Section 7

Gaussian Reduction

Paraxial Raytrace Equations

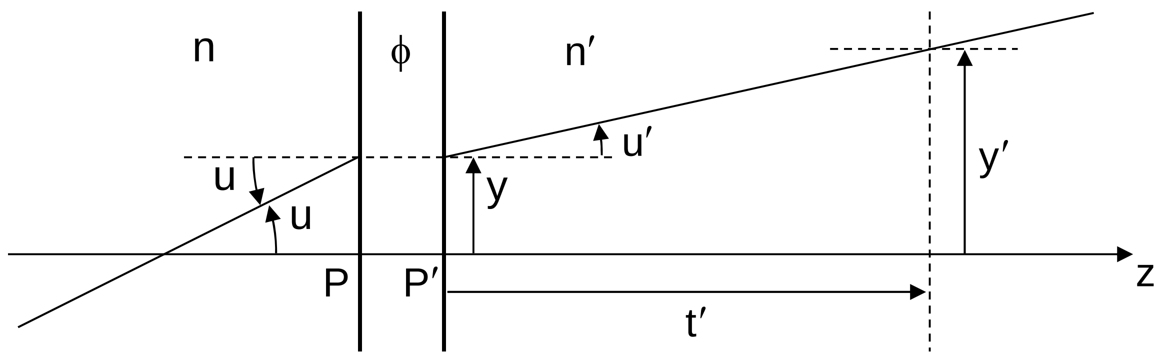
Refraction occurs at an interface between two optical spaces. The transfer distance *t*' allows the ray height *y*' to be determined at any plane within an optical space (including virtual segments).

$$\omega = nu \qquad \phi = (n' - n)C \qquad \tau = \frac{t}{n}$$

Refraction:

$$n'u' = nu - y\phi$$

$$\omega' = \omega - y\phi$$



Transfer:

$$y' = y + t'u'$$

$$y' = y + n'u'\frac{t'}{n'}$$

$$y' = y + \omega'\tau'$$

Gaussian Reduction

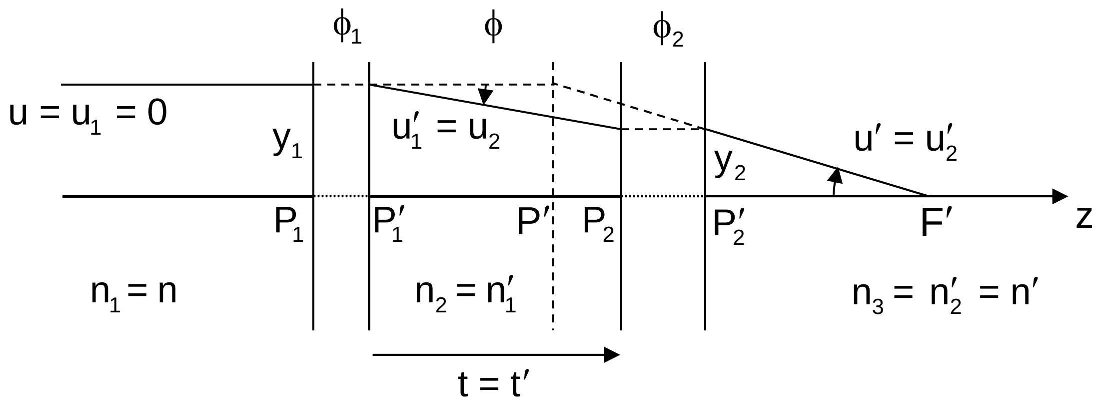
Gaussian reduction is the process that combines multiple components two at a time into a single equivalent system. The Gaussian properties (power, focal lengths, and the location of the cardinal points) are determined.

Two component system – System Power:

Trace a ray parallel to the optical axis in object space. This ray must go through the rear focal point of the system.

Paraxial raytrace: Refraction $n'u' = nu - y\phi$ $\quad \omega' = \omega - y\phi$

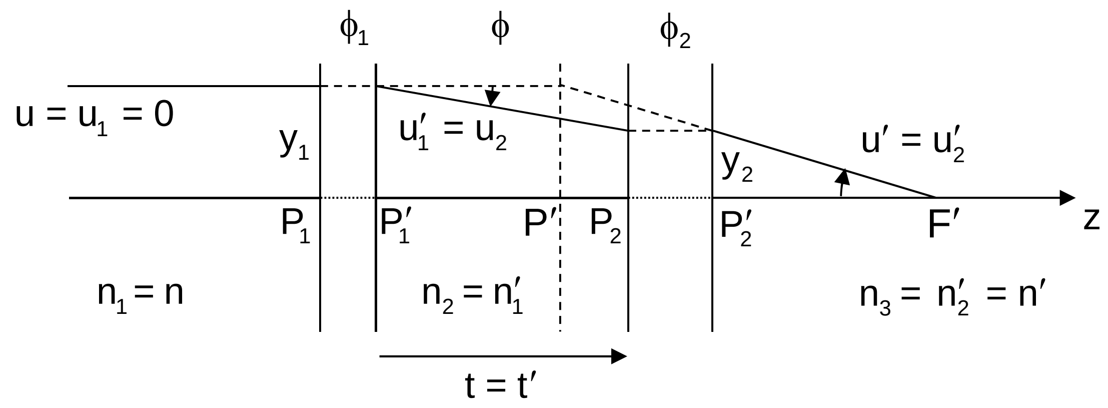
Transfer $y' = y + (n'u')(t'/n')$ $\quad y' = y + \omega'\tau'$



Define the system power by applying the refraction equation to the system:

$$\omega' = \omega_2' = \omega - y_1\phi \qquad \omega = 0 \qquad \omega' = -y_1\phi$$

Two Component System – System Power



Trace the ray:

$$\omega_2 = \omega_1' = \omega_1 - y_1\phi_1 = -y_1\phi_1$$

$$y_2 = y_1 + \omega_1'\tau$$

$$\omega' = \omega_2' = \omega_2 - y_2\phi_2 \qquad \tau = \frac{t}{n_2}$$

$$\omega' = -y_1\phi_1 - \left(y_1 + \omega_1'\tau\right)\phi_2$$

$$\omega' = -y_1\phi_1 - y_1\phi_2 - \left(-y_1\phi_1\right)\tau\phi_2 \qquad \omega' = -y_1\phi$$

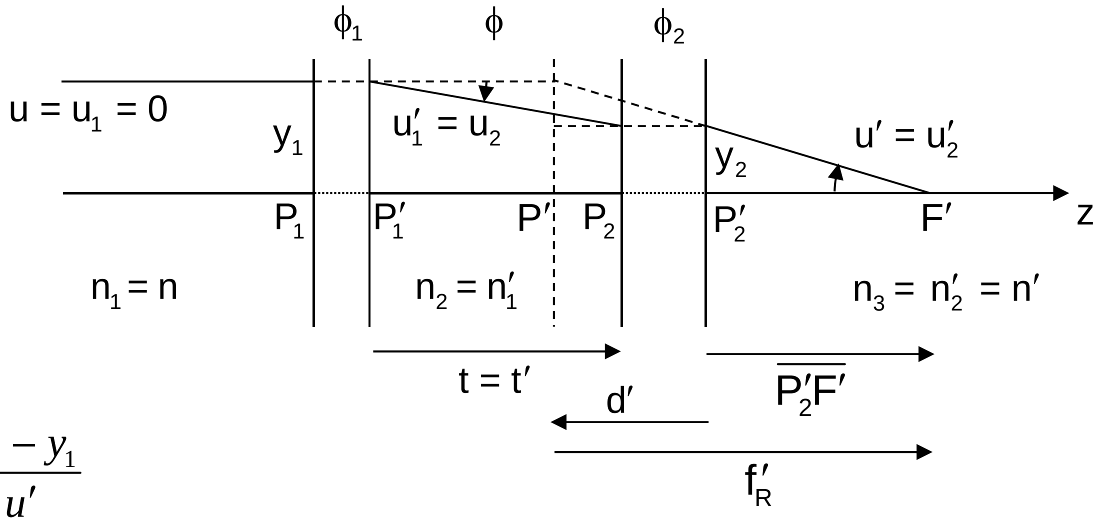
$$\omega' = -y_1\left(\phi_1 + \phi_2 - \phi_1\phi_2\tau\right) = -y_1\phi$$

System power:

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau$$

Two Component System – Rear Cardinal Points

The system rear principal plane is the plane of unit system magnification.



$$y_2 = y_1 + \omega_1' \tau$$

$$\omega_1' = \omega_2 = -y_1 \phi_1$$

$$\omega' = -y_1 \phi$$

$$d' = -\frac{y_2 - y_1}{u'}$$

$$d' = -\frac{\omega_1' \tau}{u'}$$

$$\delta' = \frac{d'}{n'} = -\frac{\omega_1' \tau}{\omega'} = -\frac{(-y_1 \phi_1)\tau}{(-y_1 \phi)}$$

$$\delta' = \frac{d'}{n'} = -\frac{\phi_1}{\phi}\tau = -\frac{\phi_1}{\phi}\frac{t}{n_2}$$

$$\overline{P_2'F'} = \frac{-y_2}{u'}$$

$$f_R' = \overline{P_2'F'} - d'$$

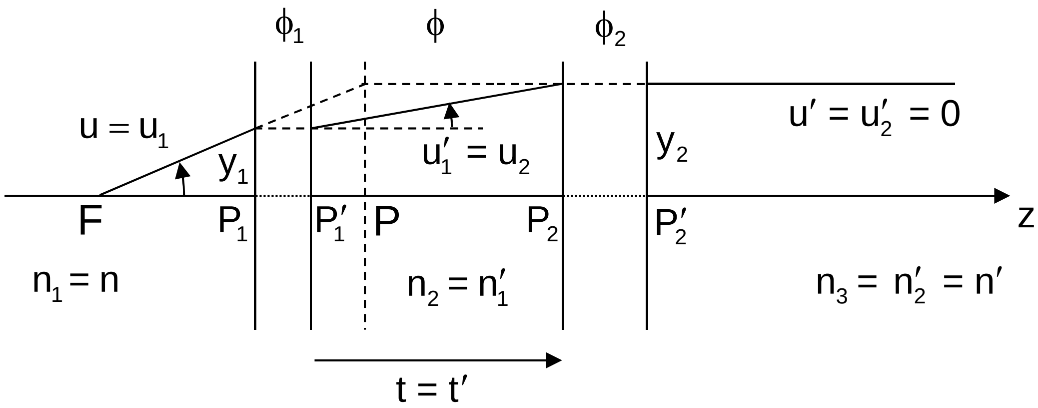
$$f_R' = \frac{n'}{\phi}$$

Note that the shift d' of the system rear principal plane P' from the rear principal plane of the second element P_2' occurs in the system image space n'.

If P_2' is the rear vertex of the system, then the distance $\overline{P_2'F'}$ is the BFD.

Two Component System – Front Properties

Repeat the process to determine the front cardinal points. Start with a ray at the system front focal point F. It will emerge from the system parallel to the optical axis.



System:

$$\omega' = \omega_2' = \omega - y_2\phi \qquad \omega' = 0$$

$$\omega = y_2\phi$$

$$\tau = \frac{t}{n_2}$$

Work the ray backwards from image space through the system.

$$\omega_2' = \omega_2 - y_2\phi_2 \qquad \omega_2' = 0$$

$$\omega_2 = y_2\phi_2$$

$$y_2 = y_1 + \omega_2\tau$$

$$y_1 = y_2 - \omega_2\tau = y_2\left(1-\phi_2\tau\right)$$

$$\omega_1' = \omega_1 - y_1\phi_1$$

$$\omega_1 = \omega_1' + y_1\phi_1 = \omega_2 + y_1\phi_1$$

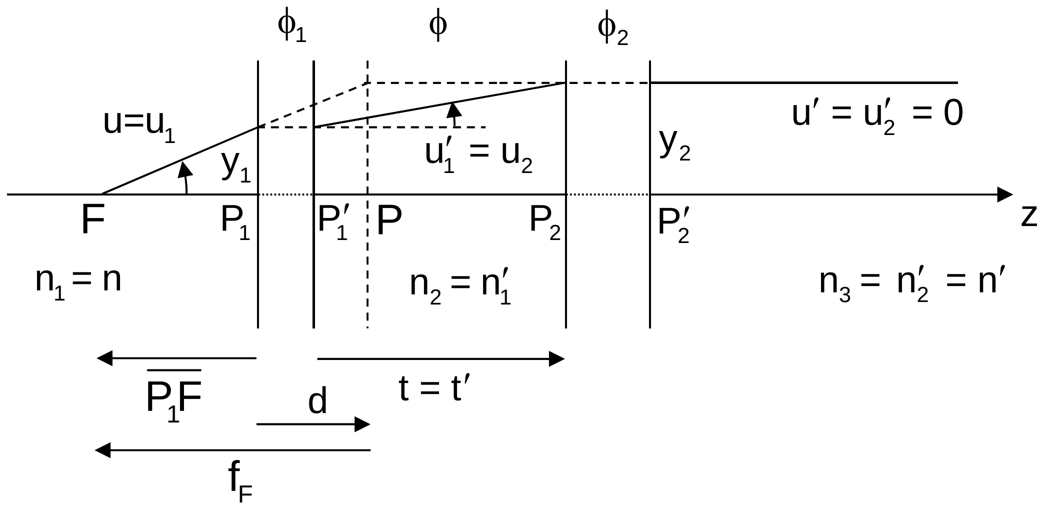
$$\omega = \omega_1 = y_2\phi_2 + y_2\phi_1\left(1-\phi_2\tau\right)$$

$$\omega = y_2\left(\phi_1 + \phi_2 - \phi_1\phi_2\tau\right) = y_2\phi$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau$$

System power: same result as for forward ray.

Two Component System – Front Cardinal Points



$$y_2 = y_1 + \omega_2 \tau$$

$$\omega_2 = y_2 \phi_2$$

$$\omega = y_2 \phi$$

$$d = \frac{y_2 - y_1}{u}$$

$$d = \frac{\omega_2 \tau}{u}$$

$$\delta = \frac{d}{n} = \frac{\omega_2 \tau}{\omega} = \frac{y_2 \phi_2 \tau}{y_2 \phi}$$

$$\delta = \frac{d}{n} = \frac{\phi_2}{\phi}\tau = \frac{\phi_2}{\phi}\frac{t}{n_2}$$

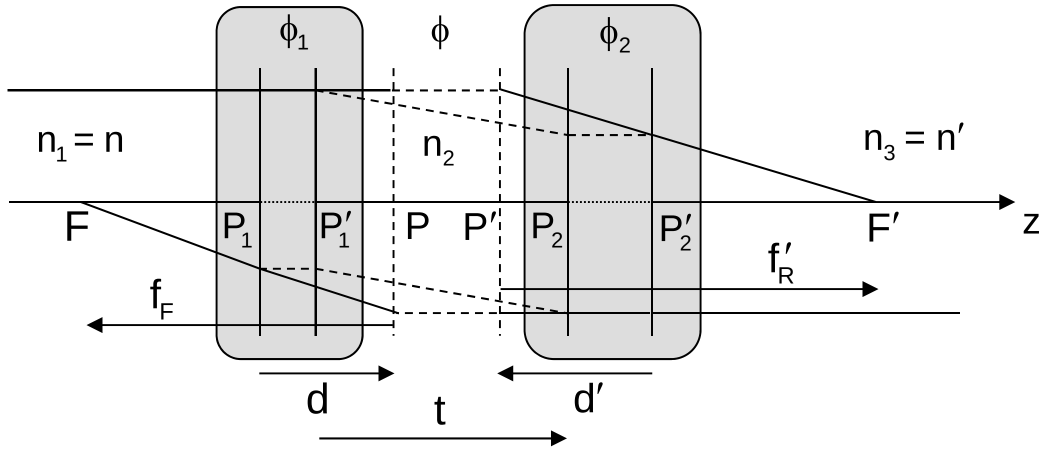
$$\overline{P_1F} = -\frac{y_1}{u}$$

$$f_F = \overline{P_1F} - d \qquad f_F = -\frac{n}{\phi}$$

Note that the shift d of the system front principal plane P from the front principal plane of the first element P_1 occurs in the system object space n.

If P_1 is the front vertex of the system, then the distance $\overline{P_1F}$ is the FFD.

Gaussian Reduction - Summary



$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau \qquad \tau = \frac{t}{n_2} \qquad f_R' = \frac{n'}{\phi} = n'f$$

$$\delta = \frac{d}{n} = \frac{\phi_2}{\phi}\tau = \frac{\phi_2}{\phi}\frac{t}{n_2} \qquad \delta' = \frac{d'}{n'} = -\frac{\phi_1}{\phi}\tau = -\frac{\phi_1}{\phi}\frac{t}{n_2} \qquad f_F = -\frac{n}{\phi} = -nf$$

$$f = f_E = \frac{1}{\phi}$$

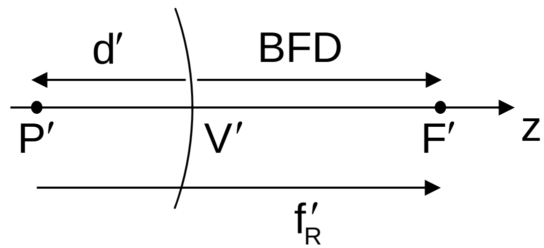
- P and P' are the planes of unit system magnification (effective refraction for the system).
- d is the shift in object space of the front system principal plane P from the front principal plane of the first system P_1 .
- d' is the shift in image space of the rear system principal plane P' from the rear principal plane of the second system P_2' .
- t is the directed distance in the intermediate optical space from the rear principal plane of the first system P_1' to the front principal plane of the second system P_2. Both of these principal planes must be in the same optical space.
- Following reduction, the two original elements and the intermediate optical space n_2 are not needed or used.

Vertex Distances

The surface vertices are the mechanical datums in a system and are often the reference locations for the cardinal points.

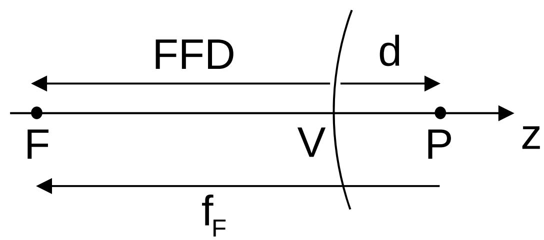
Back focal distance BFD:

$$BFD = f_R' + d'$$



Front focal distance FFD:

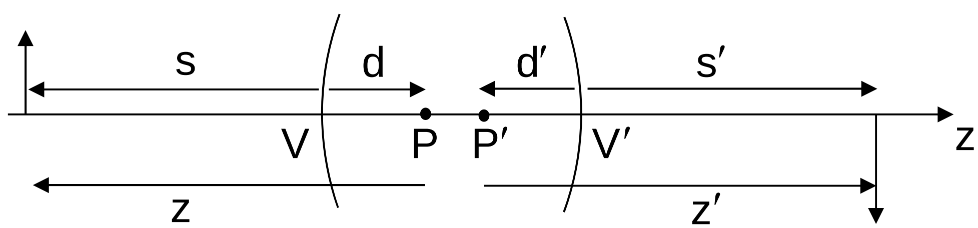
$$FFD = f_F + d$$



Object and image vertex distances are determined using the Gaussian distances:

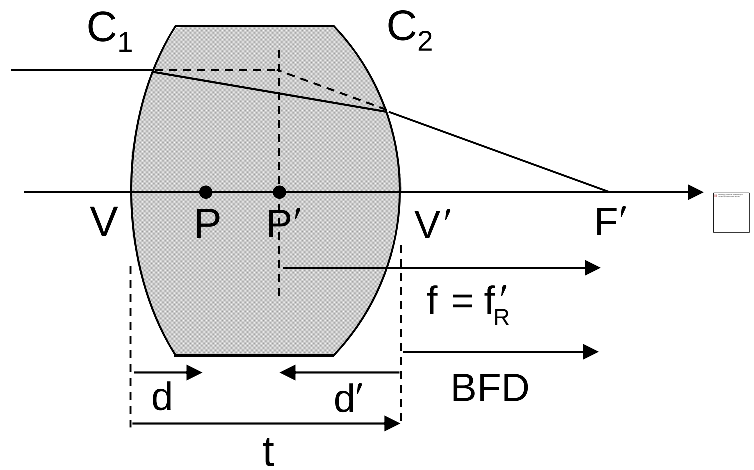
$$s = z + d$$

$$s' = z' + d'$$



Thick Lens in Air

A thick lens is the combination of two refracting surfaces.



$$\phi_1 = (n_2 - n_1) C_1$$

$$\phi_1 = (n-1) C_1$$

$$\phi_2 = (n_3 - n_2) C_2$$

$$\phi_2 = (1-n) C_2 = -(n-1) C_2$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau \qquad \tau = \frac{t}{n_2} = \frac{t}{n}$$

$$\phi = (n-1)\left[C_1 - C_2 + (n-1) C_1 C_2 t / n\right]$$

$$f_F = -\frac{n_1}{\phi} = -\frac{1}{\phi} \qquad f_R' = \frac{n_3}{\phi} = \frac{1}{\phi}$$

$$f = f_R' = -f_F = \frac{1}{\phi}$$

$$d' = \overline{V'P'} = -\frac{\phi_1}{\phi}\frac{t}{n}$$

$$BFD = f_R{}' + d'$$

$$d = \overline{VP} = \frac{\phi_2}{\phi}\frac{t}{n}$$

$$\overline{PP'} = t - d + d' = t - \frac{\phi_1 + \phi_2}{\phi}\tau$$

$$\overline{PP'} = (n-1)\tau - \frac{\phi_1\phi_2}{\phi}\tau^2$$

The nodal points are located at the respective principal planes.

<u>Thin Lens in Air</u>

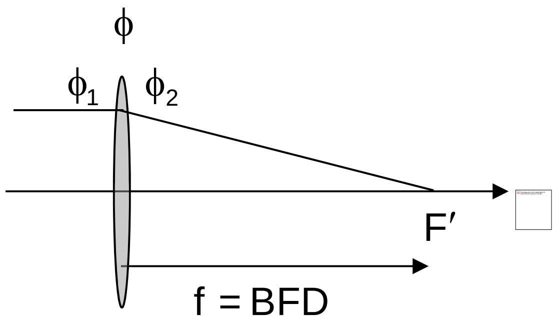
$$t \to 0 \qquad \tau \to 0$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 \tau$$

$$\phi = \phi_1 + \phi_2$$

$$\phi_1 = (n-1)C_1$$

$$\phi_2 = -(n-1)C_2$$



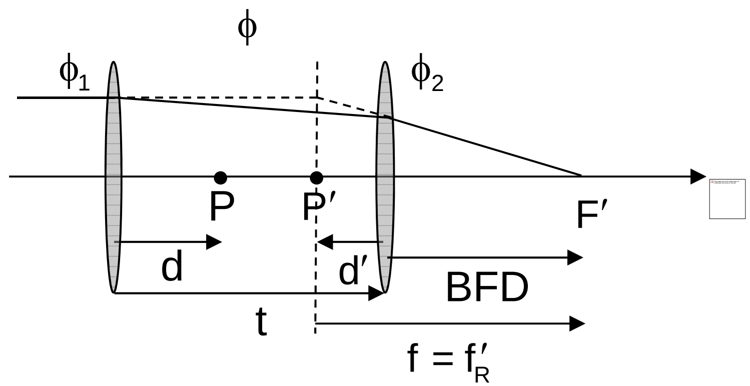
$$\phi = (n-1)(C_1 - C_2)$$

$$f = f_E = f'_R = -f_F = \frac{1}{\phi} \qquad d = d' = 0 \qquad BFD = f$$

The principal planes and nodal points are located at the lens.

Two Separated Thin Lenses in Air

$$n_2 = 1 \qquad t = \tau$$



$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

$$f = f'_R \equiv \frac{1}{\phi}$$

$$d' = -\frac{\phi_1}{\phi} t \qquad d = \frac{\phi_2}{\phi} t$$

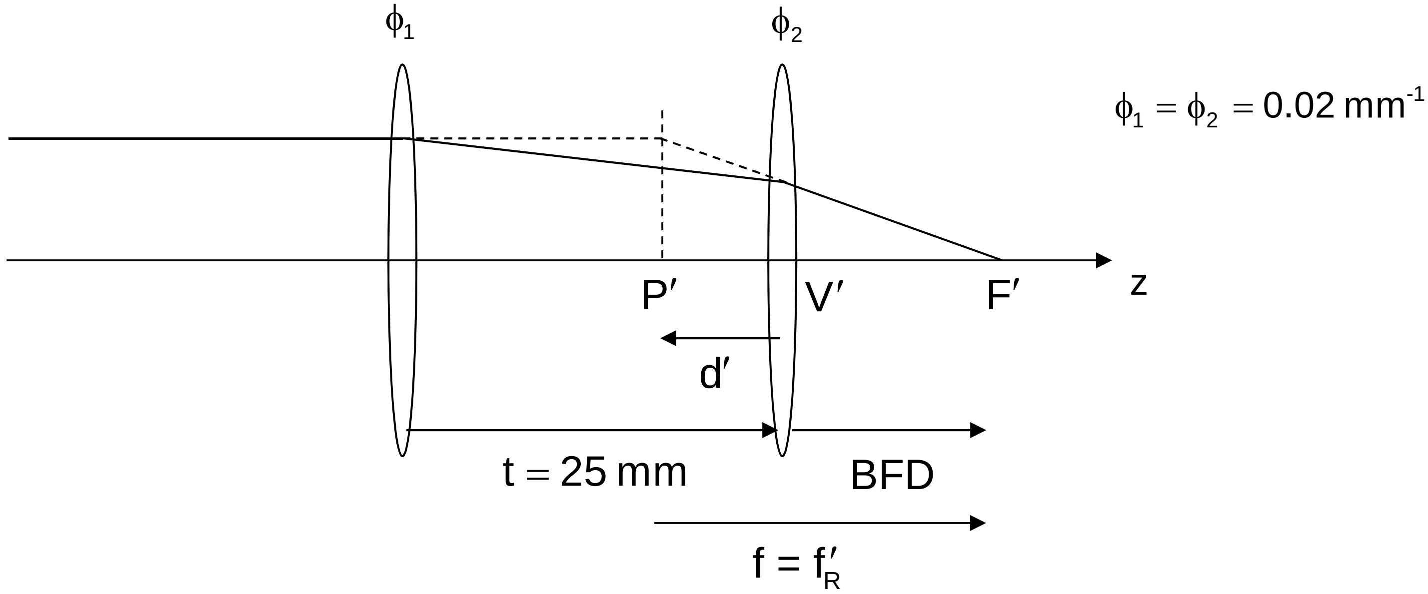
$$BFD = f'_R + d'$$

$$\overline{PP'} = t - d + d' = -\frac{\phi_1 \phi_2}{\phi} t^2$$

The nodal points are coincident with the principal planes.

Gaussian Reduction Example – Two Separated Thin Lenses in Air

Two 50 mm focal length lenses are separated by 25 mm.



$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

$$\phi = 0.02\,mm^{-1} + 0.02\,mm^{-1} - \left(0.02\,mm^{-1}\right)^2 25\,mm$$

$$\phi = 0.03\,mm^{-1}$$

$$f = f'_R = 33.333\,mm$$

$$d' = -\frac{\phi_1}{\phi} t = -\frac{0.02\,mm^{-1}}{0.03\,mm^{-1}} 25\,mm$$

$$d' = -16.667\,mm$$

$$BFD = f'_R + d'$$

$$BFD = 16.667\,mm$$

Diopters

Lens power is often quoted in diopters D.

Units are m^{-1}

$$D = \phi \quad (\text{in m}^{-1})$$

$$D = \frac{1}{f_E} \quad (f_E \text{ in m})$$

With closely spaced thin lenses, the total power is approximately the sum of the powers of the individual lenses. Focal lengths do not add.

Multi-Element Reduction

Multiple element systems are reduced two elements at a time.

A single system power and pair of principal planes results.

Given these quantities, the focal lengths and other cardinal points can be found.

There are several reduction strategies possible for multiple elements or surfaces.

$$1\ 2\ 3\ 4 \rightarrow (12)\ (34) \rightarrow (1234)$$

$$1\ 2\ 3\ 4 \rightarrow (12)\ 3\ 4 \rightarrow (123)\ 4 \rightarrow (1234)$$

The system principal planes are usually measured relative to the front and rear vertices of the systems:

- The system front principal plane is located relative to the front principal plane of the first surface or element.

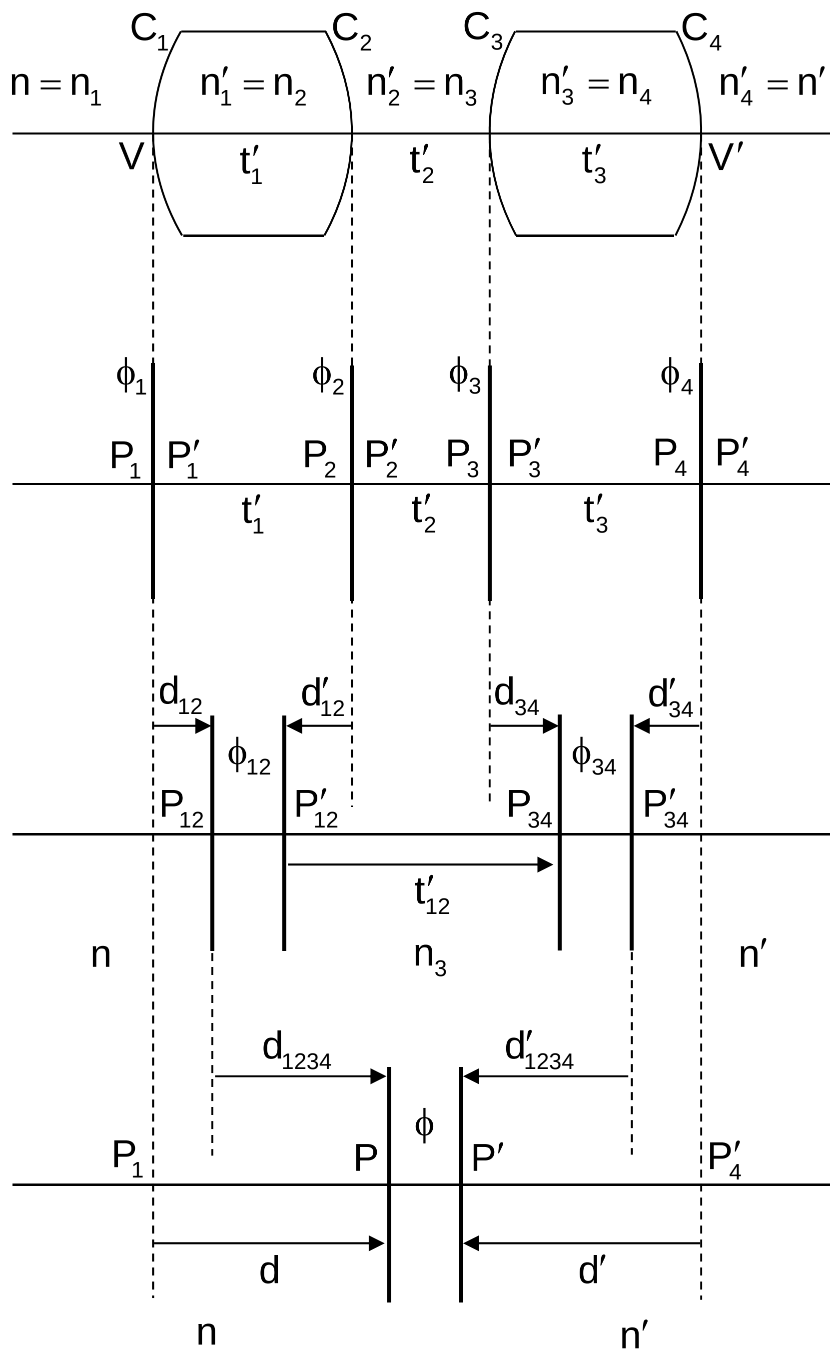
- The system rear principal plane is located relative to the rear principal plane of the last surface or element.

Reduction in Pairs

1 2 3 4

$\rightarrow$ (12) (34)

$\rightarrow$ (1234)



$$t'_{12} = t'_2 - d'_{12} + d_{34}$$

$$d = d_{12} + d_{1234}$$

$$d' = d'_{34} + d'_{1234}$$

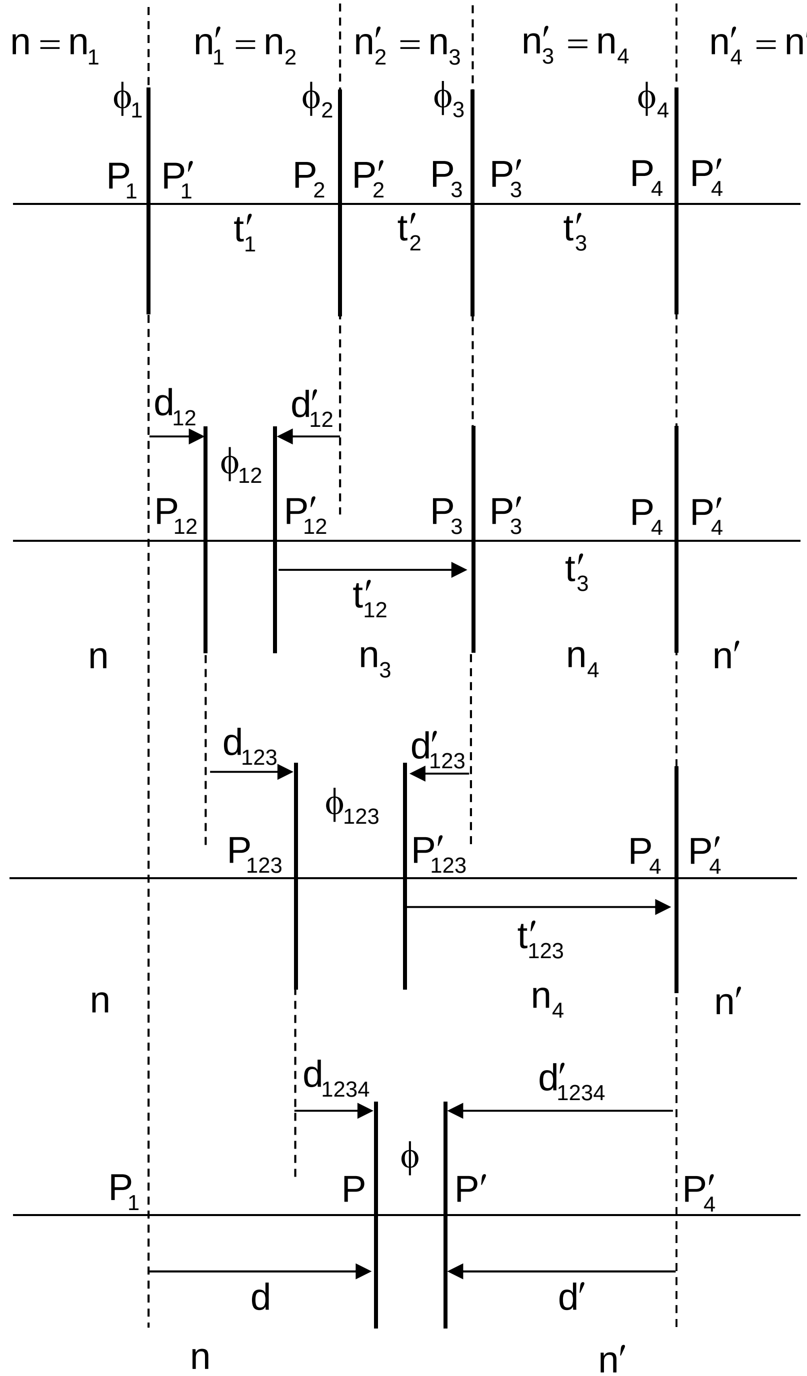
Reduction – One at a Time

1 2 3 4

→ (12) 3 4

→ (123) 4

→ (1234)



$$t_{12}' = t_2' - d_{12}'$$

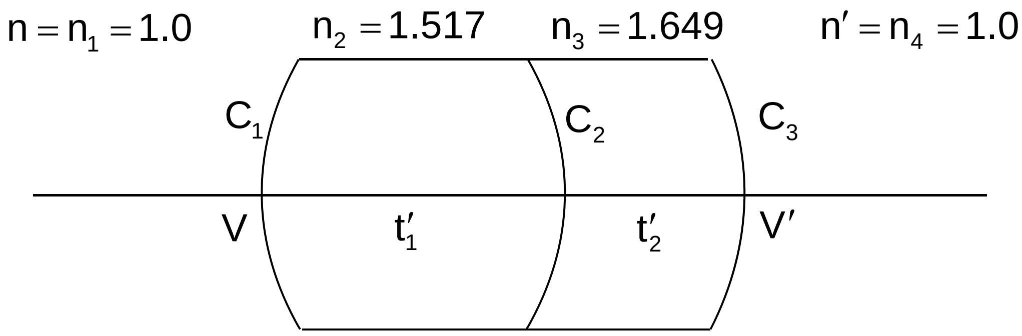
$$t_{123}' = t_3' - d_{123}'$$

$$d = d_{12} + d_{123} + d_{1234}$$

$$d' = d_{1234}'$$

Gaussian Reduction – Example

Cemented Doublet



$R_1 = 73.8950$ $R_2 = -51.7840$ $R_3 = -162.2252$

$C_1 = .0135327$ $C_2 = -.0193110$ $C_3 = -.00616427$

$\phi_1 = .00700$ $\phi_2 = -.00255$ $\phi_3 = .00400$

$t_1' = 10.5$ $t_2' = 4.0$

$$\tau_1' = \frac{t_1'}{n_2} = 6.92 \qquad \tau_2' = \frac{t_2'}{n_3} = 2.43$$

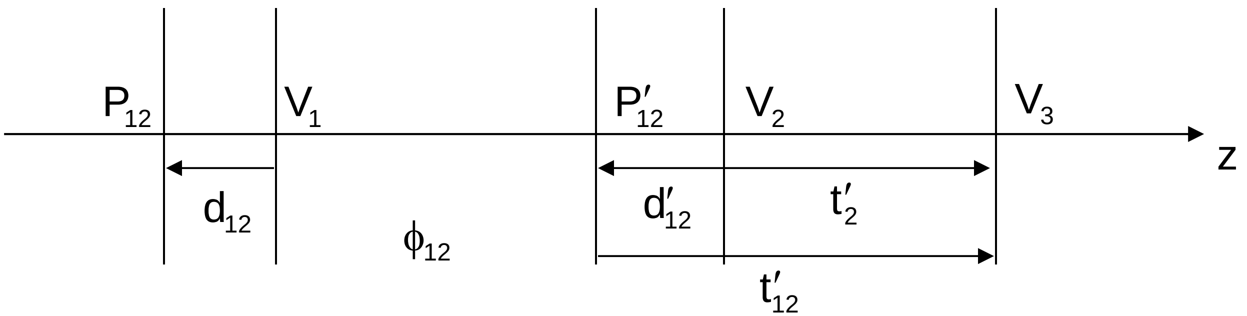
Gaussian Reduction – Example – Continued

First, reduce the first two surfaces:

$$\phi_{12} = \phi_1 + \phi_2 - \phi_1\phi_2\tau_1' = .00457$$

$$\delta_{12} = \frac{\phi_2}{\phi_{12}}\tau_1' = -3.86 \qquad d_{12} = \delta_{12}$$

$$\delta_{12}' = -\frac{\phi_1}{\phi_{12}}\tau_1' = -10.60 \qquad d_{12}' = n_3\delta_{12}' = -17.48$$



At this point, the first two surfaces are represented by ϕ_{12} and the principal planes P_{12} and P_{12}'.

$$t_{12}' = t_2' - d_{12}' = 21.48$$

$$\tau_{12}' = \frac{t_{12}'}{n_3} = \tau_2' - \delta_{12}' = 13.03$$

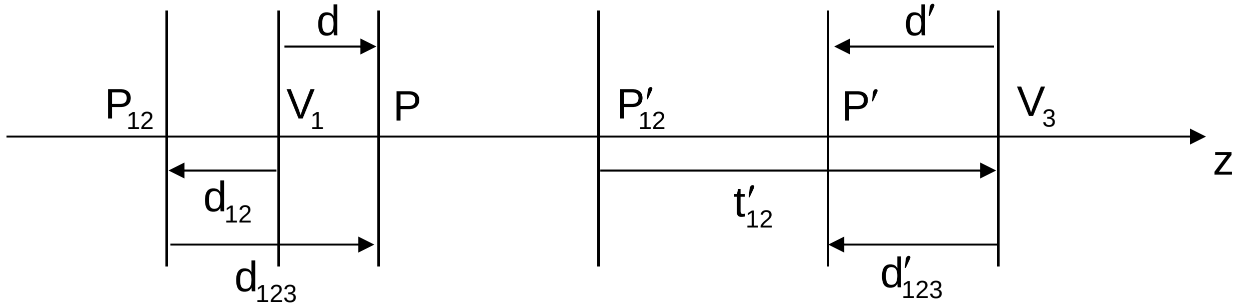
Gaussian Reduction – Example – Continued

Add the third surface:

$$\phi = \phi_{12} + \phi_3 - \phi_{12}\phi_3\tau'_{12} = .00833$$

$$d_{123} = \delta_{123} = \frac{\phi_3}{\phi}\tau'_{12} = 6.26 \qquad \text{Object space} \quad n = 1$$

$$d'_{123} = \delta'_{123} = -\frac{\phi_{12}}{\phi}\tau'_{12} = -7.15 \qquad \text{Image space} \quad n' = 1$$



$$d = \delta = \delta_{12} + \delta_{123} = d_{12} + d_{123} = 2.40$$

$$d' = \delta' = \delta'_{123} = -7.15$$

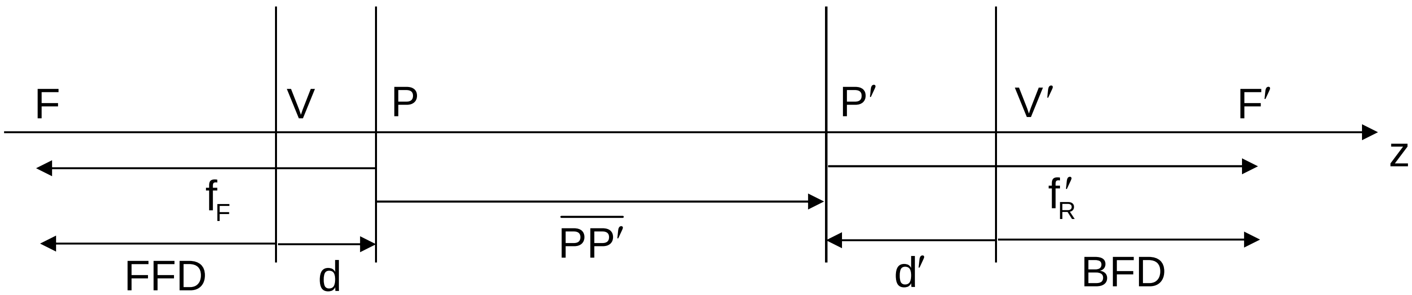
$\phi = .008333$ $\qquad f_E = 120.0$ $\qquad f'_R = -f_F = 120.0$

Gaussian Reduction – Example – Summary

$$d = \delta = \delta_{12} + \delta_{123} = d_{12} + d_{123} = 2.40$$

$$d' = \delta' = \delta_{123} = -7.15$$

$$\phi = .008333 \qquad f_E = 120.0 \qquad f'_R = -f_F = 120.0$$



$$\overline{VF} = \overline{P_1F} = f_F + d = -117.6$$ Front Focal Distance FFD

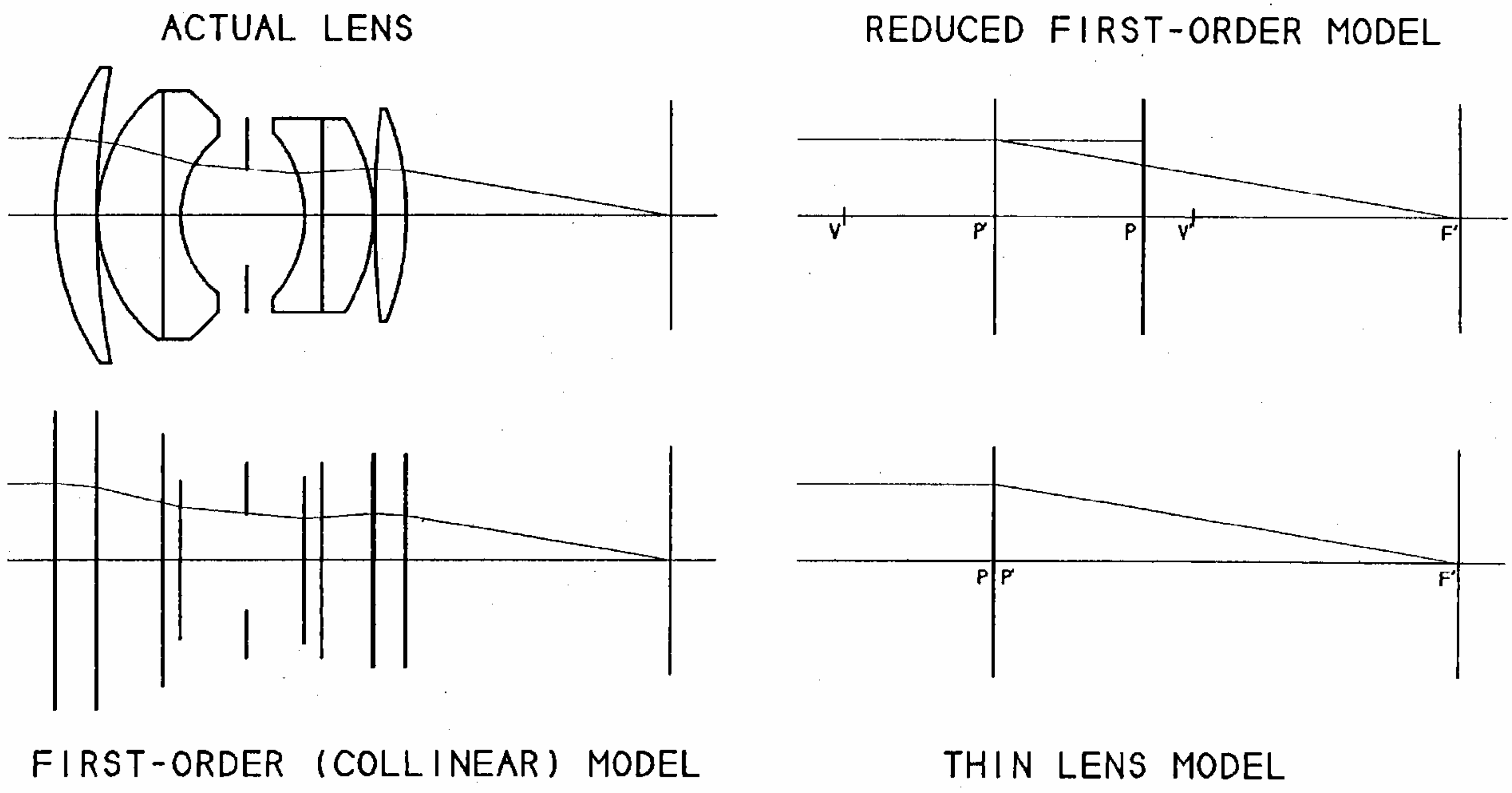
$$\overline{V'F'} = \overline{P'_3F'} = f'_R + d' = 112.85$$ Back Focal Distance BFD

Principal plane separation:

$$\overline{PP'} = \overline{VV'} - d + d' = t'_1 + t'_2 - d + d'$$

$$\overline{PP'} = 4.95$$

Real Lens to Thin Lens Model

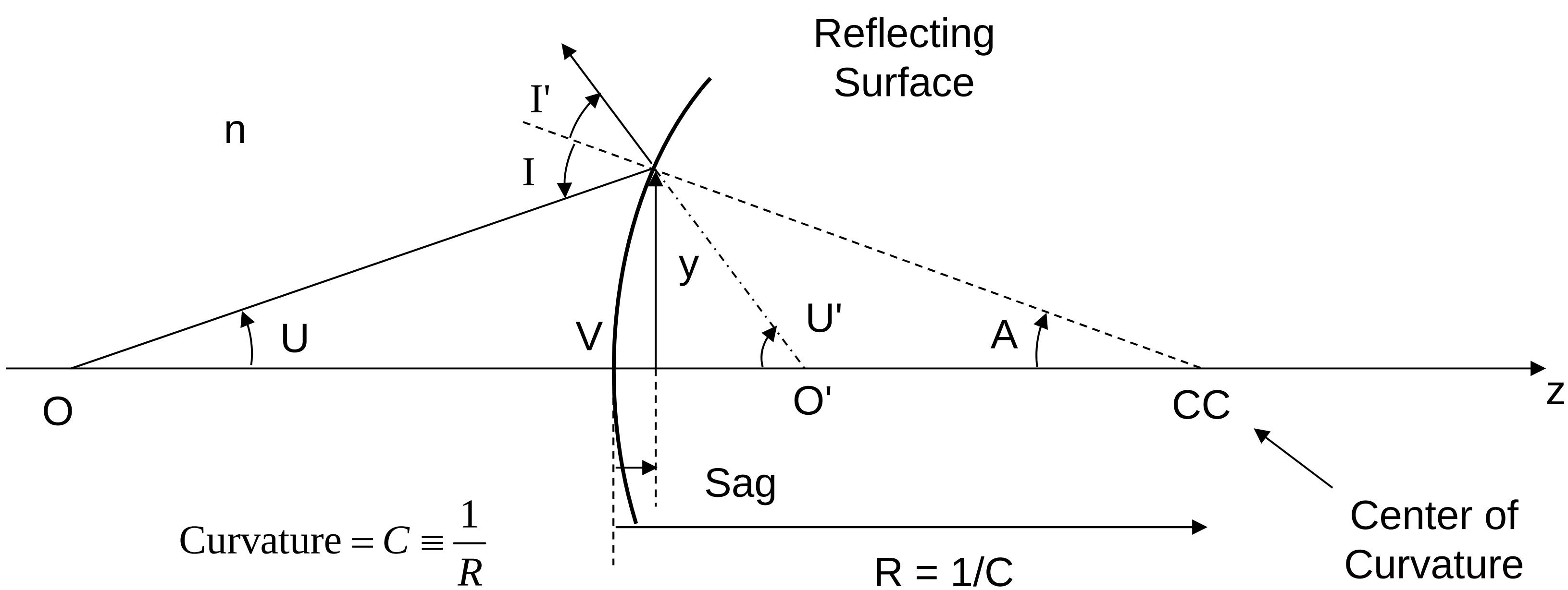


Single Reflecting Surface

Consider a single reflecting surface with a radius of curvature of R. The rays propagate in an index of refraction of n.

The angles of incidence and refraction (I and I') are measured with respect to the surface normal.

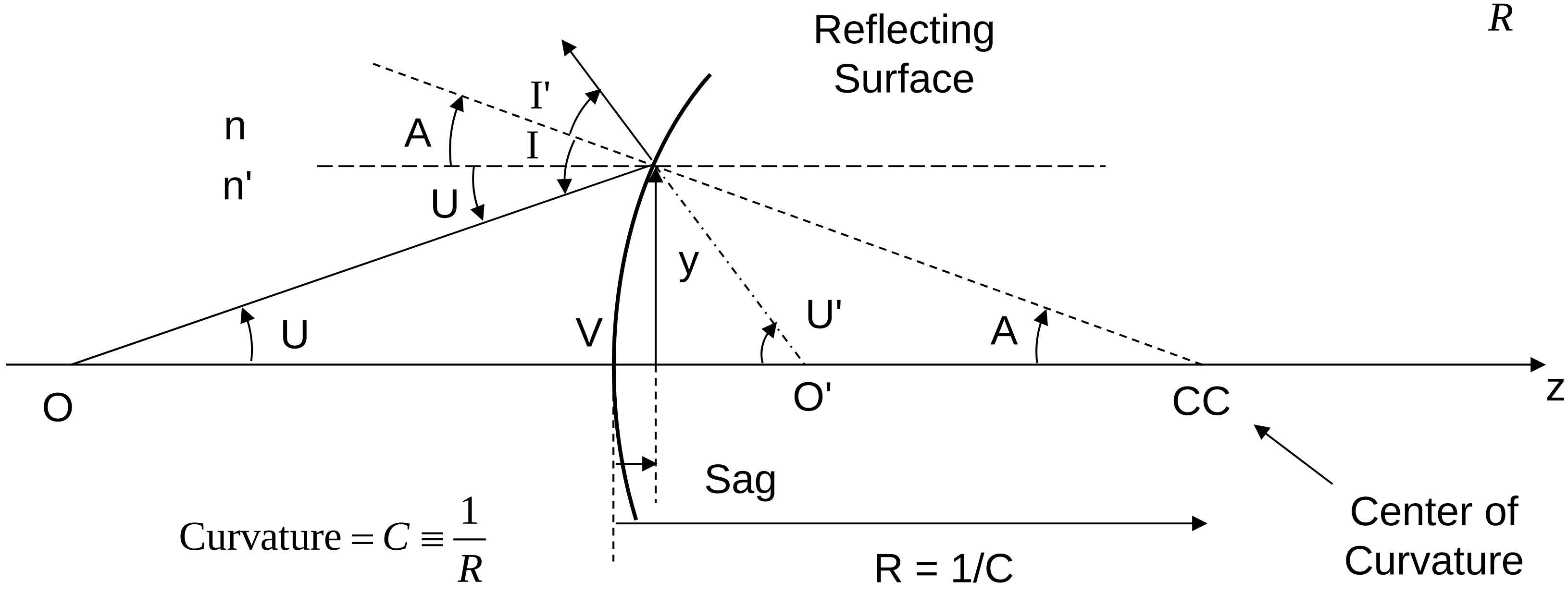
The ray angles U and U', as well as the elevation angle A of the surface normal at the ray intersection, are measured with respect to the optical axis. The usual sign conventions apply.



$$\text{Curvature} = C \equiv \frac{1}{R}$$

Single Reflecting Surface

$$\text{Curvature} = C \equiv \frac{1}{R}$$



Relating the angles at the ray intersection with the surface:

$$U' = A + I'$$

$$I = U - A \qquad I' = U' - A$$

Apply the Law of Reflection:

$$I' = -I$$

$$(U' - A) = -(U - A)$$

Single Reflecting Surface and the Law of Reflection

$$I' = -I$$

$$(U' - A) = -(U - A)$$

$$\sin(U' - A) = -\sin(U - A)$$

$$[\sin U' \cos A - \cos U' \sin A] = -[\sin U \cos A - \cos U \sin A]$$

$$\left[\sin U' - \cos U' \frac{\sin A}{\cos A}\right] = -\left[\sin U - \cos U \frac{\sin A}{\cos A}\right]$$

$$[\sin U' - \cos U' \tan A] = -[\sin U - \cos U \tan A]$$

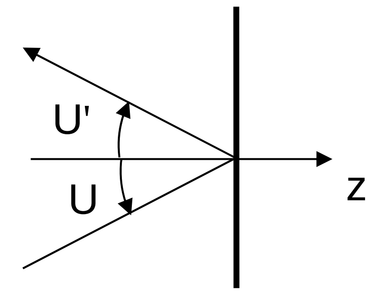
Approximation #1: $\cos U \approx \cos U'$

$$\left[\frac{\sin U'}{\cos U'} - \tan A\right] = -\left[\frac{\sin U}{\cos U} - \tan A\right]$$

$$[\tan U' - \tan A] = -[\tan U - \tan A]$$

Approximation #1 implies that magnitude of the ray angle is approximately constant.

$$|U| \approx |U'|$$



Paraxial Angles

$$\left[\tan U' - \tan A\right] = -\left[\tan U - \tan A\right]$$

$$\tan U' = -\tan U + 2\tan A$$

Reformulate in terms of the paraxial angles or ray slopes:

$$u \equiv \tan U \qquad u' \equiv \tan U' \qquad \alpha \equiv \tan A$$

$$u' = -u + 2\alpha$$

$$\alpha = \tan A = -\frac{y}{(R - Sag)}$$

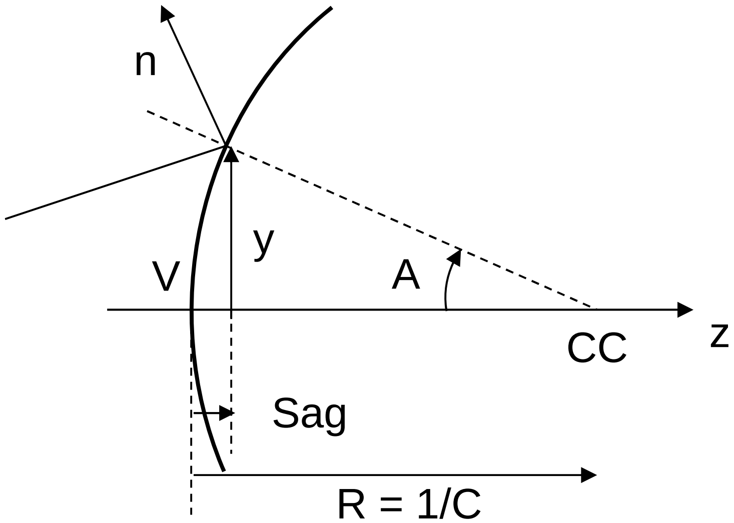
Approximation #2: $|\mathrm{Sag}| << |\mathrm{R}|$

$$\alpha \approx -\frac{y}{R}$$

$$u' = -u - 2\frac{y}{R}$$

$$u' = -u - 2yC$$

This is the Paraxial Reflection Equation.



Approximation #2 implies that the sag of the surface at the ray intersection is much less than the radius of curvature of the surface.

Reflection and Refraction

Refraction: $n'u' = nu - y\phi = nu - y(n' - n)C$

if $n' = -n$

$$nu' = -nu + y\phi = -nu - 2nyC$$

$$u' = -u - 2yC$$

$$\phi = (n' - n)C$$
$$n' = -n$$
$$\phi = -2nC$$

Reflection Equals Refraction with $n' = -n$

$$\phi_{\text{REFLECTION}} = -2nC$$

$$n'u' = nu - y\phi$$

Note that a reflector with a positive curvature has a negative power.

Reflection: $f_F = -\dfrac{n}{\phi} = \dfrac{1}{2C} = \dfrac{R}{2}$

$$f'_R = \frac{n'}{\phi} = -\frac{n}{\phi} = f_F$$

$$f_E = f \equiv \frac{1}{\phi} = -\frac{f_F}{n} = \frac{f'_R}{n'} = -\frac{f'_R}{n}$$

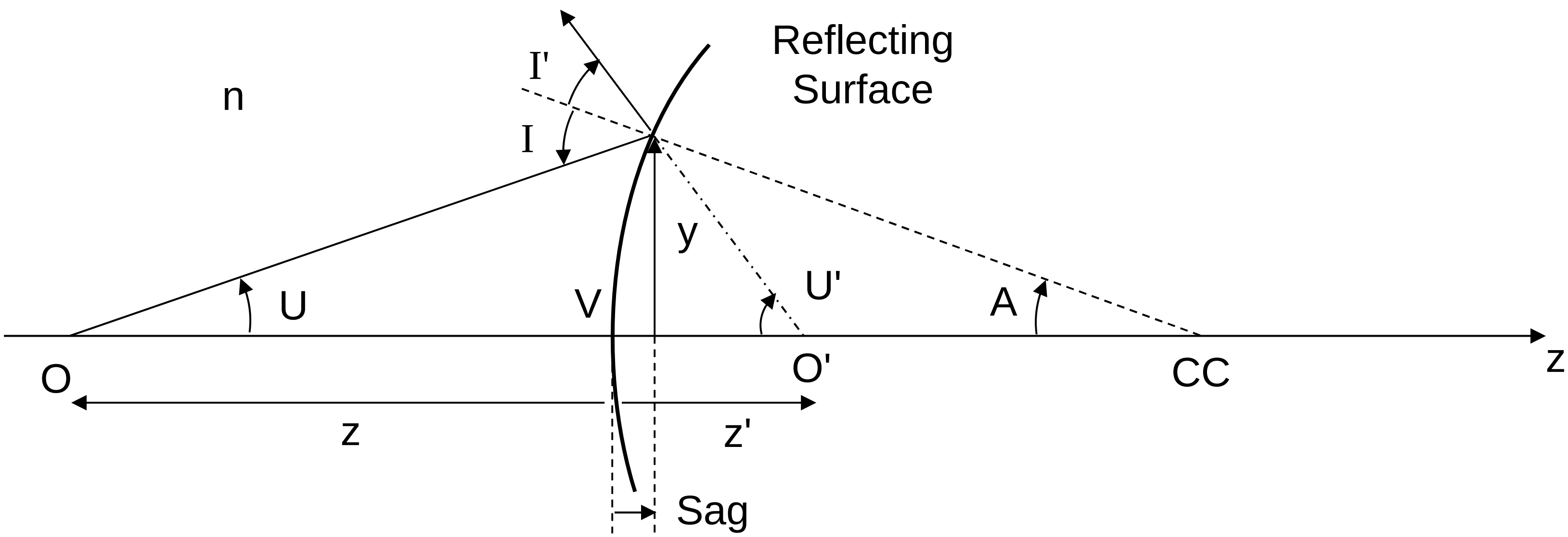
$$f'_R = f_F \qquad n' = -n$$

$$f_F = f'_R = -\frac{n}{\phi} = -nf_E = \frac{1}{2C} = \frac{R}{2}$$

The front and rear focal lengths are equal to half the radius of curvature.

Object and Image distances for a Single Reflecting Surface

The object and image distances (z and z') are also both measured from the surface vertex.



Approximation #3: The object and image distances are much greater than the sag of the surface at the ray intersection.

$$|Sag| << |z| \qquad |Sag| << |z'|$$

$$u = \tan U = -\frac{y}{(z - Sag)} \approx -\frac{y}{z}$$

$$u' = \tan U' = -\frac{y}{(z' - Sag)} \approx -\frac{y}{z'}$$

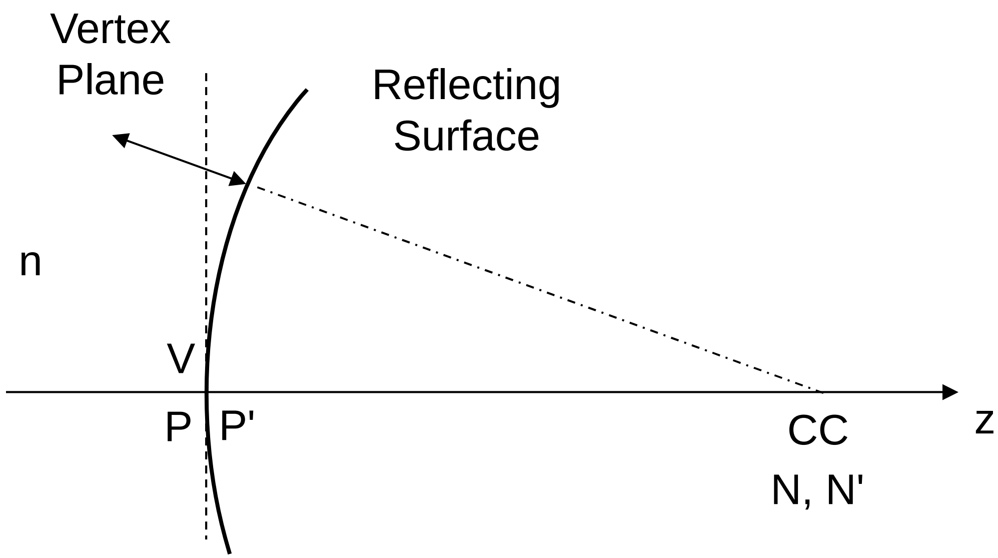
Surface Vertex Plane and Principal Planes

The same set of approximations hold for paraxial analysis of a reflecting surface as for a refraction surface:

> The surface sag is ignored and paraxial reflection occurs at the surface vertex.
>
> The ray bending at each surface is small.

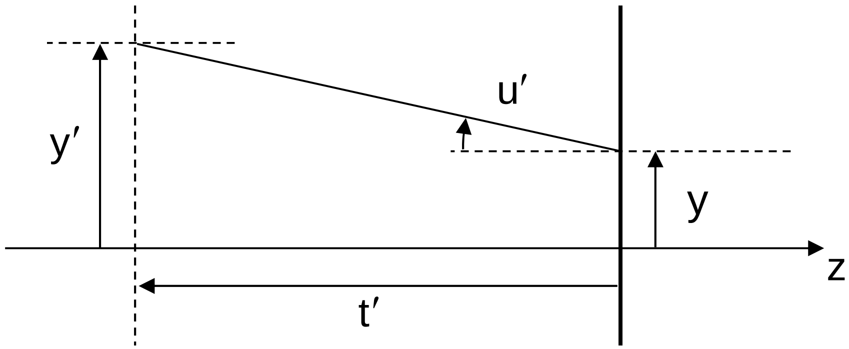
By ignoring the surface sag in paraxial optics, the planes of effective refraction for the single reflecting surface are located at the surface vertex plane V. The Front and Rear Principal Planes (P and P') of the surface are both located at the surface.

The nodal points of a reflecting surface are located at its center of curvature as a ray perpendicular to the surface is reflected back on itself.



<u>Transfer After Reflection</u>

Transfer after reflection works exactly the same as for a refractive system, except that the distance to the next surface (to the left) is negative.



$$t' < 0$$

$$u' = \frac{y' - y}{t'}$$

$$t'u' = y' - y$$

$$y' = y + t'u'$$

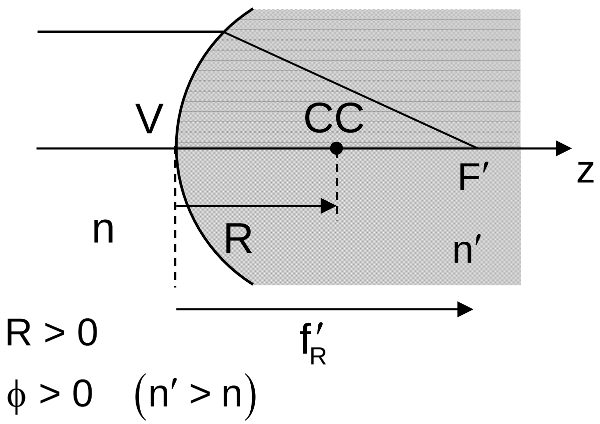
$$y' = y + \tau'\omega'$$

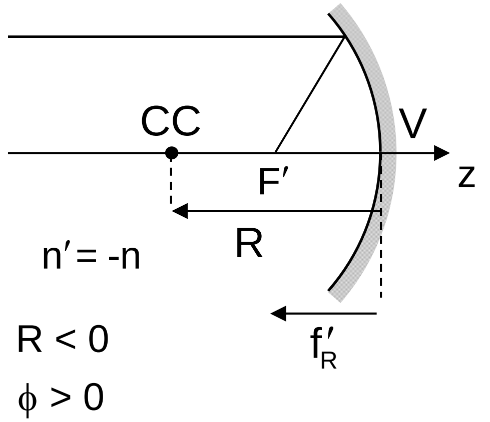
The transfer equation is independent of the direction of transfer.

After reflection, the signs of ω and τ are opposite those of the corresponding u and t. A drawing done in reduced distances and optical angles will unfold the mirror system and show a thin lens equivalent system.

Sign conventions and reflection:
- Use directed distances as defined by the usual sign convention. A distance to the left is negative, and a distance to the right is positive.
- The signs of all indices of refraction following a reflection are reversed.

Optical Surfaces





The front and rear principal planes of an optical surface are coincident and located at the surface vertex V. Both nodal points of a single refractive or reflective surface are located at the center of curvature of the surface.

$$\phi = (n' - n)C = \frac{(n' - n)}{R} \qquad C = \frac{1}{R}$$

$$f = f_E \equiv \frac{1}{\phi}$$

$$f_F = -\frac{n}{\phi} = -nf_E \qquad f'_R = \frac{n'}{\phi} = n'f_E$$

$$f_E = -\frac{f_F}{n} = \frac{f'_R}{n'} \qquad -\frac{f'_R}{f_F} = \frac{n'}{n}$$

A reflective surface is a special case with $n' = -n$

$$\phi = -2nC = -\frac{2n}{R} \qquad f_F = f'_R = -\frac{n}{\phi} = -nf_E = \frac{R}{2} = \frac{1}{2C}$$

Refractive and Reflective Surfaces

Power of a refractive surface:

$$\phi = (n' - n)C$$

Assume $n = 1$ and $n' = 1.5$

$$\phi = (0.5)C = C/2$$

Power of a reflective surface:

$$\phi = (n' - n)C = -2nC$$

Assume $n = 1$

$$\phi = -2C$$

For the same optical power, a reflective surface requires approximately one quarter of the curvature of a refractive surface. However the signs of the powers are opposite.

This is one advantage of using reflective surfaces.

Optical Surfaces – Cardinal points

$$\phi = (n' - n)C = \frac{(n' - n)}{R} \qquad C = \frac{1}{R}$$

$$f_F = -\frac{n}{\phi} = -nf_E \qquad f'_R = \frac{n'}{\phi} = n'f_E$$

$P,\ P'$ at surface vertex

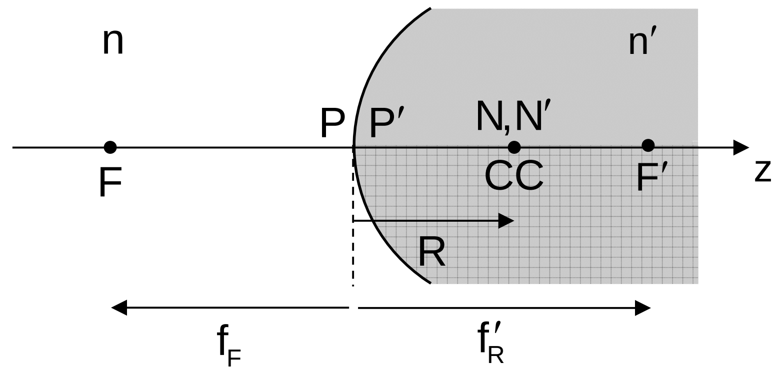
$N,\ N'$ at center of curvature

Positive Refracting:

$\phi > 0$

$n' > n$

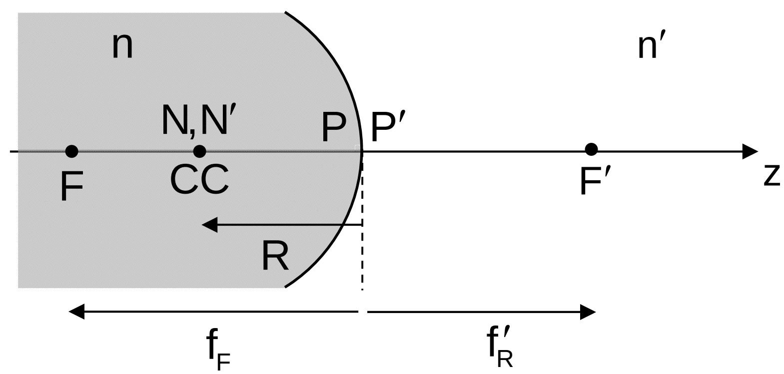
$C > 0$



$\phi > 0$

$n' < n$

$C < 0$



Optical Surfaces – Cardinal points

$$\phi = (n' - n)C = \frac{(n' - n)}{R} \qquad C = \frac{1}{R}$$

$$f_F = -\frac{n}{\phi} = -n f_E \qquad f'_R = \frac{n'}{\phi} = n' f_E$$

$P,\ P'$ at surface vertex

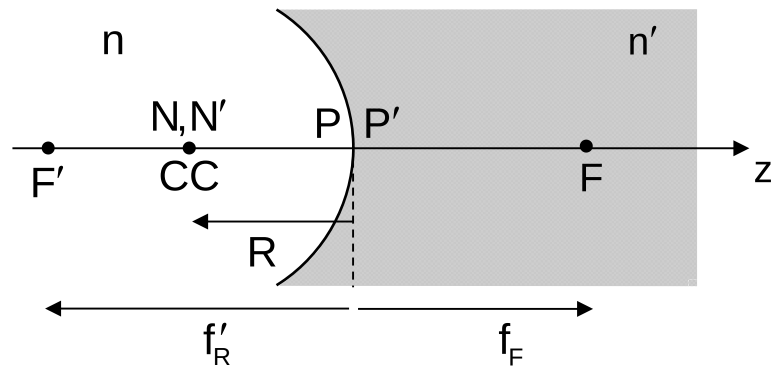
$N,\ N'$ at center of curvature

Negative Refracting:

$\phi < 0$

$n' > n$

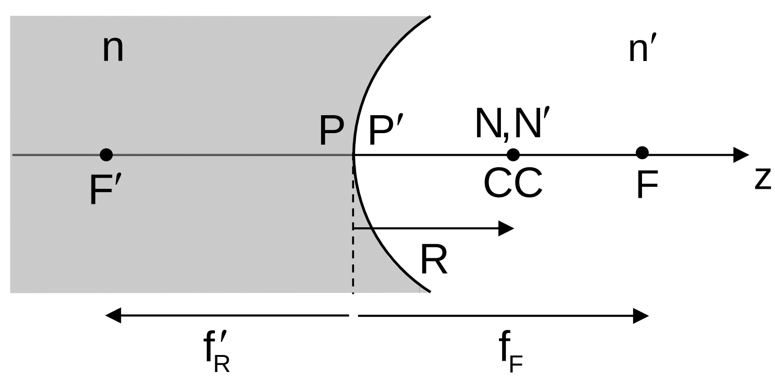
$C < 0$



$\phi < 0$

$n' < n$

$C > 0$



Optical Surfaces – Cardinal points

$$\phi = -2nC = \frac{-2n}{R} \qquad C = \frac{1}{R}$$

$P,\ P'$ at surface vertex

$N,\ N'$ at center of curvature

$$n' = -n$$

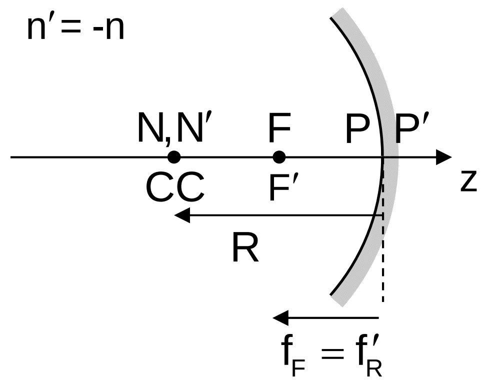
$$f'_R = \frac{n'}{\phi} = -\frac{n}{\phi} = f_F \qquad f_F = f'_R = -nf_E \qquad f_F = f'_R = \frac{R}{2}$$

Positive Reflecting:

$\phi > 0$

$C < 0$

$n > 0$

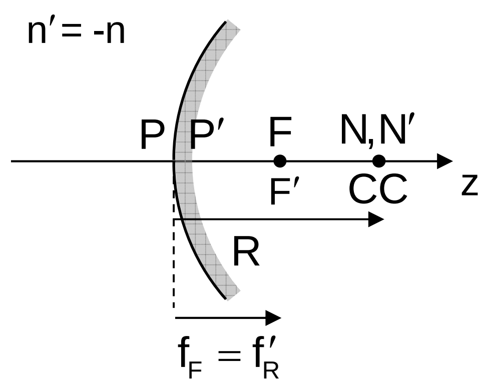


Negative Reflecting:

$\phi < 0$

$C > 0$

$n > 0$

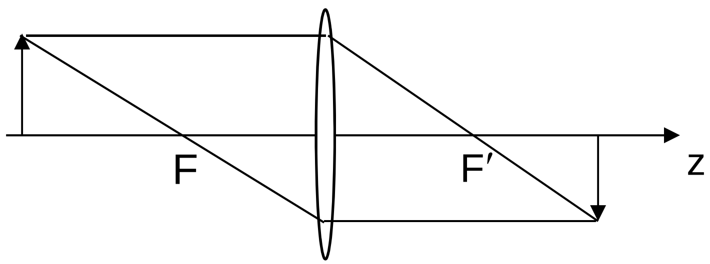


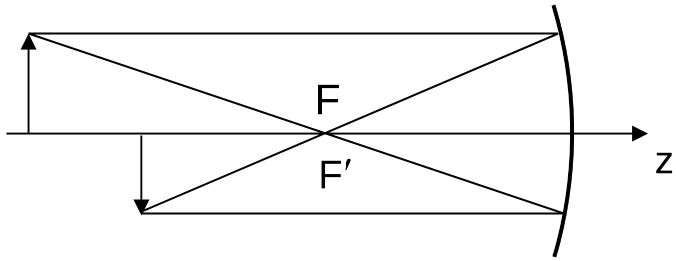
A concave mirror has a positive power and focal length, but negative front and rear focal lengths.
A convex mirror has a negative power and focal length, but positive front and rear focal lengths.

Real and Virtual Images

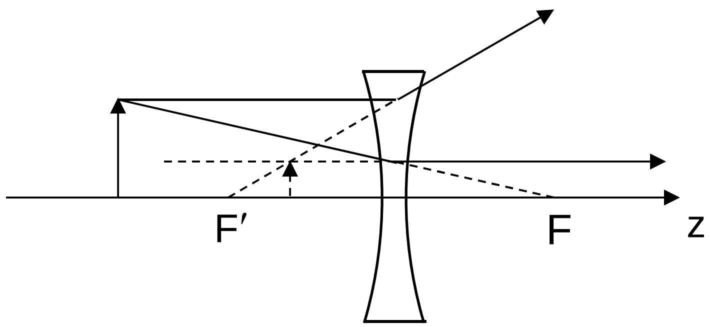
Real images can be projected and made visible on a screen; virtual images cannot.

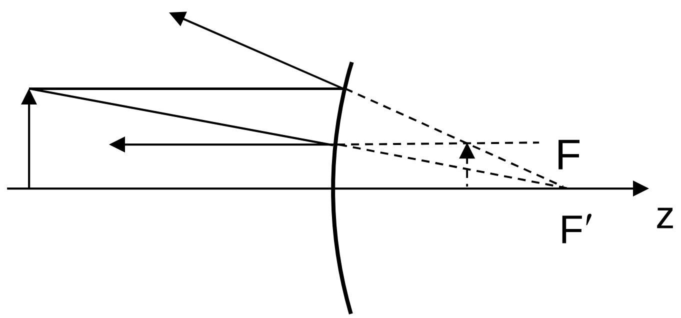
Real images– the actual rays in image space head towards the image.





Virtual – the actual rays in image space head away from the image. The rays must be projected backwards to find the image (virtual ray segments).



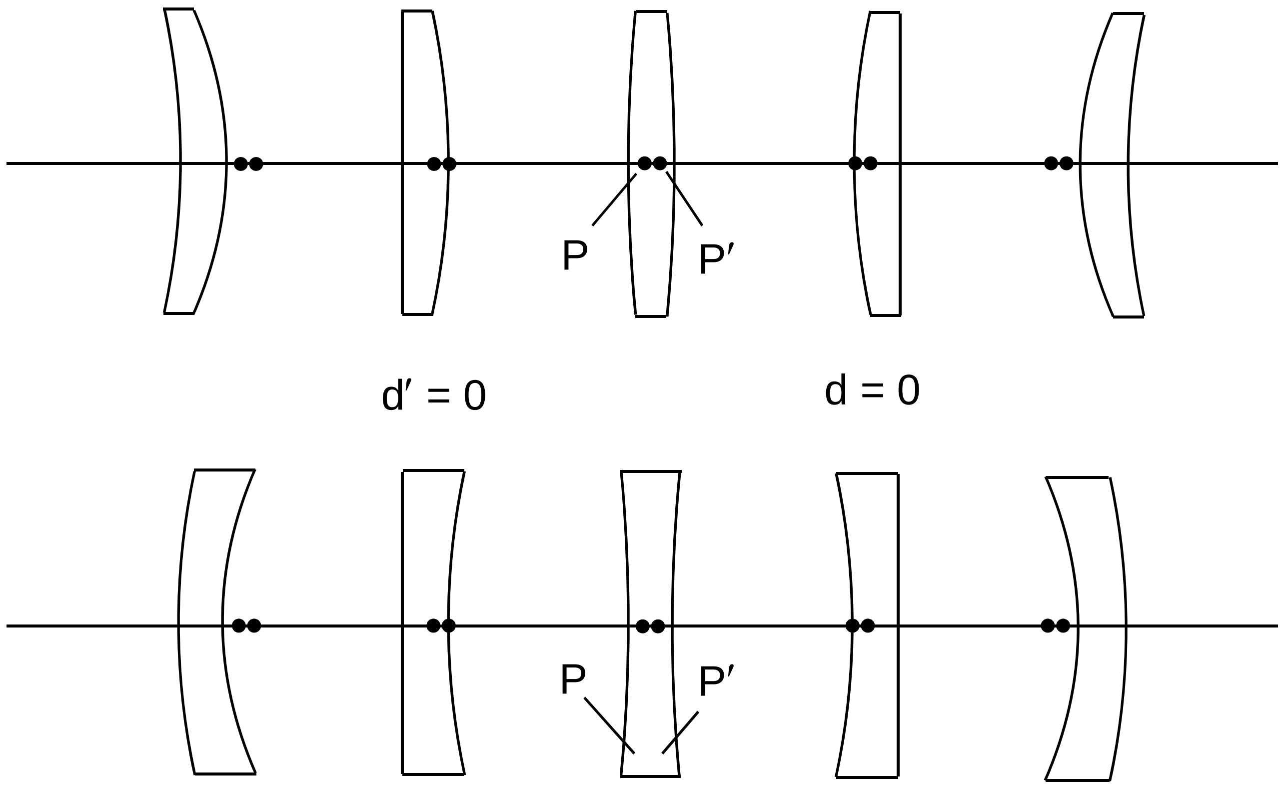


Lens Bending

The power of a thin lens is proportional to the difference in the surface curvatures:

$$\phi = (n-1)(C_1 - C_2)$$

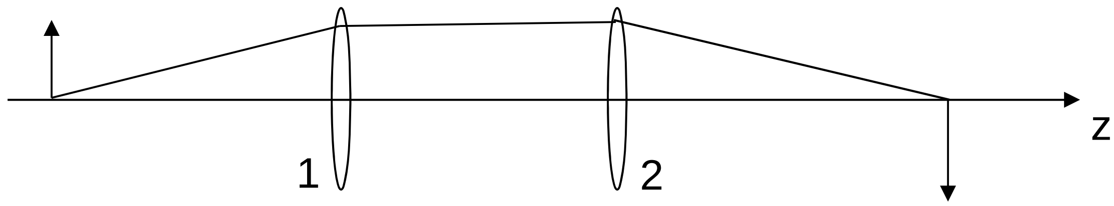
Even for a thick element, different shape lenses can be used to get the same power of focal length. The locations of the principal planes shift.



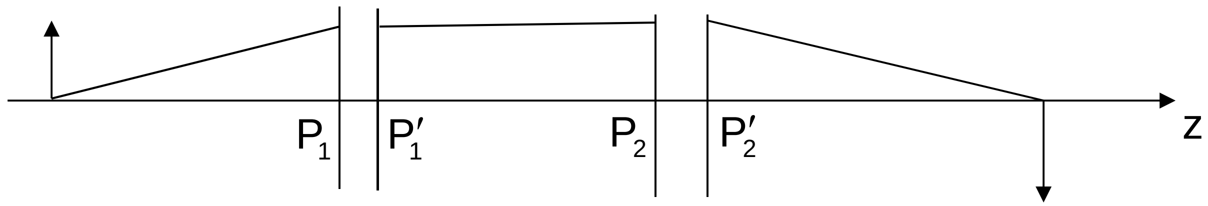
For a relatively thin lens, the principal plane separation is independent of the lens bending.

System Design Using Thin Lenses

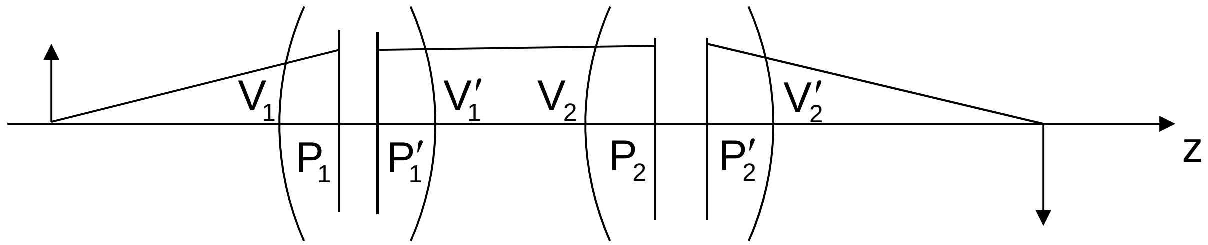
1) Obtain the thin lens solution to the problem:



2) Include the principal plane separations of real elements:



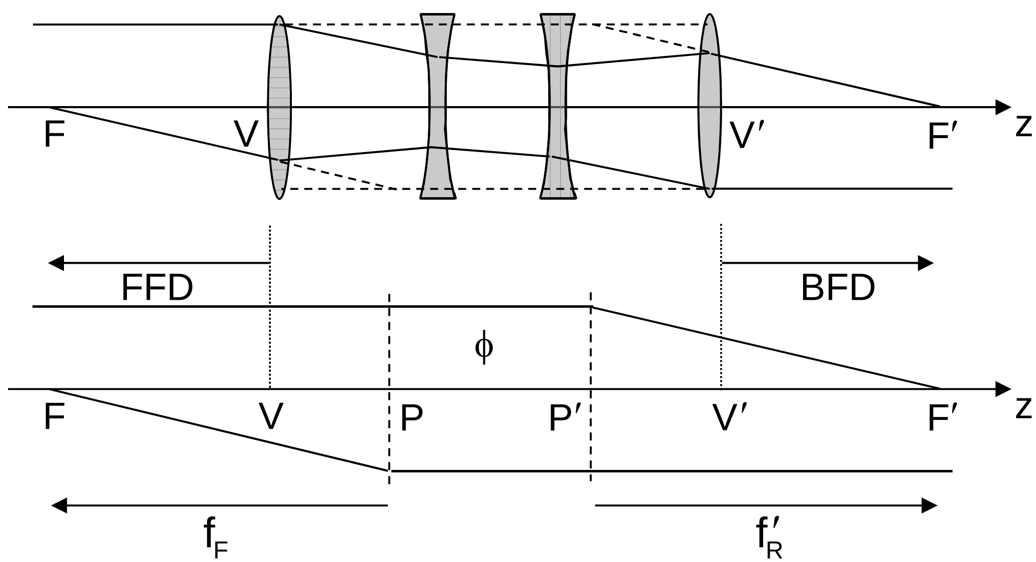
3) Locate the vertices of the real components:



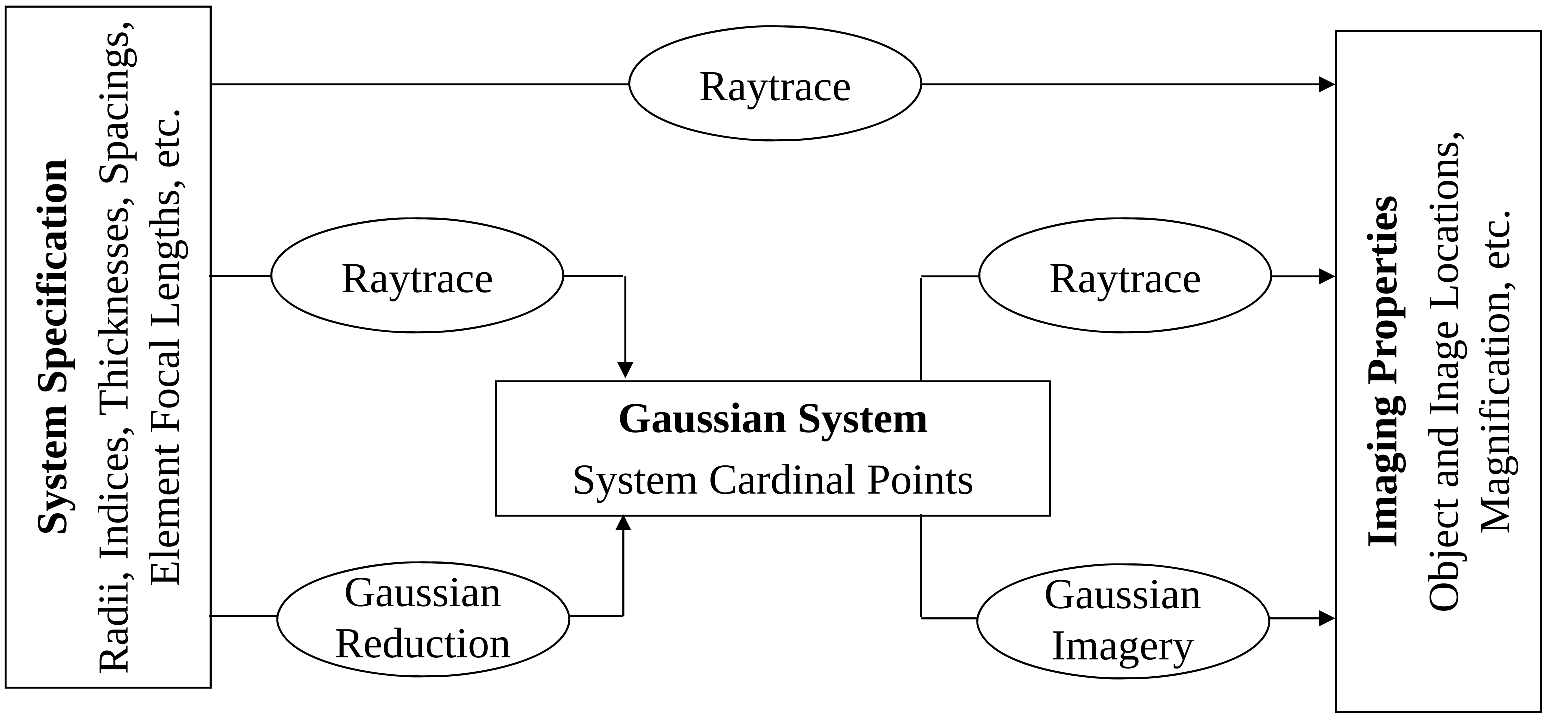
The vertices and vertex-to-vertex separations are the mechanical datums for the system.

Gaussian Imagery and Gaussian Reduction

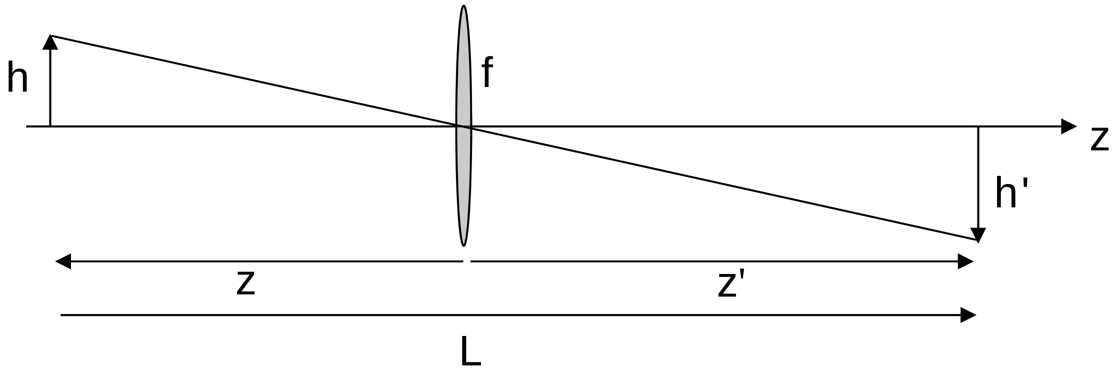
The utility of Gaussian optics and Gaussian reduction is that the imaging properties of any combination of optical elements can be represented by a system power or focal length, a pair of principal planes and a pair of focal points. In initial design, the P-P' separation is often ignored (i.e. a thin lens model).



Methods of System Analysis



Thin Lens Design – Overall Object-to-Image Distance



$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_E} \qquad m = \frac{h'}{h}$$

$$z = \left(\frac{1-m}{m}\right) f_E \qquad z' = (1-m)\, f_E$$

$$L = z' - z = (1-m)\, f_E - \left(\frac{1-m}{m}\right) f_E$$

$$L = -\frac{(m-1)^2}{m} f_E$$

Real object and real image: $m < 0$ $\quad z < -f_E$ $\quad z' > f_E$

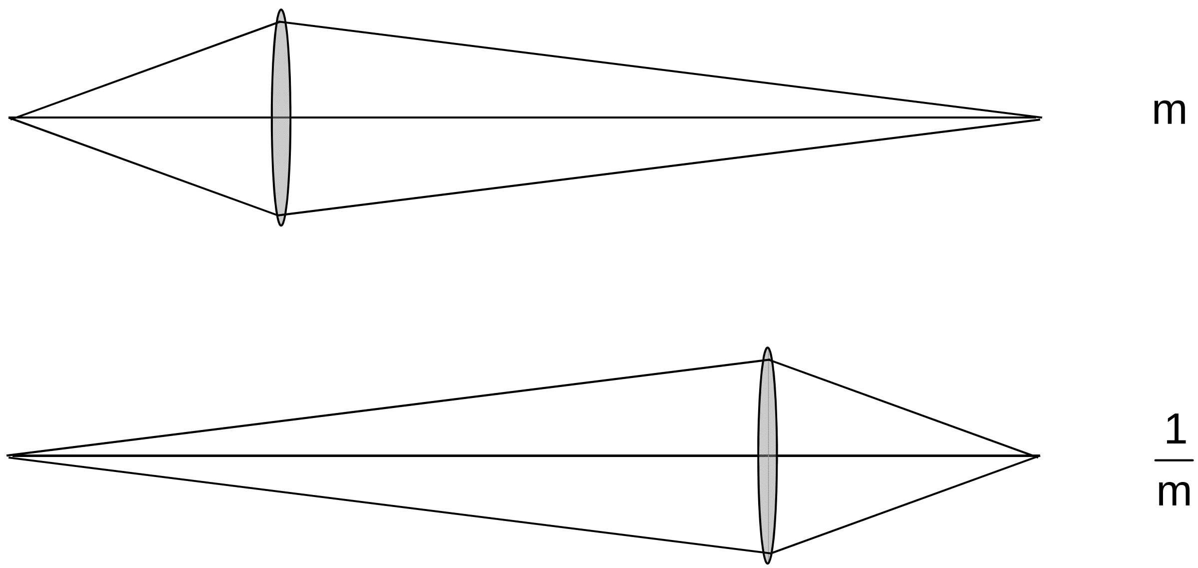
Minimum object to image distance: $m = -1$ $\quad L = 4 f_E$

Reciprocal Magnifications

Overall object-to-image distance:

$$L = -\frac{(m-1)^2}{m} f_E$$

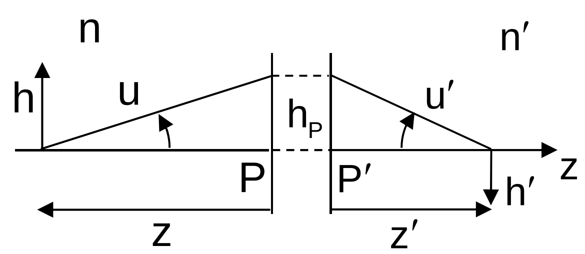
For each L, there are two possible magnifications and conjugates: Reciprocal magnifications.



$$\frac{L}{f_E} = -\frac{(m-1)^2}{m} = -\frac{\left(1/m - 1\right)^2}{1/m}$$

Magnification Properties

The Gaussian Magnification may also be determined from the object and image ray angles.



$$m \equiv \frac{h'}{h} = \frac{z'/n'}{z/n}$$

$$u = -\frac{h_P}{z} \qquad u' = -\frac{h_P}{z'}$$

$$nu = \omega = -\frac{h_P}{z/n} \qquad n'u' = \omega' = -\frac{h_P}{z'/n'}$$

$$z/n = -\frac{h_P}{\omega} \qquad z'/n' = -\frac{h_P}{\omega'}$$

$$m = \frac{h'}{h} = \frac{z'/n'}{z/n} = \frac{h_P/\omega'}{h_P/\omega} = \frac{\omega}{\omega'}$$

$$m = \frac{\omega}{\omega'} = \frac{nu}{n'u'}$$

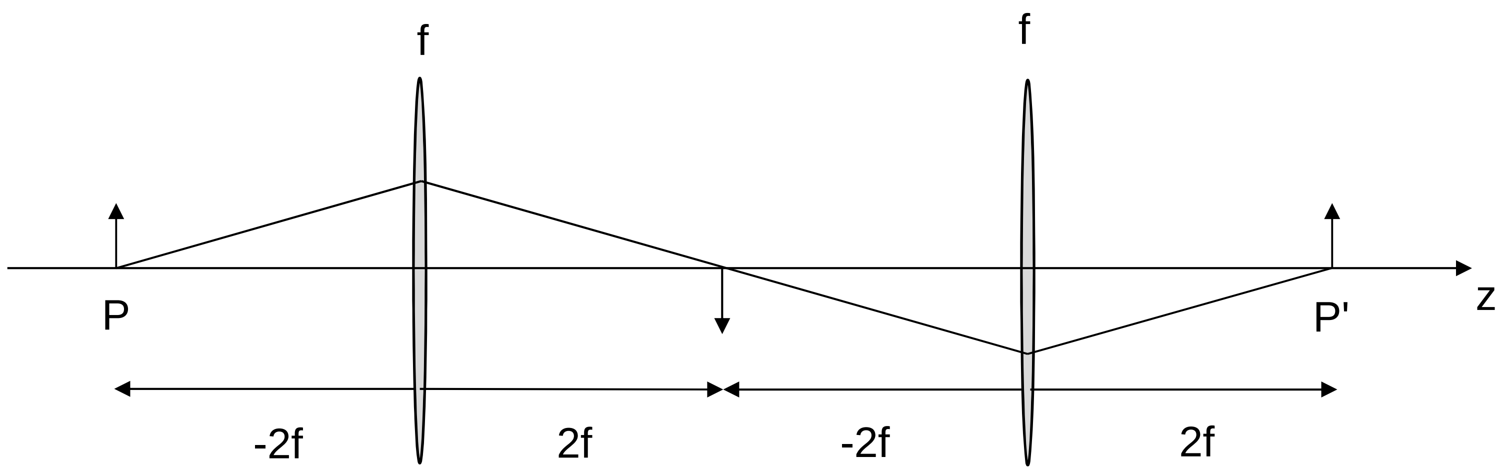
This angle relationship holds for all rays passing through on-axis conjugate points.

Cardinal Points Example

The power and the relative locations of the cardinal points of a system completely define the imaging mapping.

Different combinations of elements can produce interesting situations.

As an example consider this true 1:1 imaging system consisting of cascaded 2f-2f systems. An inverted intermediate image is formed.

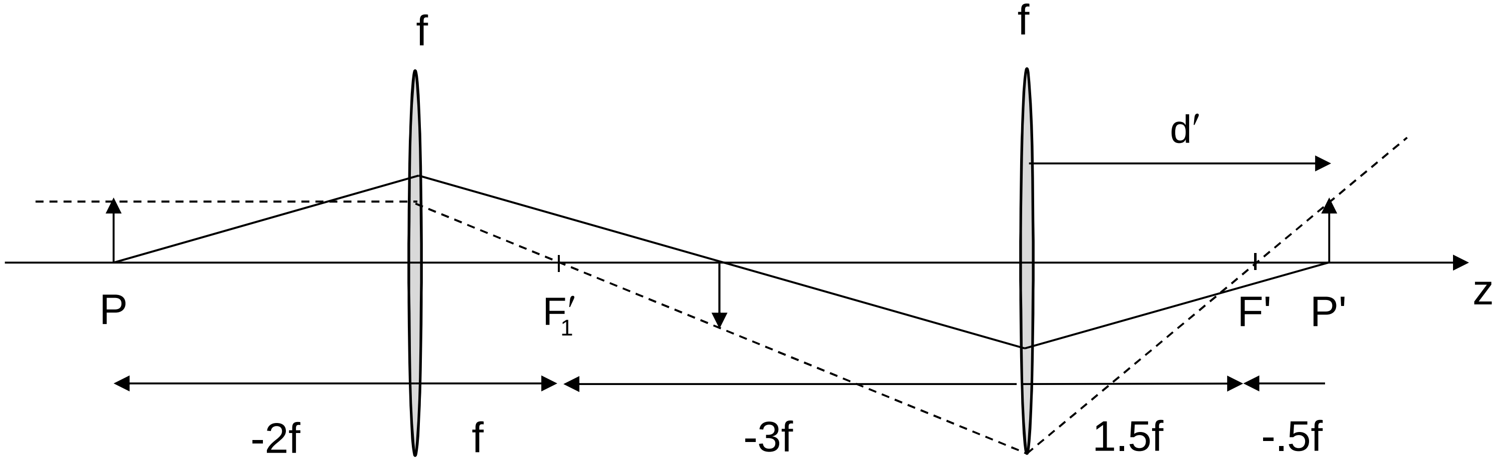


Because the object and image planes are planes of unit magnification, the system front and rear principal planes are coincident with the object and image planes.

Where are the focal points?

Cardinal Points Example - Continued

To find the rear focal point of the system, launch a ray parallel to the axis:



The system rear focal point is to the left of the system rear principal plane, and the system power is negative! This infinity ray diverges from the rear principal plane.

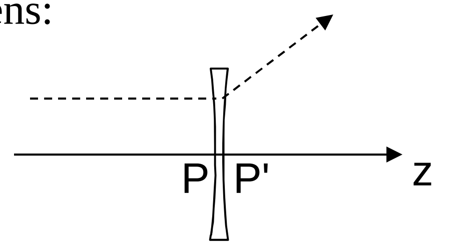
$$\phi_1 = \phi_2 = \frac{1}{f} \qquad t = 4f$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t = \frac{2}{f} - \frac{4f}{f^2} = -\frac{2}{f}$$

$$f_{SYSTEM} = f'_R = -0.5f$$

$$d' = -\frac{\phi_1}{\phi}t = -\frac{1/f}{-2/f}4f = 2f$$

In a simplified Gaussian model that ignores the P-P' separation, this system looks just like a negative thin lens:



By symmetry, the front focal point of the system is to the right of the system front principal plane.

Mini Quiz

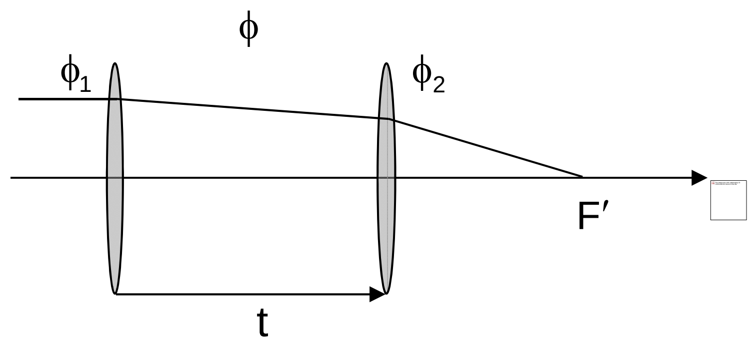
Two 100 mm focal length thin lenses are separated by 50 mm. What is the focal length of this combination of lenses?

[ ] a. 66.67 mm
[ ] b. 200 mm
[ ] c. 50 mm
[ ] d. Infinity

Mini Quiz – Solution

Two 100 mm focal length thin lenses are separated by 50 mm. What is the focal length of this combination of lenses?

[X] a. 66.67 mm
[ ] b. 200 mm
[ ] c. 50 mm
[ ] d. Infinity



$$f_1 = f_2 = 100\,mm \qquad \phi = \frac{1}{f} \qquad \phi_1 = \phi_2 = 0.01\,mm^{-1}$$

$$t = 50\,mm$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t$$

$$\phi = 0.015\,mm^{-1}$$

$$f = 66.67\,mm$$