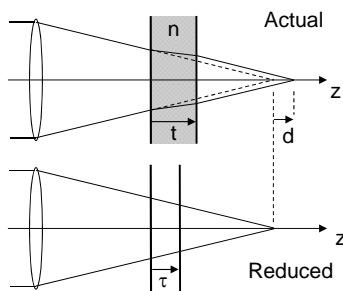
$\frac{h}{-b} = \frac{nh}{-t} \quad b = \frac{t}{n} = \tau \equiv \text{Reduced Thickness}$

Image Shift: $d = t - \tau \quad d = t - \frac{t}{n}$

$d = t \left(\frac{n-1}{n} \right)$

Reduced Thickness

The reduced thickness τ gives the air-equivalent thickness of the glass plate. A reduced diagram shows the amount of air path needed to fit the plate in the system, and no refraction is shown at the faces of the plate. Reduced diagrams can be placed directly onto system layout drawings to determine the required prism aperture sizes for a given FOV.



$t - d = t - \left(\frac{n-1}{n} \right) t = \frac{t}{n} \equiv \tau$

Reduced Thickness = $\tau = \frac{t}{n}$

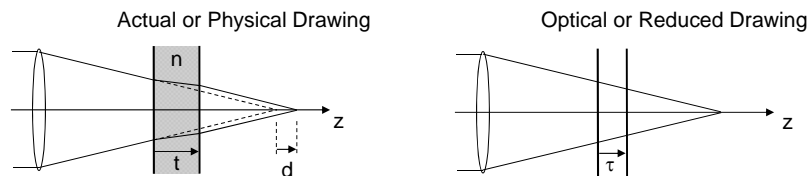
The reduced thickness is useful to determine whether a certain size plate or prism will fit into the available airspace in an optical system (between elements or between the final element and the image plane). Since the plate makes some extra room for itself by pushing back the image plane, the required space is less than the actual plate thickness.

Reduced Thickness vs. Optical Path Length

Reduced Thickness (t/n) is a geometrical concept. It relates to the amount of air space that is used when inserting a plate of index n into a system. It is the “air-equivalent thickness” of the glass plate. A reduced diagram shows the amount of air path needed to fit the plate in the system. The reduced drawing is an optical representation of the system masks the refractions at the surfaces and provides a simplified view of the system. This optical model is not a mechanical representation of the system.

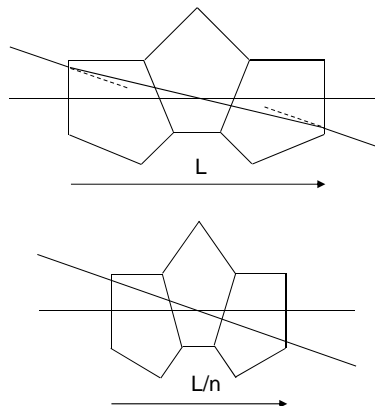
Optical Path Length (nL) is a physical property of the optical system that is proportional to the propagation time, and it can be thought of as the “air-equivalent propagation distance.” The OPL of the optical system increases greatly when a prism or glass plate is inserted in the system.

- The actual propagation distance increases by d
- The index of refraction slows down the propagation velocity



Reduced Tunnel Diagram

A reduced tunnel diagram shortens the length of a tunnel diagram by $1/n$ to show the air-equivalent length of the prism. The diagram is foreshortened only along the direction of propagation (the optical axis).

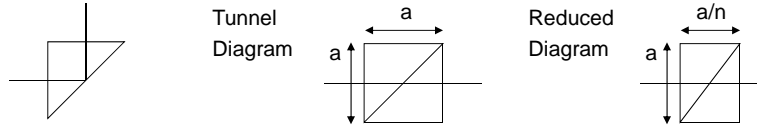


When reduced diagrams are used, no refraction is shown at the surfaces.

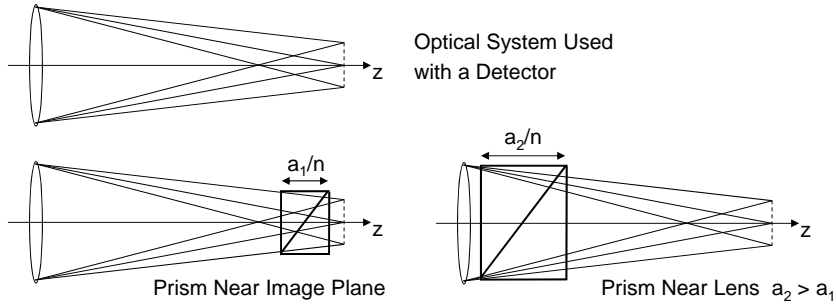
Example of the Use of the Reduced Tunnel Diagram

2-37

Determine the required size of a right angle prism that is to be placed behind a lens.



The reduced tunnel diagram can be overlaid scaled on the system drawing to determine the required prism size a for different prism locations. The refractions at the surfaces are not shown in these optical representations.

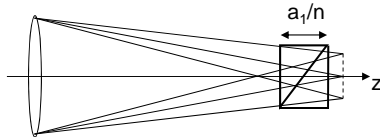


Remember that these are optical representations and the unfolded mechanical drawing would use the tunnel diagram and would show the refractions at the surface.

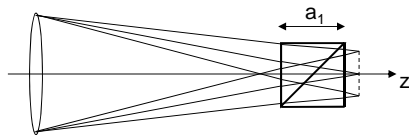
Optical Drawing and the Mechanical Drawing

2-38

Optical drawing with the reduced tunnel diagram:



Mechanical drawing of the unfolded system with the tunnel diagram – refractions are included:



The full mechanical drawing of the system can be obtained by folding the light paths and showing the reflections from the prism:

