Section 9

Paraxial Raytracing

YNU Raytrace

Refraction (or reflection) occurs at an interface between two optical spaces. The transfer distance t' allows the ray height y' to be determined at any plane within an optical space (including virtual segments).

$$\omega = nu \qquad \phi = (n' - n)C \qquad \tau' = \frac{t'}{n'}$$

Refraction or Reflection: $n'u' = nu - y\phi$ $\qquad \omega' = \omega - y\phi$

Transfer: $y' = y + u't'$ $\qquad y' = y + \omega'\tau'$

This type of raytrace is called a YNU raytrace. All rays propagate from object space to image space.

A reverse raytrace allows the ray properties to be determined in the optical space upstream of a known ray segment. A ray can then be worked back to its origins in object space.

Refraction or Reflection (reverse): $nu = n'u' + y\phi$ $\qquad \omega = \omega' + y\phi$

Transfer (reverse): $y = y' - u't'$ $\qquad y = y' - \omega'\tau'$

Paraxial Raytrace Equations - System

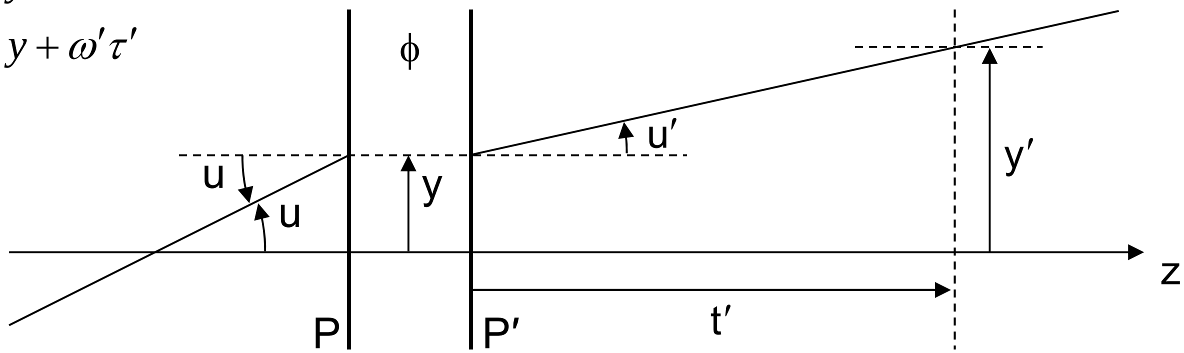
Paraxial refraction equation: $n'u' = nu - y\phi$ $\qquad \phi \equiv \frac{1}{f_E}$

$\omega' = \omega - y\phi$

Refraction at an optical system effectively occurs at the principal planes of the system. The ray emerges from the rear principal plane at the same height, but with a different angle.

Transfer: $y' = y + u't'$

$y' = y + \omega'\tau'$



The transfer distance *t'* allows the ray height *y'* to be determined at any plane within an optical space (including virtual segments).

Paraxial refraction occurs at the vertex plane of a surface. The surface sag is ignored.

For a system represented by a power and a pair of principal planes, paraxial refraction occurs at the principal planes.

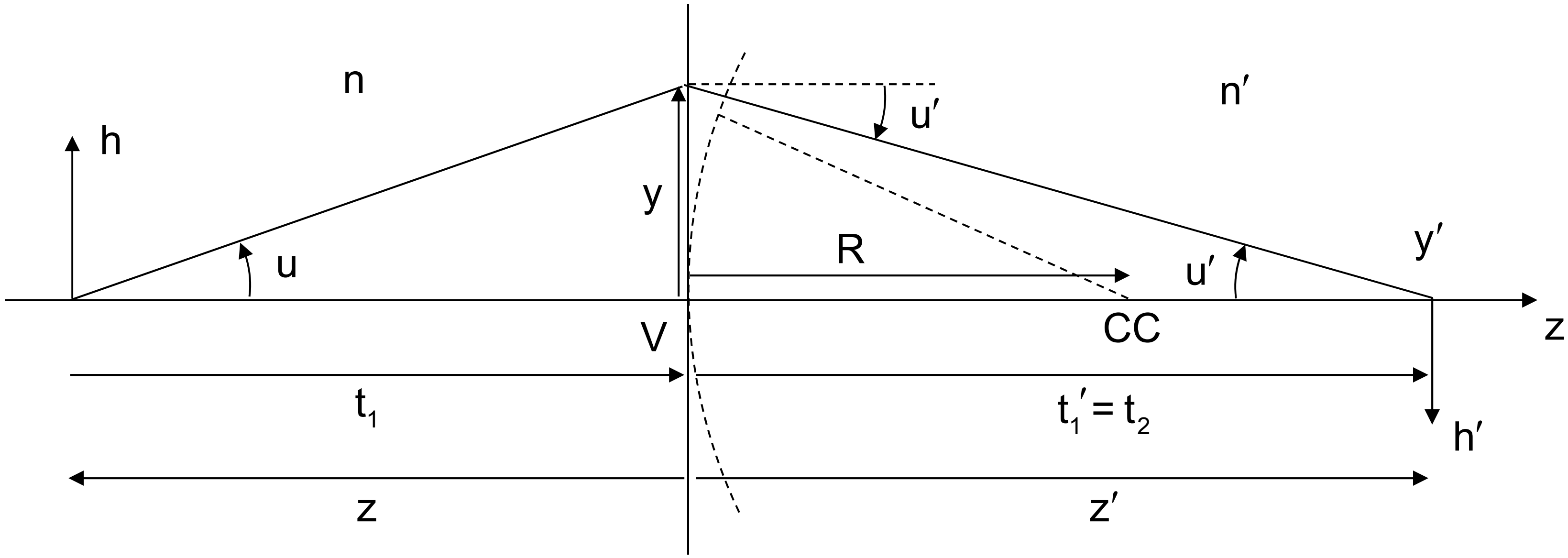
Paraxial Raytrace – Single Surface

Paraxial refraction occurs at the vertex plane of the surface.

The surface sag is ignored.

The image location is found by solving for a ray height of zero.

$$n'u' = nu - y\phi$$
$$y' = y + u't'$$



$$\phi = \frac{(n'-n)}{R}$$

$$y = ut_1$$
$$n'u' = nu - y\phi$$

$$y' = 0 = y + u't_1'$$
$$t_2 = t_1' = -\frac{y}{u'}$$

$$\frac{n'}{z'} = \frac{n}{z} + \phi$$

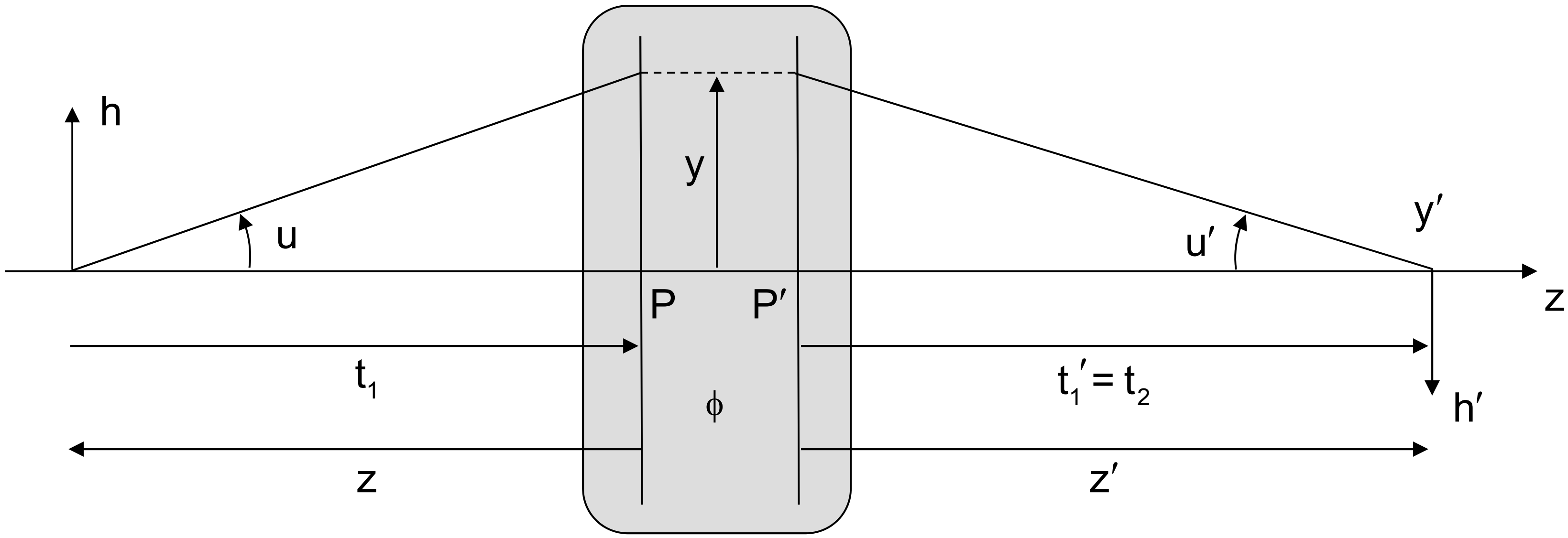
$$m = \frac{h'}{h} = \frac{z'/n'}{z/n} = -\frac{t_2/n'}{t_1/n} = \frac{nu}{n'u'}$$

A raytrace spacing is the distance from the current surface to the next surface.

Paraxial Raytrace – Single Component (in air)

The principal planes are the locations of effective refraction.

$$u' = u - y\phi$$
$$y' = y + u't'$$



$$y = ut_1$$
$$u' = u - y\phi$$

$$y' = y + u't_1' = 0$$
$$t_2 = t_1' = -\frac{y}{u'}$$

$$\frac{1}{z'} = \frac{1}{z} + \phi$$

$$m = \frac{h'}{h} = \frac{z'}{z} = -\frac{t_2}{t_1} = \frac{u}{u'}$$

These relationships also apply to a thin lens in air.

General Raytrace Equations

Transfer: $y_{j+1} = y_j + u'_j t'_j$ $\qquad y_{j+1} = y_j + \omega'_j \tau'_j$

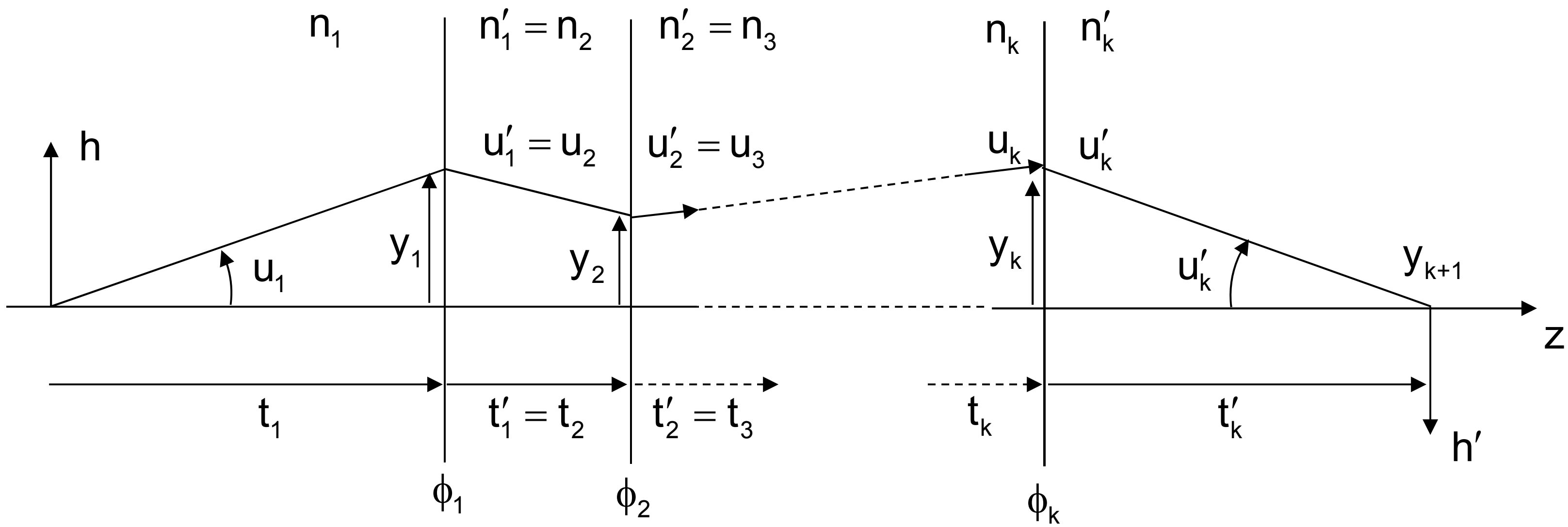
$$u_{j+1} = u'_j$$

Refract: $n'_j u'_j = n_j u_j - y_j \phi_j$ $\qquad \omega'_j = \omega_j - y_j \phi_j$

Refraction occurs at each surface. The amount of ray deviation depends on the surface power and the ray height.

Transfer occurs between surfaces. The ray height change depends on the ray angle and the spacing between surfaces.

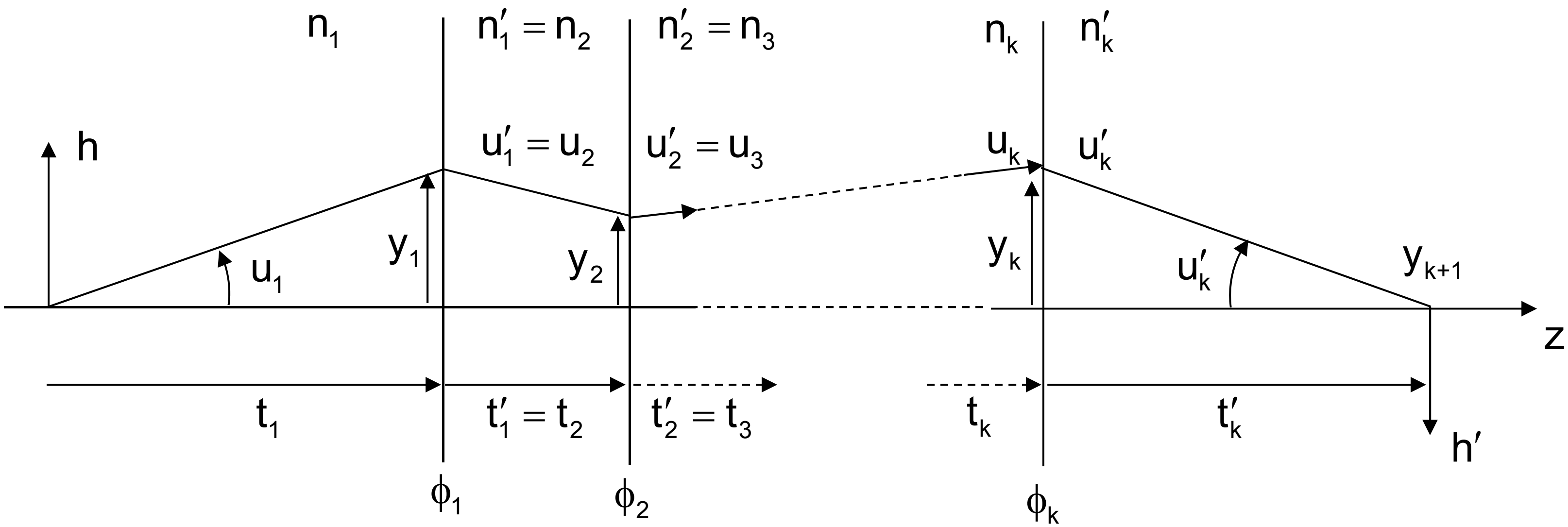
The image location is found by solving for a ray height of zero in image space.



Paraxial Raytrace – Series of Surfaces

Transfer: $y_{j+1} = y_j + u'_j t'_j$

Refract: $n'_j u'_j = n_j u_j - y_j \phi_j$



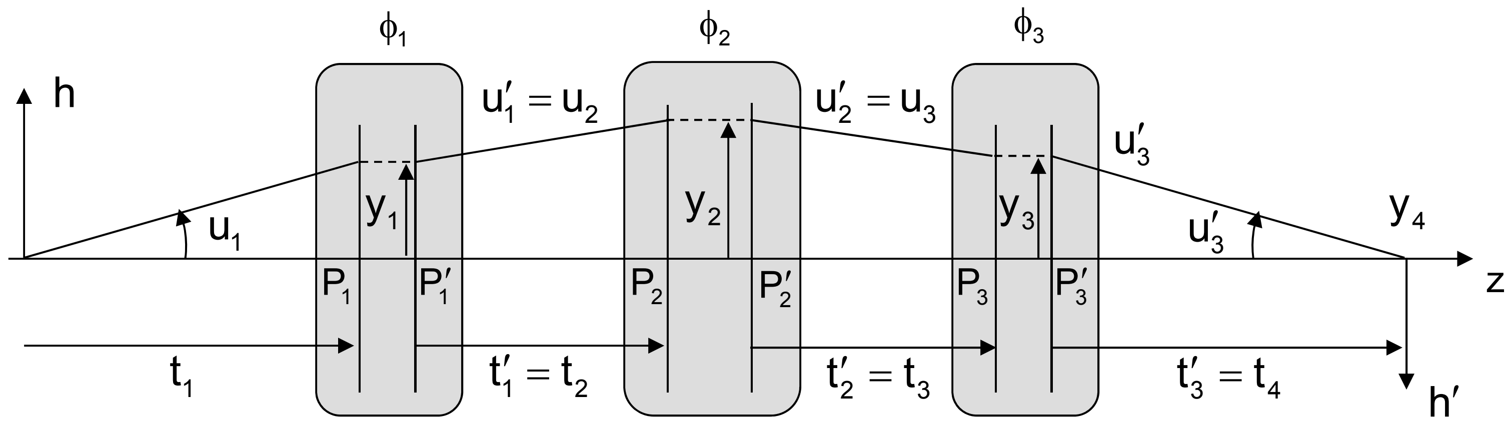
$y_1 = u_1 t_1$ $\qquad y_2 = y_1 + u'_1 t'_1$ $\qquad y_k = y_{k-1} + u'_{k-1} t'_{k-1}$ $\qquad y_{k+1} = y_k + u'_k t'_k$

$n'_1 u'_1 = n_1 u_1 - y_1 \phi_1$ $\qquad n'_2 u'_2 = n_2 u_2 - y_2 \phi_2$ $\qquad n'_k u'_k = n_k u_k - y_k \phi_k$

$n_2 u_2 = n'_1 u'_1$ $\qquad n_3 u_3 = n'_2 u'_2$

Image Location and Magnification: $y_{k+1} = 0 \longrightarrow t'_k = -\dfrac{y_k}{u'_k}$ $\qquad m = \dfrac{h'}{h} = \dfrac{n_1 u_1}{n'_k u'_k} = \dfrac{\omega_1}{\omega'_k}$

Paraxial Raytrace – Series of Components (in air)



The general raytrace equations hold (in air):

$$y_{j+1} = y_j + u_j' t_j'$$

$$u_{j+1} = u_j'$$

$$u_j' = u_j - y_j \phi_j$$

Each element or component refracts the ray, and the principal planes are the locations of effective refraction.

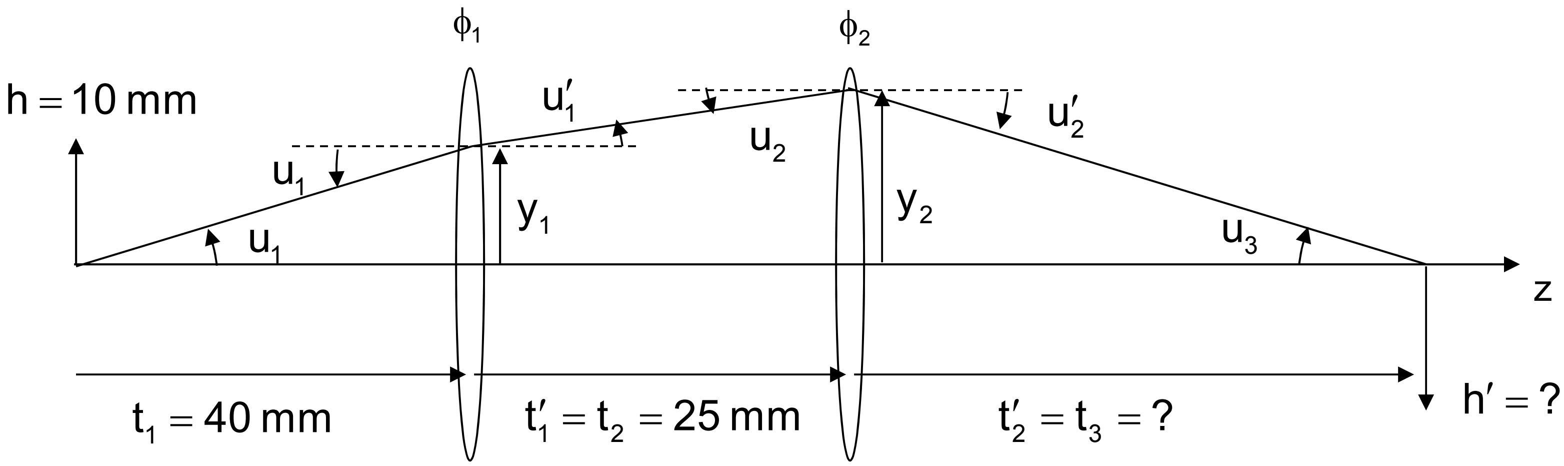
Transfer occurs between the rear principal plane of one component and the front principal plane of the next.

Image location and magnification: $y_4 = 0 \longrightarrow t_4 = t_3' = -\dfrac{y_3}{u_3'}$ $\quad m = \dfrac{h'}{h} = \dfrac{u_1}{u_3'}$

Raytrace Example – Two Separated Thin Lenses in Air

Two 50 mm focal length lenses are separated by 25 mm.
A 10 mm high object is 40 mm to the left of the first lens.

$\phi_1 = \phi_2 = 0.02\ \text{mm}^{-1}$



$$y_0 = 0$$
$$u_1 = 0.1\,(\text{Arbitrary})$$
$$y_1 = u_1 t_1 = 4.0\ \text{mm}$$

$$u_1' = u_1 - y_1\phi_1$$
$$u_1' = 0.02$$

$$y_2 = y_1 + u_1' t_1'$$
$$y_2 = 4.5\ \text{mm}$$

$$u_2 = u_1' = 0.02$$
$$u_2' = u_2 - y_2\phi_2$$
$$u_2' = -0.07$$

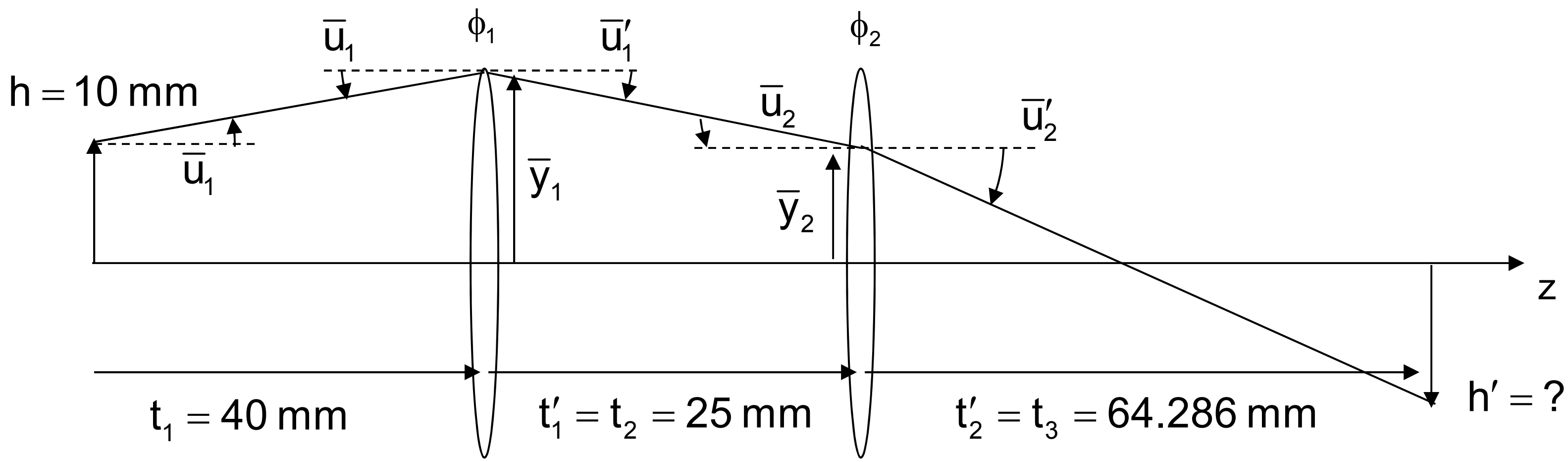
$$y_3 = y_2 + u_2' t_2' = 0$$
$$t_2' = t_3 = 64.286\ \text{mm}$$

$$u_3 = u_2' = -0.07$$
$$m = \frac{h'}{h} = \frac{u_1}{u_3} = \frac{0.1}{-0.07} = -1.429$$
$$h' = -14.29\ \text{mm}$$

Raytrace Example (Continued)– Two Separated Thin Lenses in Air

A second ray can be traced to determine the image size.

$\phi_1 = \phi_2 = 0.02\ \text{mm}^{-1}$



$\bar{y}_0 = h = 10.0$

$\bar{u}_1 = 0.1$ (Arbitrary)

$\bar{y}_1 = \bar{y}_0 + \bar{u}_1 t_1 = 14.0\ \text{mm}$

$\bar{y}_2 = \bar{y}_1 + \bar{u}_1' t_1'$

$\bar{y}_2 = 9.5\ \text{mm}$

$\bar{y}_3 = \bar{y}_2 + \bar{u}_2' t_2'$

$h' = \bar{y}_3 = -14.29\ \text{mm}$

$\bar{u}_1' = \bar{u}_1 - \bar{y}_1 \phi_1$

$\bar{u}_1' = -0.18$

$\bar{u}_2 = \bar{u}_1' = -0.18$

$\bar{u}_2' = \bar{u}_2 - \bar{y}_2 \phi_2$

$\bar{u}_2' = -0.37$

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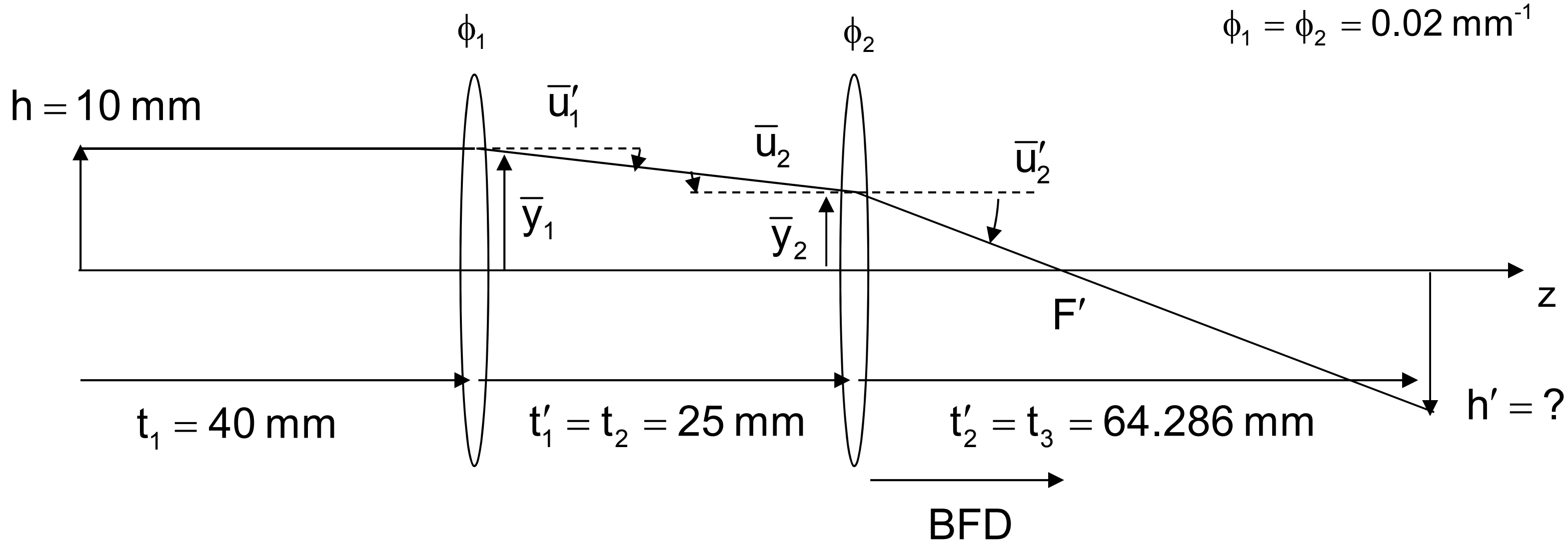
Raytrace Example (Continued)– Two Separated Thin Lenses in Air

If the arbitrary initial angle of the second ray is chosen to be zero, the location of the rear focal point of the system can also be determined.



$\bar{y}_0 = h = 10.0$

$\bar{u}_1 = 0$

$\bar{y}_1 = \bar{y}_0 + \bar{u}_1 t_1 = 10.0\ \text{mm}$

$\bar{y}_2 = \bar{y}_1 + \bar{u}_1' t_1'$

$\bar{y}_2 = 5.0\ \text{mm}$

$\bar{y}_3 = \bar{y}_2 + \bar{u}_2' t_2'$

$h' = \bar{y}_3 = -14.29\ \text{mm}$

$\bar{u}_1' = \bar{u}_1 - \bar{y}_1 \phi_1$

$\bar{u}_1' = -0.2$

$\bar{u}_2 = \bar{u}_1' = -0.2$

$\bar{u}_2' = \bar{u}_2 - \bar{y}_2 \phi_2$

$\bar{u}_2' = -0.3$

BFD (transfer to $\bar{y}_3 = 0$):

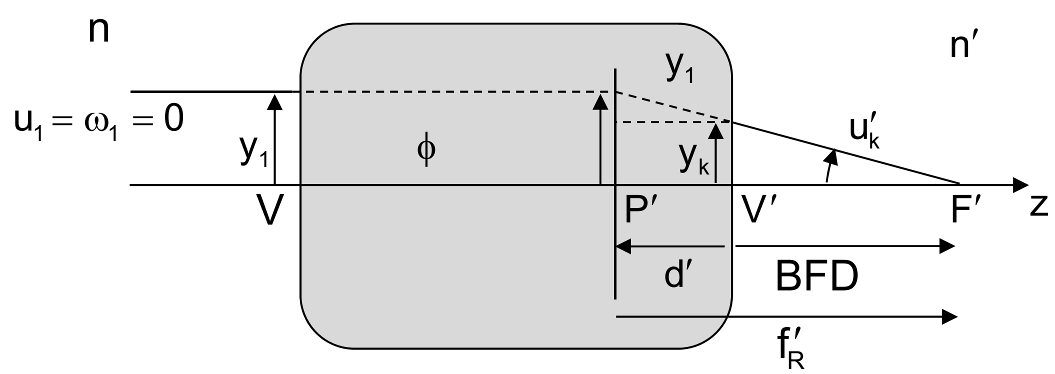
$\bar{y}_3 = \bar{y}_2 + \bar{u}_2'\, BFD = 0$

$BFD = 16.67\ \text{mm}$

Cardinal Points from a Raytrace – Rear Points

The Gaussian properties of an optical system can be determined using a paraxial raytrace with particular rays.

Rear cardinal points: Trace a ray parallel to the axis in object space. This ray must go through the rear focal point of the system. The k[th] surface is the final surface in the system.



System: $\omega'_k = \omega_1 - y_1\phi$

$\omega'_k = -y_1\phi \qquad \omega'_k = n'u'_k$

$$\phi = -\frac{\omega'_k}{y_1} = -\frac{n'u'_k}{y_1} \qquad f_E = \frac{1}{\phi} \qquad f'_R = \frac{n'}{\phi}$$

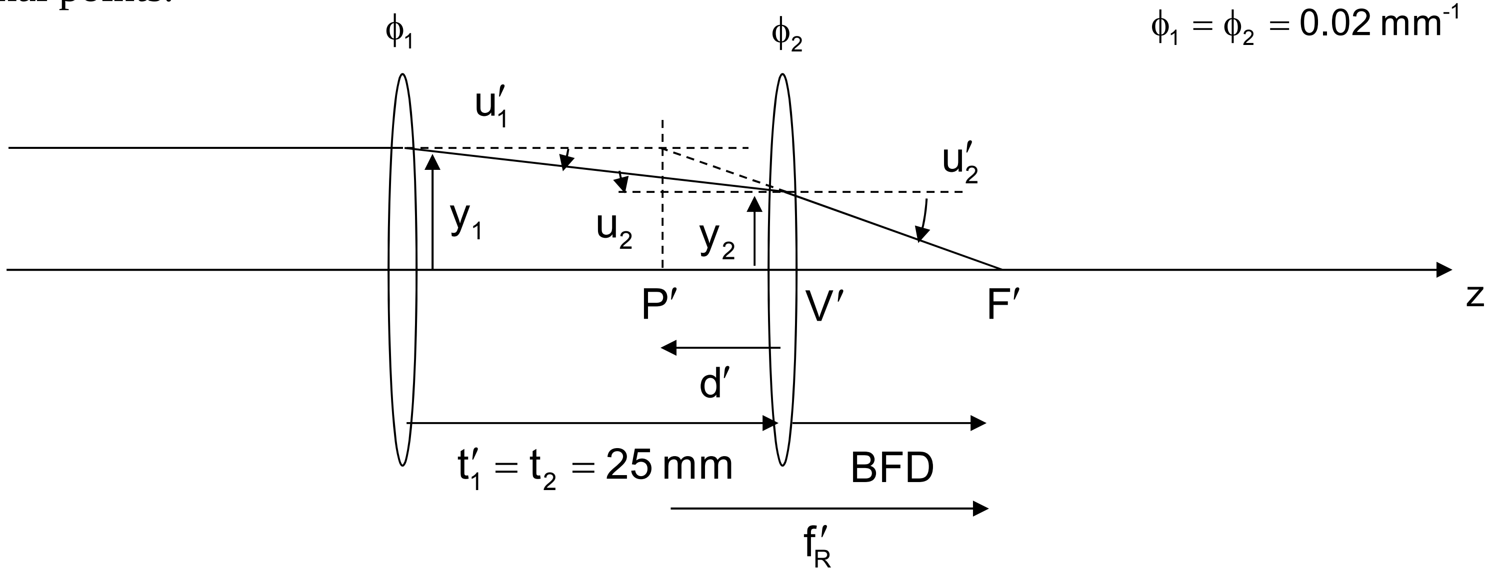
$$BFD = \overline{V'F'} = -\frac{y_k}{u'_k} = -\frac{n'y_k}{\omega'_k} \qquad d' = \overline{V'P'} = \frac{y_1 - y_k}{u'_k} \qquad d' = BFD - f'_R$$

As transfers: $F':\quad y_{k+1} = 0 = y_k + u'_k BFD \qquad P':\quad y_{k+1} = y_1 = y_k + u'_k d'$

Raytrace Example (Continued)– Two Separated Thin Lenses in Air

A ray from an axial object at infinity can be used to determine the rear cardinal points.

$\phi_1 = \phi_2 = 0.02\ \text{mm}^{-1}$



$y_1 = 1.0$ $\quad u_1' = u_1 - y_1\phi_1$ $\quad y_2 = y_1 + u_1' t_1'$ $\quad u_2 = u_1' = -0.02$ $\quad y_3 = y_2 + u_2' \, BFD = 0$

$u_1 = 0$ $\quad u_1' = -0.02$ $\quad y_2 = 0.5 \text{ mm}$ $\quad u_2' = u_2 - y_2\phi_2$ $\quad BFD = -\dfrac{y_2}{u_2'} = 16.666 \text{ mm}$

$u_2' = -0.03$

$$\phi = -\frac{\omega_2'}{y_1} = -\frac{n'u_2'}{y_1} = -\frac{u_2'}{y_1} = 0.03\ mm^{-1}$$

$$f = f_R' = \frac{1}{\phi} = 33.333\ mm$$

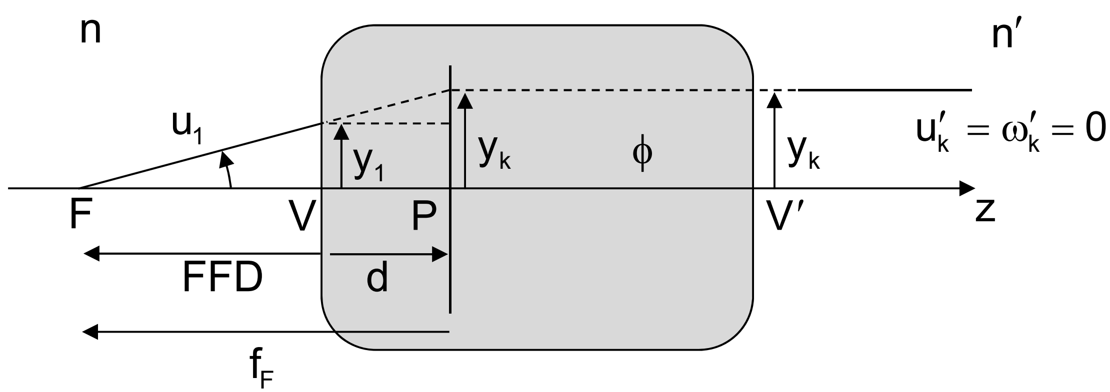
$$d' = \overline{V'P'} = \frac{y_1 - y_2}{u_2'} = -16.666\ mm$$

or

$$d' = BFD - f_R' = BFD - f = -16.666\ mm$$

Cardinal Points from a Raytrace – Front Points

Trace a ray from the system front focal point that emerges parallel to the axis in image space. The reverse raytrace equations are used to work from image space back to object space.



System: $\omega_k' = \omega_1 - y_k \phi = 0$

$$\phi = \frac{\omega_1}{y_k} = \frac{nu_1}{y_k} \qquad f_E = \frac{1}{\phi} \qquad f_F = -\frac{n}{\phi}$$

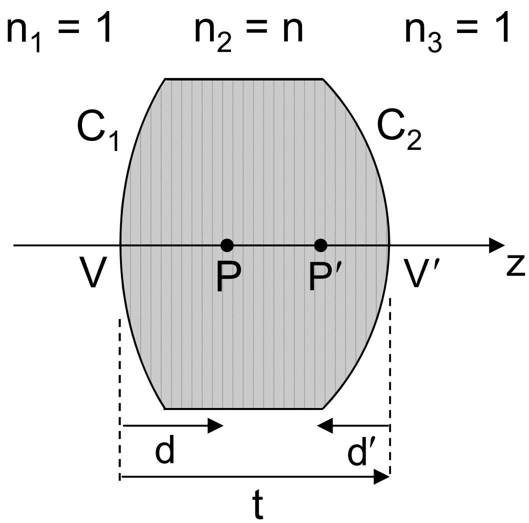
$$FFD = \overline{VF} = -\frac{y_1}{u_1} = -\frac{ny_1}{\omega_1} \qquad d = \overline{VP} = \frac{y_k - y_1}{u_1} \qquad d = FFD - f_F$$

As transfers: $F: \quad y_1 = y_0 + u_1(-FFD) = -u_1 FFD \qquad P: \quad y_k = y_1 + u_1 d$

$$y_0 = 0$$

Example – Thick Lens in Air

$C_1 = 0.02/\text{mm}$ $R_1 = 50$ mm
$C_2 = -0.01/\text{mm}$ $R_2 = -100$ mm
$t = 10$ mm
$n = 1.5$



From Gaussian optics (for comparison):

$\phi_1 = 0.01/\text{mm}$
$\phi_2 = 0.005/\text{mm}$

$\phi = 0.01467/\text{mm}$
$f_E = 68.16$ mm
$f_F = -68.16\,\text{mm}$
$f'_R = 68.16\,\text{mm}$

$d = 2.27$ mm
$d' = -4.54$ mm
$\overline{PP'} = 3.19\,\text{mm}$

There are several different spreadsheet forms that can be used to facilitate the raytrace.

Raytrace Example – Forward Ray

	Object Surface	Space 1	Surface 1	Space 2	Surface 2	Space 3	Image Surface
C t n							
-ϕ t/n							
y nu u							
y nu u							

Raytrace Example – Forward Ray

First, trace a ray parallel to the axis in object space to determine the rear focal point and rear principal plane.

F'

	Object Surface	Space 1	Surface 1	Space 2	Surface 2	Space 3	Image Surface
C			0.02		-0.01		
t		∞		10		?	
n		1.0		1.5		1.0	
-φ			-.01		-.005		
t/n		∞		6.667		63.63	
y	Ray parallel to axis		1		.9333		0
nu		0		-.01		-.01467	
u		0				-.01467	
y							
nu							
u							

Solve to obtain y = 0 at F'

y_1 arbitrarily chosen to equal 1

$$\omega' = \omega - y\phi$$

$$y' = y + \omega'\tau'$$

$$-.01467\tau_3 + .9333 = 0$$

Raytrace Example – From the Trace of the Forward Ray

$$\frac{\overline{V'F'}}{n_3} = \overline{V'F'} = 63.63\ mm$$

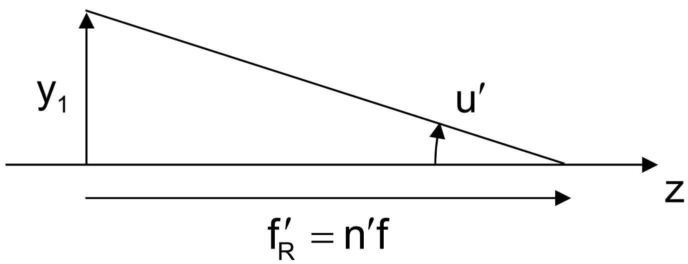
$$\omega' = u' = -.01467$$

$$y_1 = 1$$

$$\phi = -\frac{\omega'}{y_1} = .01467 / mm$$

$$f_E = 68.16\ mm$$

$$f'_R = 68.16\, mm$$



$$u' = -\frac{y_1}{f'_R} = -\frac{y_1}{n'f} = -\frac{y_1 \phi}{n'}$$

$$\phi = -\frac{n'u'}{y_1} = -\frac{\omega'}{y_1}$$

$$BFD = \overline{V'F'} = 63.63\ mm$$

$$d' = BFD - f'_R = -4.54\ mm$$

Raytrace Example – Front Properties

Now, trace a ray from the front focal point that emerges parallel to the axis in image space to determine the front focal point and front principal plane.

	F Object Surface	Space 1	Surface 1	Space 2	Surface 2	Space 3	Image Surface	
C			0.02		-0.01			
t		?		10		∞		
n		1.0		1.5		1.0		
-ϕ			-.01		-.005			
t/n		$\overline{FV}$		6.667		∞		
y	0		b		1		1	Ray parallel to axis
nu		a		c		0		
u								

$$a\,\overline{FV} = b$$

$$-.01b + a = c$$

$$6.667c + b = 1$$

$$-.005 + c = 0 \longrightarrow c = .005 \;>\; b = .9667 \;>\; a = .01467 \;>\; \overline{FV} = 65.89$$

Raytrace Example – Reverse Ray

Use the reverse raytrace equations.

	F Object Surface	Space 1	Surface 1	Space 2	Surface 2	Space 3	Image Surface
C			0.02		-0.01		
t		?		10		∞	
n		1.0		1.5		1.0	
-ϕ			-.01		-.005		
t/n		65.89		6.667		∞	
y	0		.9667		1		1
nu		.01467		.005		0	
u		.01467					
y							
nu							
u							

Ray parallel to axis

$$\omega = \omega' + y\phi$$

$$y = y' - \omega'\tau'$$

Solve for τ_1: $.9667 - .01467\,\tau_1 = 0$

Raytrace Example – From the Trace of the Reverse Ray

$$\frac{\overline{FV}}{n_1} = \overline{FV} = 65.89\ mm$$

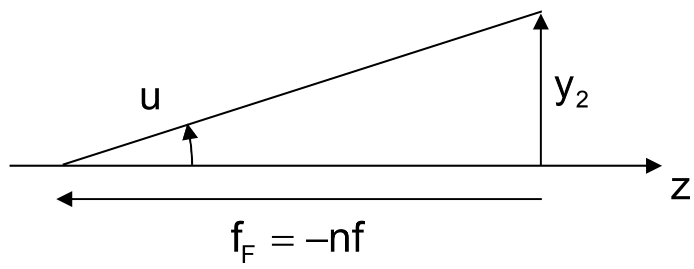
$$\omega = u = .01467$$

$$y_2 = 1$$

$$\phi = \frac{\omega}{y_2} = .01467 / mm$$

$$f_E = 68.16\ mm$$

$$f_F = -68.16\,mm$$



$$u = \frac{y_2}{-f_F} = \frac{y_2}{nf} = \frac{y_2\phi}{n}$$

$$\phi = \frac{nu}{y_2} = \frac{\omega}{y_2}$$

$$FFD = \overline{VF} = -\overline{FV} = -65.89\ mm$$

$$d = FFD - f_F = 2.27\ mm$$

$$\overline{PP'} = t - d + d' = 10.0 - 2.27 - 4.54 = 3.19\,mm$$

Raytrace Example – For a Finite Object

$h = 1 \qquad \overline{OV} = 200 \qquad s = -\overline{OV} = -200$

	Object Surface	Space 1	Surface 1	Space 2	Surface 2	Space 3	Image Surface	
C			0.02		-0.01			
t		200		10		?		
n		1.0		1.5		1.0		
-φ			-.01		-.005		Solve	
t/n		200		6.667		98.30		
y	0		20		19.33		0	Image Location
nu		.1*		-.1		-.1966		
u		.1				-.1966		
y	1		1		.9333		-.51	Image Size
nu		0*		-.01		-.01467		
u		0				-.01467		

* arbitrary

$h' = -.51 mm$

$s' = \overline{V'I'} = 98.30\, mm$

$m = \frac{-.51}{1} = -.51 \qquad m = \frac{nu}{n'u'} = \frac{.1}{-.1966} = -.51$

Gaussian check: $d = 2.27\, mm \qquad d' = -4.54\, mm$

$\phi = .01467 \qquad f_E = 68.16\, mm$

$z = s - d = -202.27\, mm \qquad z' = 102.8\, mm$

$s' = \overline{V'I'} = z' + d' = 98.3\, mm$

YNU Raytrace Form

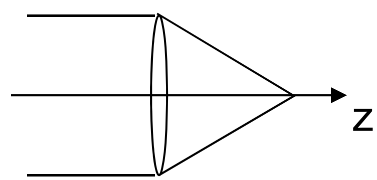
Surface	0	1	2	3	4	5	6	7
C								
t								
n								
-ϕ								
t/n								
y								
nu								
u								
y								
nu								
u								
y								
nu								
u								

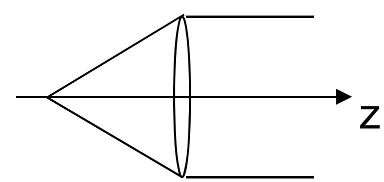
Raytrace Example – YNU Raytrace Form

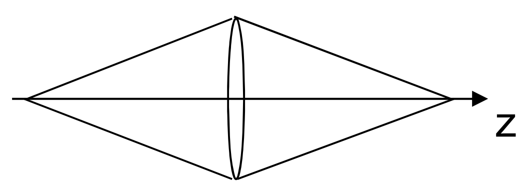
Surface	0		1		2		3
C			.02		-.01		
t		∞/?/200		10		?/∞/?	
n		1.0		1.5		1.0	
-ϕ			-.01		-.005		
t/n		∞/?/200		6.667		?/∞/?	
							63.63
y	1		1		.9333		0
nu		0		-.01		-.01467	
u		0				-.01467	
		65.89					
y	0		.9667		1		1
nu		.01467		.005		0	
u		.01467				0	
							98.30
y	0		20		19.33		0
nu		.1		-.1		-.1966	
u		.1				-.1966	

Curvatures, powers and ray heights are associated with optical surfaces.

Thicknesses, indices and angles are associated with optical spaces.







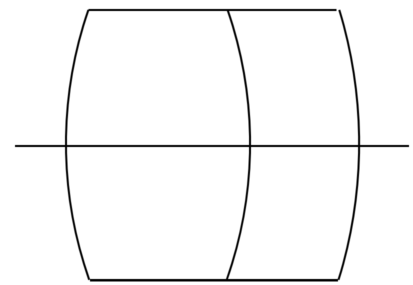
Cemented Doublet

$R_1 = 73.8950$ $\quad R_2 = -51.7840$ $\quad R_3 = -162.2252$

$C_1 = .0135327$ $\quad C_2 = -.0193110$ $\quad C_3 = -.00616427$

$n_1 = 1.517$ $\quad n_2 = 1.649$

$t_1 = 10.5$ $\quad t_2 = 4.0$



Surface	0		1		2		3		4
C			.013533		-.019311		-.006164		
t		∞		10.5		4.0		?	
n		1.0		1.517		1.649		1.0	
-φ			-.00700		.00255		-.00400		
t/n		∞		6.92		2.43		112.85	
y	2		2		1.903		1.881		0
nu		0		-.01400		-.00914		-.01667	
u		0						-.01667	

$$\frac{\overline{V'F'}}{n'} = \overline{V'F'} = BFD = 112.85$$

$\omega' = u' = -.01667 \qquad y_1 = 2$

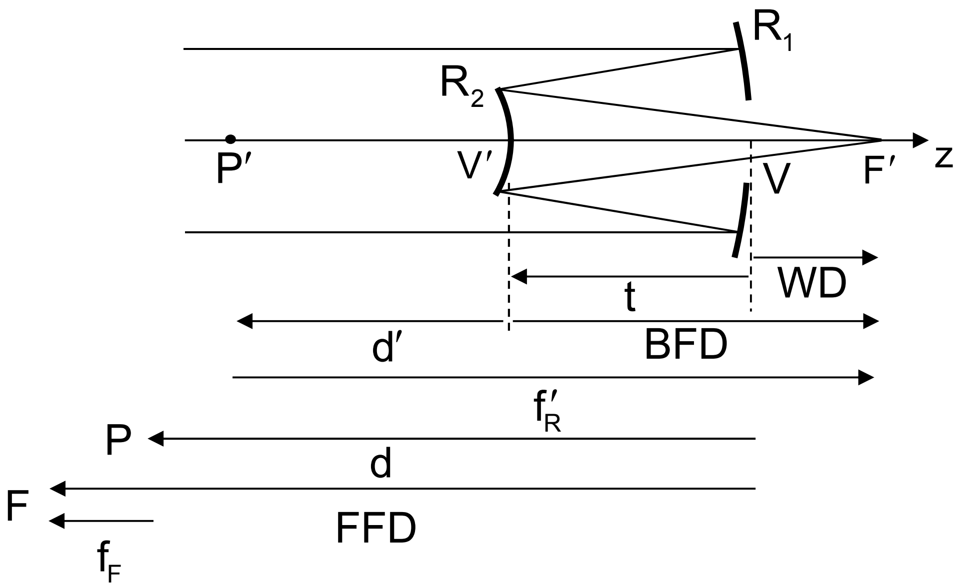
$$\phi = -\frac{\omega'}{y_1} = .008333$$

$f_E = 120.0$

$f'_R = 120.0$

$d' = BFD - f'_R = -7.15$

Mirror System – Cassegrain Telescope



$$R_1 = -200\,mm$$
$$t = -80\,mm$$
$$R_2 = -50\,mm$$
$$n_1 = n = 1$$
$$n_2 = -1$$
$$n_3 = n' = 1$$

$$C_1 = -.005 \qquad C_2 = -.02$$
$$\phi_1 = (n_2 - n_1)C_1 \qquad \phi_2 = (n_3 - n_2)C_2$$
$$\phi_1 = .01/mm \qquad \phi_2 = -.04/mm$$

Gaussian Reduction: $\tau = \dfrac{t}{n_2} = \dfrac{-80\,mm}{-1} = 80\,mm$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau$$
$$\phi = .01 - .04 - (.01)(-.04)(\frac{-80}{-1})$$
$$\phi = .002/mm \qquad f_E = f'_R = 500\,mm$$
$$f_F = -500\,mm$$

$$d' = -\frac{\phi_1}{\phi}\tau = -\frac{.01}{.002}\left(\frac{-80}{-1}\right) = -400\,mm$$
$$d = \delta = \frac{\phi_2}{\phi}\tau = \frac{-.04}{.002}\left(\frac{-80}{-1}\right) = -1600\,mm$$

$$BFD = f'_R + d' = 100\,mm$$
$$FFD = f_F + d = -2100\,mm$$
$$WD = BFD + t = 20\,mm$$

The front focal point and both principal planes are well in front of the system.

Mirror System – Cassegrain Telescope – Raytrace

Surface	0		1		2		3
C			-.005		-.02		
t		∞/?		-80		?/∞	
n		1.0		-1.0		1.0	
-ϕ			-.01		.04		
t/n		∞/?		80		?/∞	
						(100)	
y	1		1		.20		0
→ nu		0		-.01		-.002	
u		0		.01		-.002	
		(2100)					
y	0		4.2		1		1
← nu		.002		-.04		0	
u		.002		.04		0	

$$BFD = \overline{V'F'} = 100\,mm$$

$$\omega' = u' = u_2' = -.002 \qquad y_1 = 1$$

$$\phi = -\frac{\omega'}{y_1} = -\frac{u'}{y_1} = .002/mm$$

$$f_E = f_R' = 500\,mm$$

$$d' = BFD - f_R' = -400\,mm$$

$$WD = BFD + t = 20\,mm$$

$$\overline{FV} = 2100\,mm$$

$$FFD = -\overline{FV} = -2100\,mm$$

$$\omega = u = .002 \qquad y_2 = 1$$

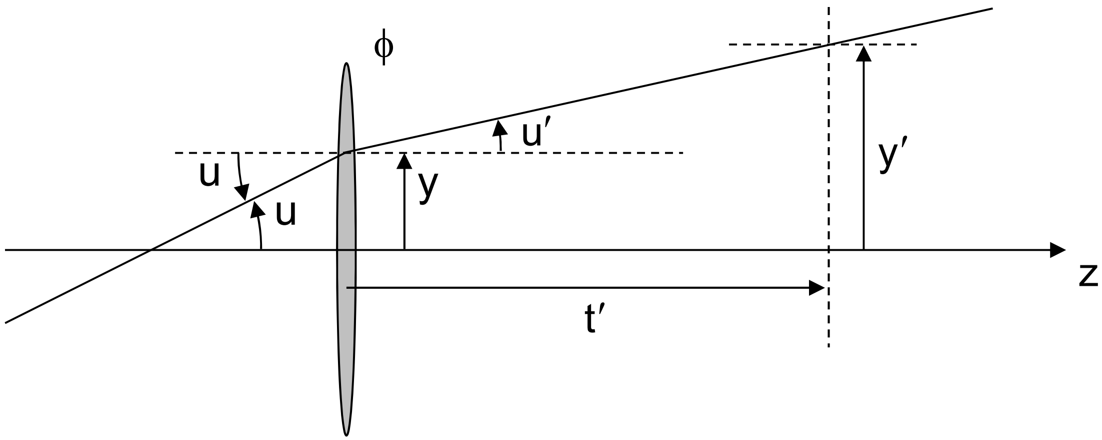
$$\phi = \frac{\omega}{y_2} = \frac{u}{y_2} = .002/mm$$

$$f_F = -500\,mm \qquad f_E = 500\,mm$$

$$d = FFD - f_F = -1600\,mm$$

Paraxial Raytrace – Thin Lens in Air

The principal planes of a thin lens are both located in the plane of the lens.



$n = n' = 1$

Power: $\phi \equiv \frac{1}{f}$

Refraction: $u' = u - y\phi$

Transfer: $y' = y + u't'$

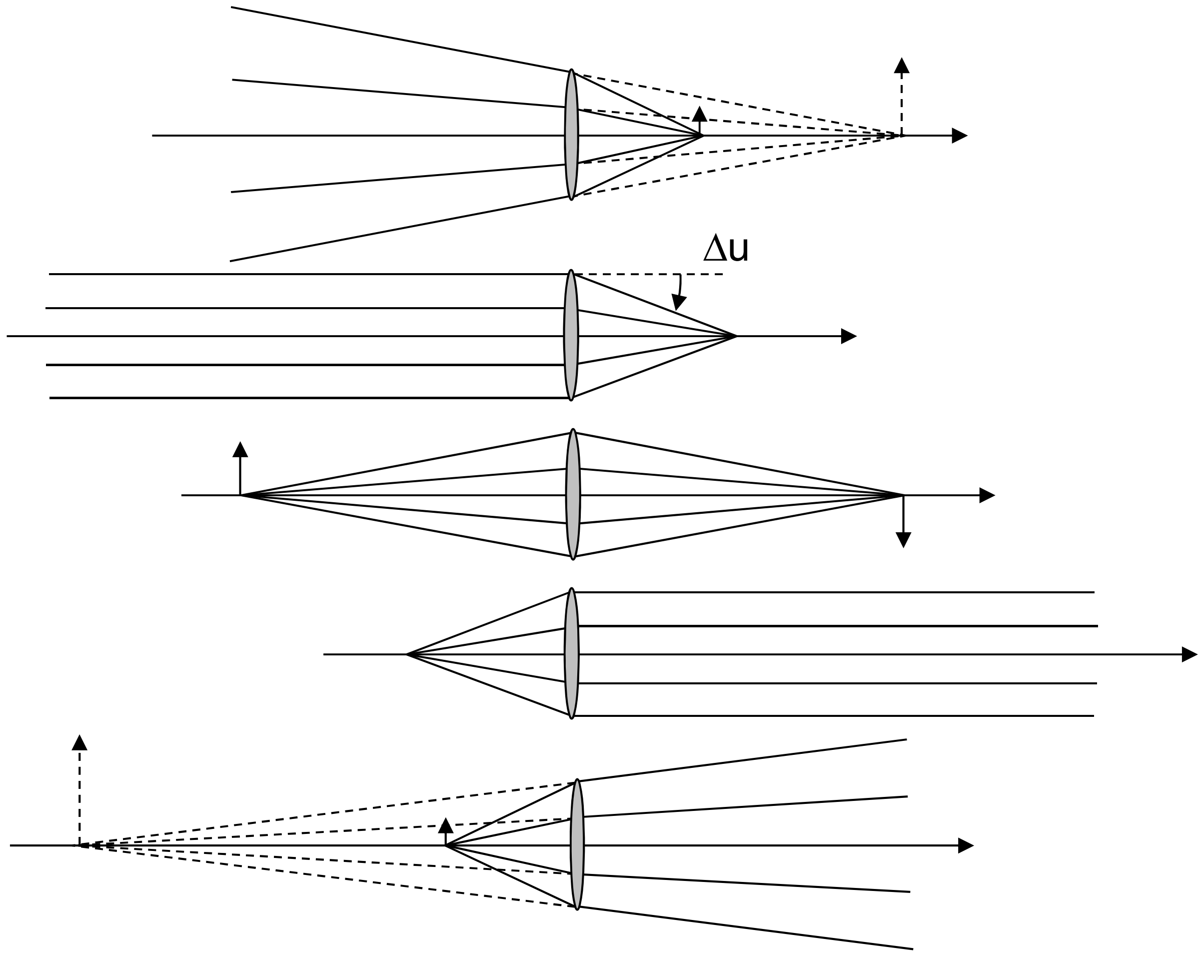
Ray Deviation – Thin Lens in Air

The paraxial ray deviation introduced by a thin lens is independent of the object-image conjugates. Remember that paraxial angles are actually ray slopes.

It depends only on the ray height at the lens and the lens power or focal length:

$$u' = u - y\phi$$

$$\Delta u = u' - u = -y\phi$$



Thin Lens YU Raytrace

Surface	0	1	2	3	4	5	6	7
f								
-ϕ								
t								
y								
u								
y								
u								
y								
u								

$$u' = u - y\phi$$

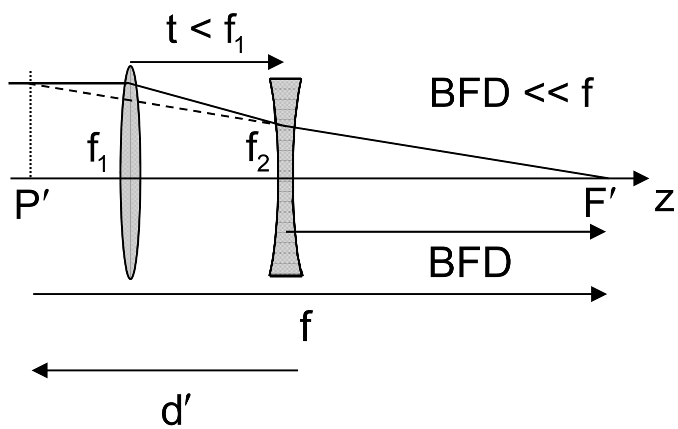
$$y' = y + t'u'$$

Thin Lens Telephoto Lens

$f_1 = 100mm \qquad \phi_1 = .01/mm$

$t = 50mm$

$f_2 = -75mm \qquad \phi_2 = -.01333/mm$



$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t$

$\phi = .00333/mm$

$f = f_E = 300\ mm$

$f_R' = 300mm$

$f_F = -300mm$

$$d' = -\frac{\phi_1}{\phi}t = -150mm$$

$$d = \frac{\phi_2}{\phi}t = -200mm$$

$BFD = f_R' + d' = 150mm$

$FFD = f_F + d = -500mm$

Thin Lens Telephoto Lens - Raytrace

Surface	0		1		2		
f			100		-75		
-ϕ			-.01		.01333		
t		∞/?		50		?/∞	
						(150)	
→ y	1		1		.5		0
u		0		-.01		-.00333	
		(500)					
← y	0		1.667		1		1
u		.00333		-.01333		0	

$$BFD = \overline{V'F'} = 150\,mm$$

$$u' = u_2' = -.00333 \qquad y_1 = 1$$

$$\phi = -\frac{u'}{y_1} = .00333/mm$$

$$f_E = f_R' = 300\,mm$$

$$d' = BFD - f_R' = -150\,mm$$

$$\overline{FV} = 500\,mm$$

$$FFD = -\overline{FV} = -500\,mm$$

$$u = .00333 \qquad y_2 = 1$$

$$\phi = \frac{u}{y_2} = .00333/mm$$

$$f_F = -300\,mm \qquad f_E = 300\,mm$$

$$d = FFD - f_F = -200\,mm$$

Raytrace Comments

In a paraxial raytrace, *t* is the directed distance from the current surface to the next surface. As a result, real objects will usually have a positive distance to the first surface, as opposed to the typical negative Gaussian object distance *z*.

Surfaces are raytraced in optical order, not physical order. All planes of interest in an optical space must be analyzed before transferring to a reflective or refractive surface and entering the next optical space. Within an optical space, transfers move back or forth along the ray in that space without changing the ray angle. Real and virtual segments of the space can be accessed.

YNU Raytrace Form

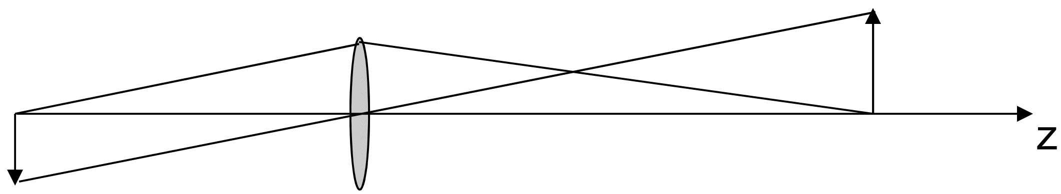
Surface	0	1	2	3	4	5	6	7
C								
t								
n								
-ϕ								
t/n								
y								
nu								
u								
y								
nu								
u								
y								
nu								
u								

Thin Lens YU Raytrace Form

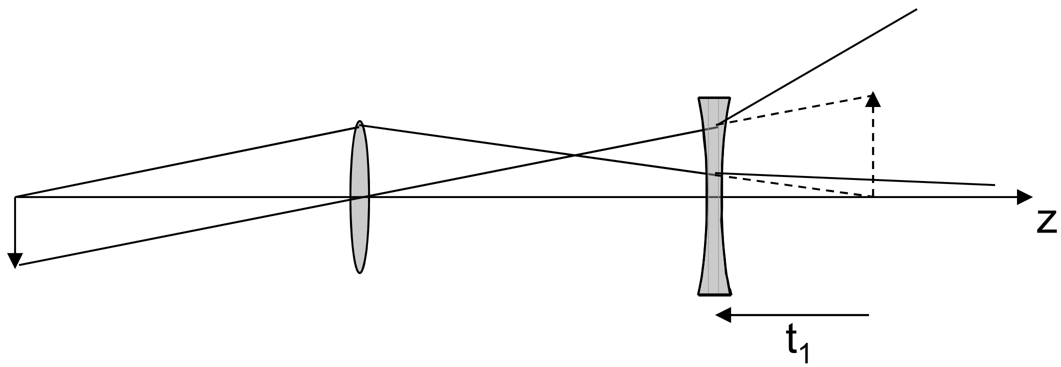
Surface	0	1	2	3	4	5	6	7
f								
-ϕ								
t								
y								
u								
y								
u								
y								
u								

Virtual Objects and Raytraces

Consider an optical system or projector forming a real image. The ray heights and angles at the image are easy to determine.



A second lens is now placed between the first lens and its image. The original image no longer exists, but it now serves as a virtual object for the second lens. A final system image is formed by the second lens working with the first lens.



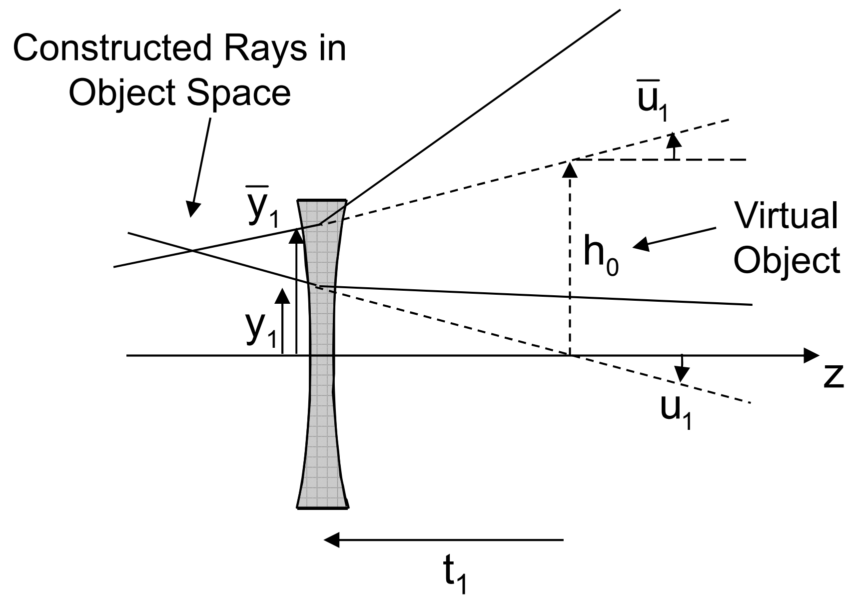
Once again, determining the ray heights and angles in the system image space is straightforward. Transfer from the first lens to the second lens and refract.

Virtual Objects and Raytraces

A common situation is to be given the size and location of the virtual object for a lens or system without any information about the optical system used to produce it (the first lens on the previous slide). Rays must be created corresponding to this intermediate image (the virtual object). These rays exist in the optical space corresponding to the virtual object, which is the object space of the optical system.

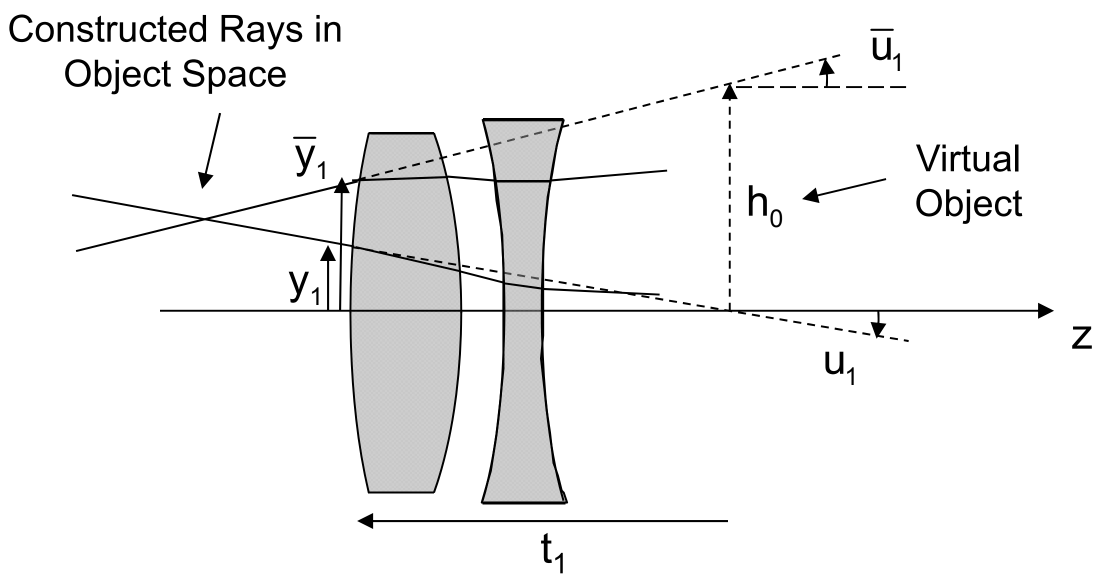
Once the intermediate space rays are defined, these rays are transferred by t_1 back to the entry vertex of the optical system. The ray heights and angles are now known at the first vertex of the optical system, and they are in the system object space. These constructed rays can then be propagated through the system to the system image space.

Pick two rays, one through the top of the virtual object, the other through the axial object point. The angles are arbitrary.



Virtual Objects and Raytraces

When transferring back to the front vertex, these rays defining the virtual object are not refracted by the optical system. These rays are already in object space. The negative thickness t_1 transfers to the left along the virtual segments of the rays. Regardless of the physical order, rays are traced in optical order: from object space to image space. The thicknesses are the directed distances as defined by the sign conventions.



The transfer distance t_1 is the directed distance from the virtual object to the front vertex of the system. This transfer distance is in object space and is associated with the index of refraction of object space. The powers and indices of the elements must not be used in transferring back to the front vertex of the system.

If a system represented just by its power and principal planes is to be analyzed, the transfer distance is the distance from the virtual object to the front principal plane of the system P.

Virtual Object Example

A 10 mm high virtual object is located 40 mm to the right of the first surface of the following 100 mm focal length convex-plano thick lens:

R1 = 51.7 mm   R2 = Infinity   t = 10 mm   n = 1.517
C1 = 0.01934/mm   C2 = 0

Determine the image location and size.

Surface	Obj	1	2	Image
C		0.01934	0	
t	-40	10	?	
n	1.0	1.517	1.0	
-ϕ		-0.0100	0	
t/n	-40	6.592	21.98	
y	0	4.0	3.0771	0
nu	-0.1*	-0.1400	-0.1400	
u	-0.1	-0.0923	-0.1400	
$\bar{y}$	10	10	9.341	7.143
$n\bar{u}$	0*	-0.10	-0.10	
$\bar{u}$	0	-0.659	-0.10	

* Arbitrary

Launch rays from the axial object location and from the top of the object at arbitrary angles.

Transfer these ray heights to the first surface of the lens (t_0 = -40 mm). There is no refraction associated with this transfer as these are object-space rays.

Solve for the image location and the image height:

The image is located 21.98 mm to the right of the rear surface of the lens. The image height is 7.143 mm.

This is a virtual object producing a real image.