



## Section 3

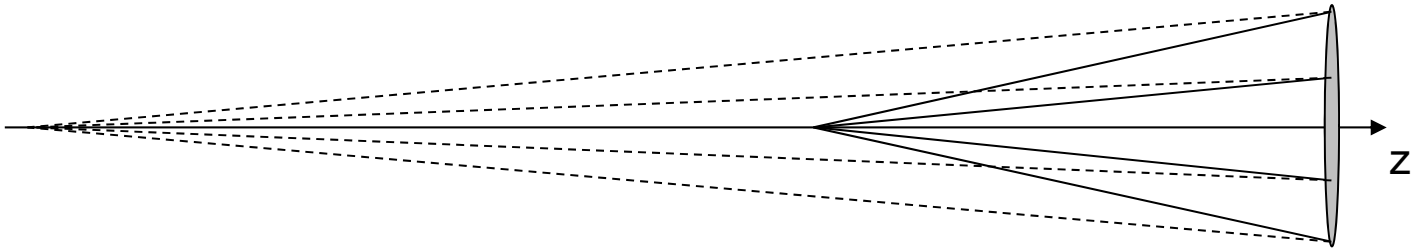
# Imaging With A Thin Lens

## Object at Infinity

An object at infinity produces a set of collimated set of rays entering the optical system.

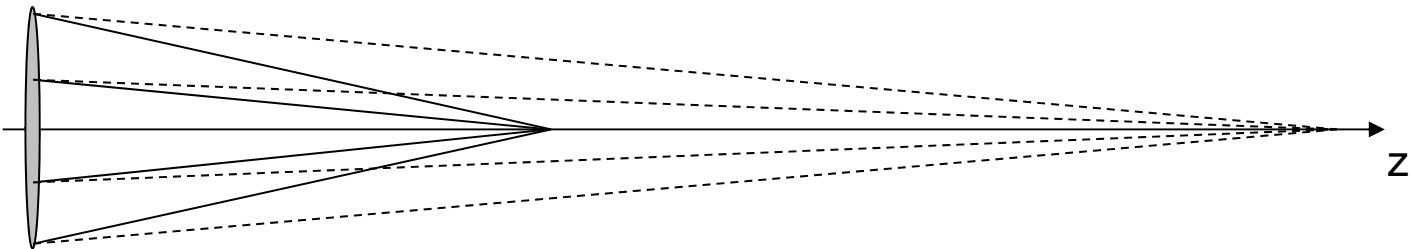
Consider the rays from a finite object located on the axis.

When the object becomes more distant the rays become more parallel.



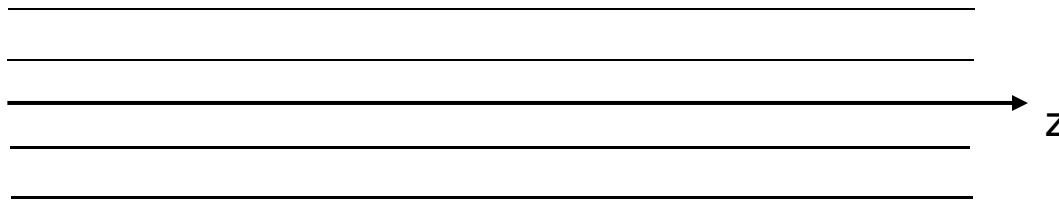
When the object goes to infinity, the rays become parallel or collimated.

An image at infinity is also represented by collimated rays.



## Parallel Rays

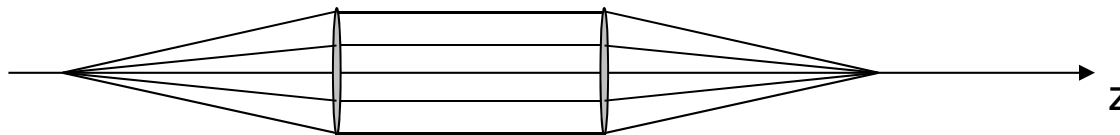
Parallel rays are used to represent either an object at infinity or an image at infinity.



One way to think of this is that parallel lines intersect at a point at infinity. Without a lens, these rays could represent either an object at negative infinity or an image at positive infinity. With a lens, it is easy to tell an object from an image:



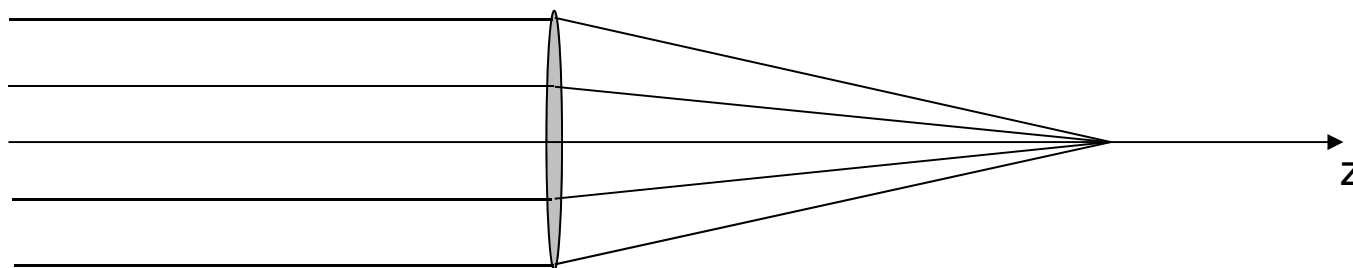
But an image at positive infinity can serve as an object at negative infinity:



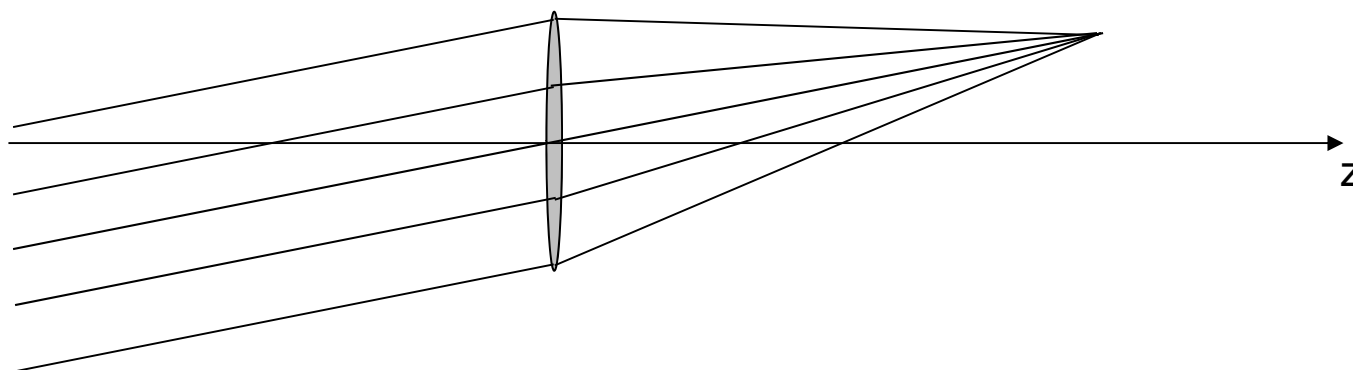
Infinity is infinity and it does not matter if we are discussing positive or negative infinity.

## On Axis and Off Axis – Object at Infinity

An on-axis object is aligned with the optical axis and the image will be on the optical axis:



For an off-axis object, collimated light still enters the optical system, but at an angle with respect to the optical axis. The image is formed off the optical axis.



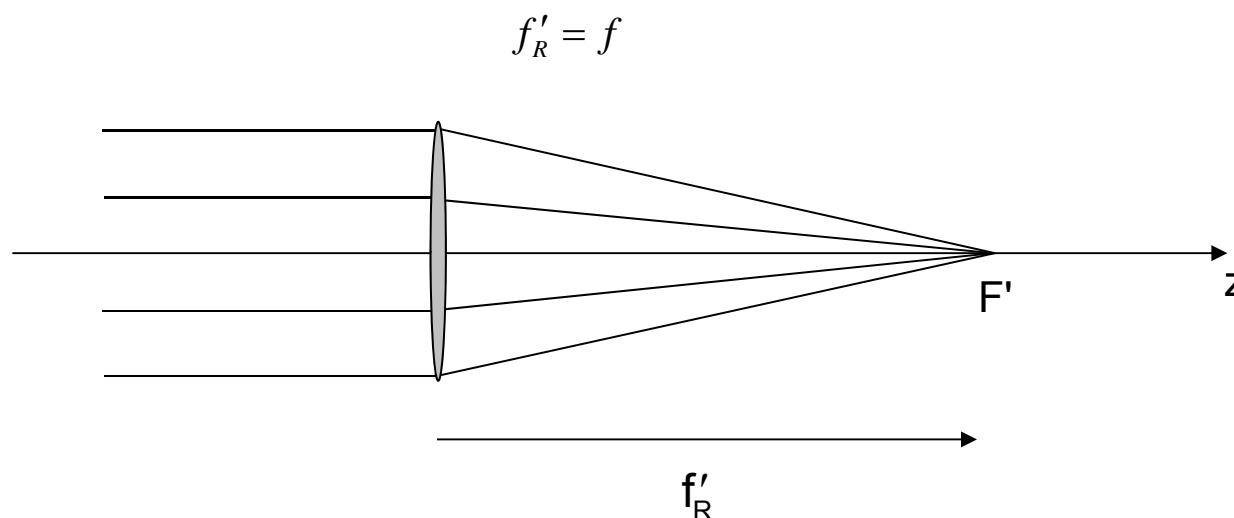
In practice, no object is ever really at infinity – just at a great distance. The rays from that distant object point are “approximately” collimated, but the approximation is incredibly good!

## Imaging with a Thin Lens in Air

Many optical systems are first modeled as a thin lens\*. A thin lens is an optical element with zero thickness that has refracting power. It is almost always used in air ( $n = n' = 1.0$ ) and is characterized by its focal length  $f$ .

An object at infinity is imaged to the *Rear Focal Point* of the lens  $F'$ .

- All rays parallel to the axis produce rays that cross at the rear focal point.
- The distance from the lens to the rear focal point is the *rear focal length* of the lens.



\* This discussion should actually refer to “paraxial lenses” which are perfect first-order lenses of zero thickness. Thin lenses, which also have zero thickness, may have aberrations. However, the term “thin lens” is commonly used to describe both of these idealized elements.

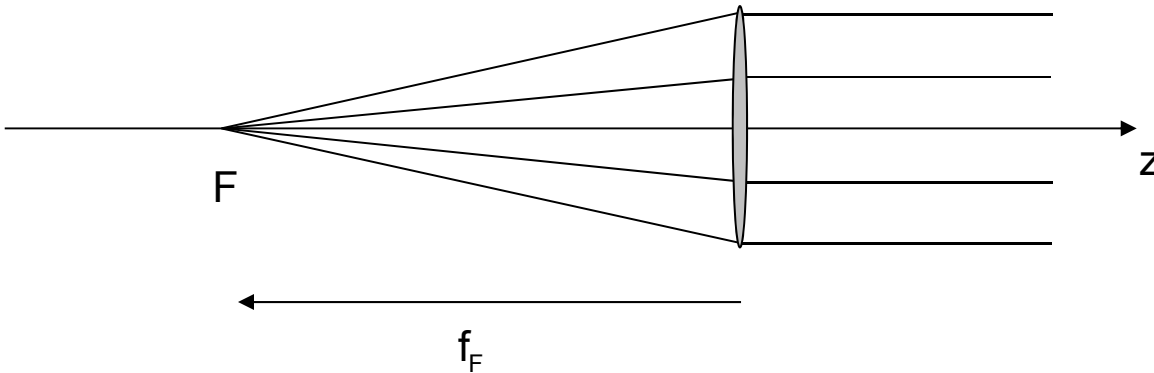


## Thin Lens – Front Focal Point

An object at the *Front Focal Point* of the lens  $F$  is imaged to infinity.

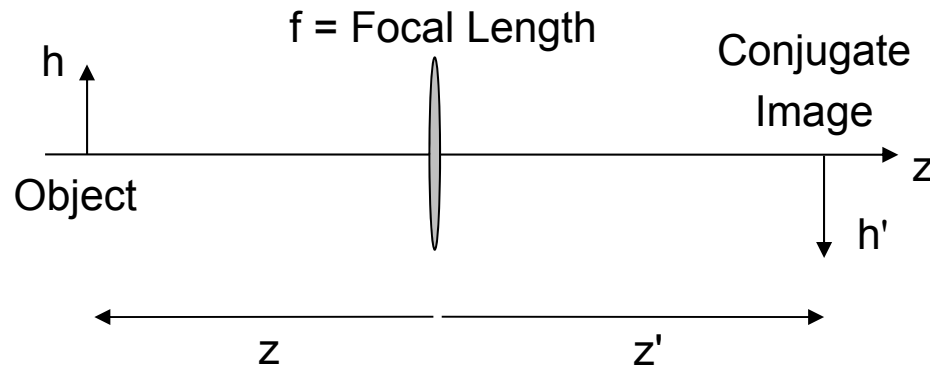
- All rays crossing at the front focal point emerge from the lens parallel to the axis.
- The distance from the lens to the front focal point is the *front focal length* of the lens which equals minus the focal length.

$$f_F = -f$$



## Thin Lens - Conjugates

An object and its image are conjugate, and the respective distances from the lens are called conjugate distances ( $z$  and  $z'$ ) or object and image distances.



Because of sign conventions, an object to the left of the lens has a negative object distance.

The magnification of this object and image is defined as the ratio of the image height to the object height:

$$m \equiv \frac{h'}{h}$$

In this figure,  $h'$  is negative so that the magnification is also negative.

A pair of conjugate planes (the object and its image) is related by their magnification.

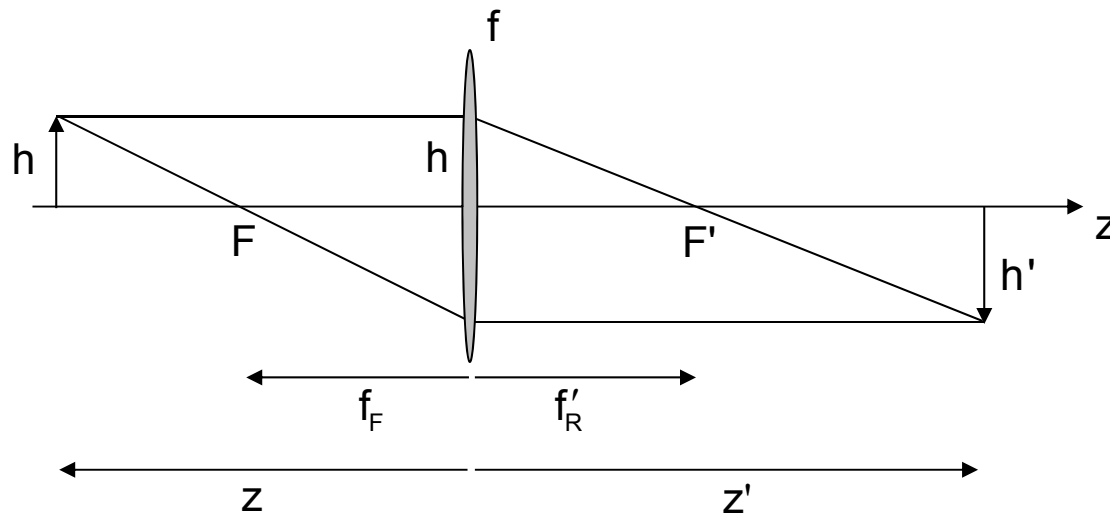
- A given magnification defines a unique pair of conjugate planes.
- For every object position  $z$ , there is a single image location  $z'$ .
- Each object position (or image position) has a unique, associated magnification.



## Thin Lens – Imaging Relationships

The relationships between the object position, the image position, the magnification, and the focal length can be determined from the properties of the focal points.

- A point is defined by the intersection of two rays.
- Construct two rays intersecting at the object point.
- Determine the two conjugate rays.
- The image point is found at the intersection of these two rays.



$$f_F = -f$$

$$f'_R = f$$

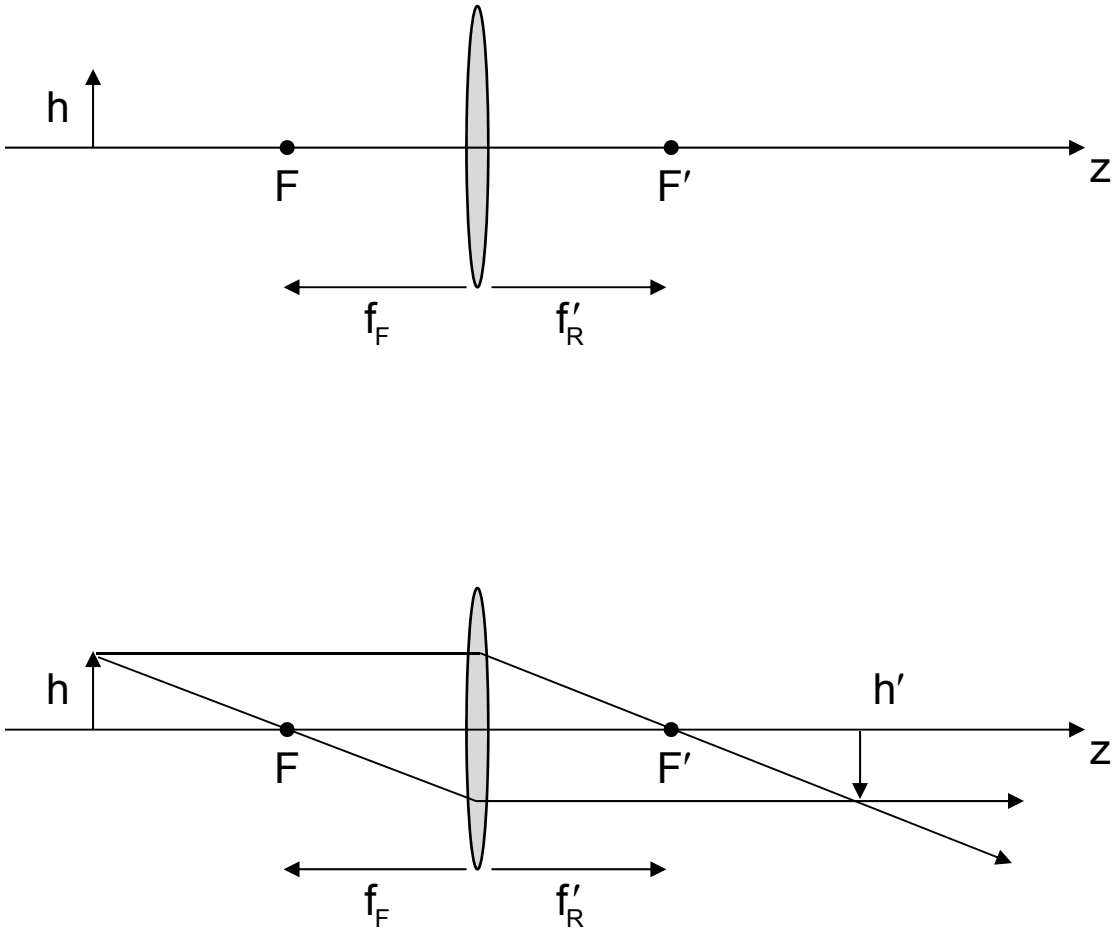
The two rays that are normally used are:

- A ray parallel to the optical axis emerges through the rear focal point.
- A ray through the front focal point emerges parallel to the optical axis.



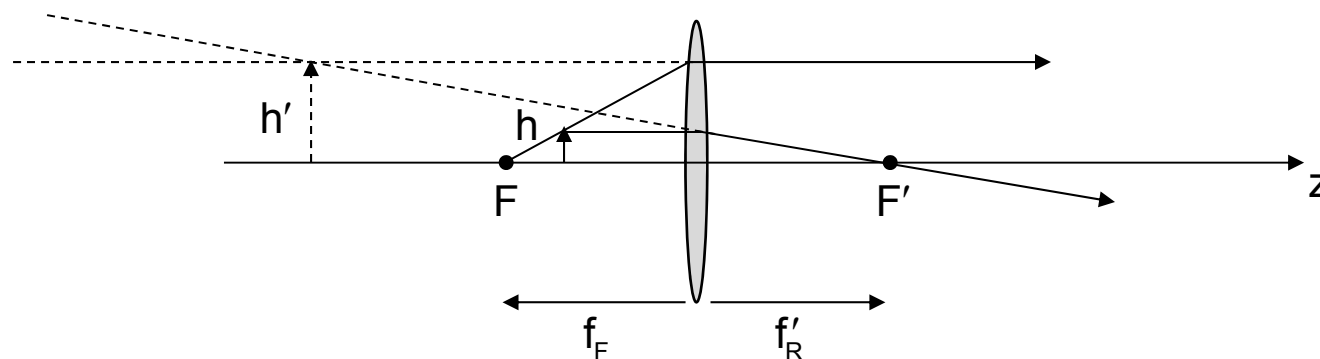
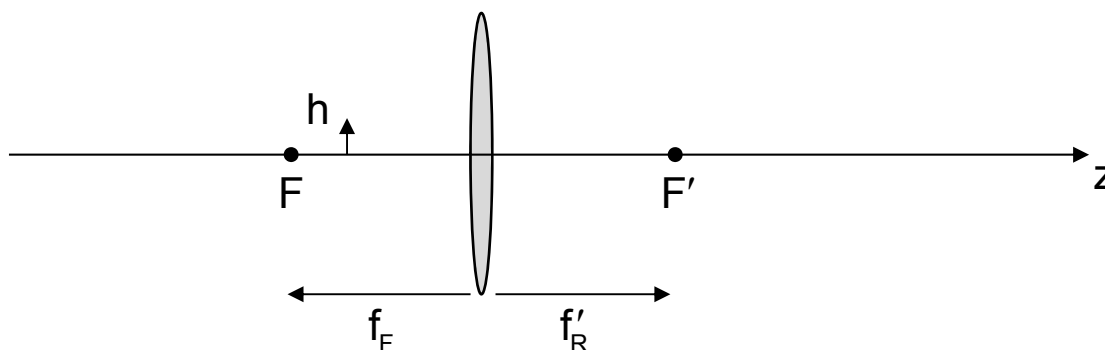
## Locating an Image with the Focal Points – Example 1

Positive Lens– Real object to the left of the front focal point  $F$



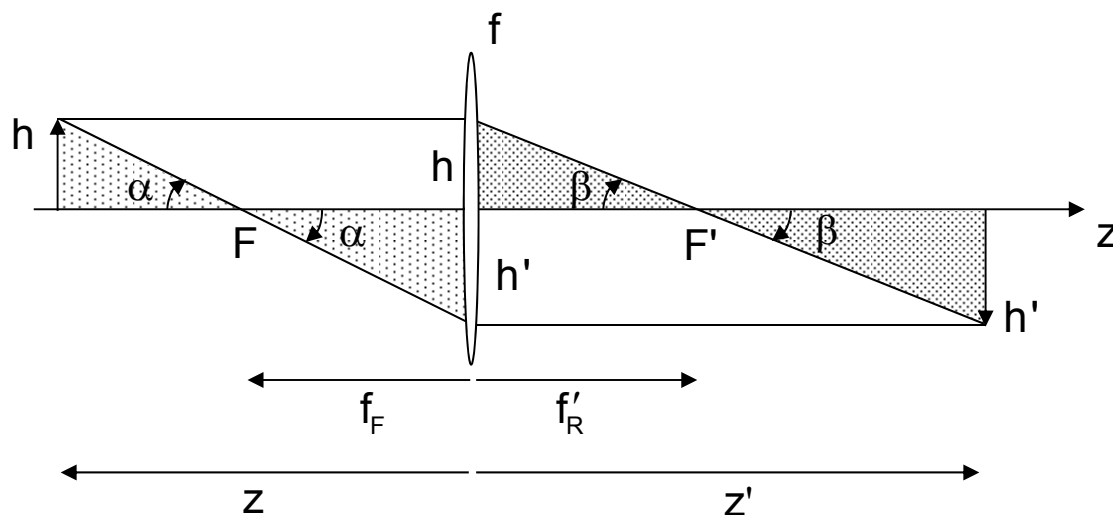
## Locating an Image with the Focal Points – Example 2

Positive Lens– Real object to the right of the front focal point  $F$



The two image rays diverge and have a virtual crossing. An enlarged, erect virtual image is produced. The image is in image space.

## Thin Lens – Imaging Relationships - Derivation



$$f_F = -f$$

$$f'_R = f$$

$$\alpha < 0$$

$$\beta < 0$$

Similar triangles:

$$\frac{h}{z - f_F} = \tan \alpha = \frac{h'}{-f_F}$$

$$\frac{h}{-f'_R} = \tan \beta = \frac{h'}{z' - f'_R}$$

$$m \equiv \frac{h'}{h} = \frac{-f_F}{z - f_F}$$

$$m \equiv \frac{h'}{h} = \frac{z' - f'_R}{-f'_R}$$

$$\frac{1}{m} = 1 - \frac{z}{f_F} = 1 + \frac{z}{f}$$

$$m = 1 - \frac{z'}{f'_R} = 1 - \frac{z'}{f}$$

$$\frac{z}{f} = \frac{1}{m} - 1 \quad \frac{z}{f_F} = 1 - \frac{1}{m}$$

$$\frac{z'}{f} = \frac{z'}{f'_R} = 1 - m$$

These equations give the conjugate distances for a particular magnification.

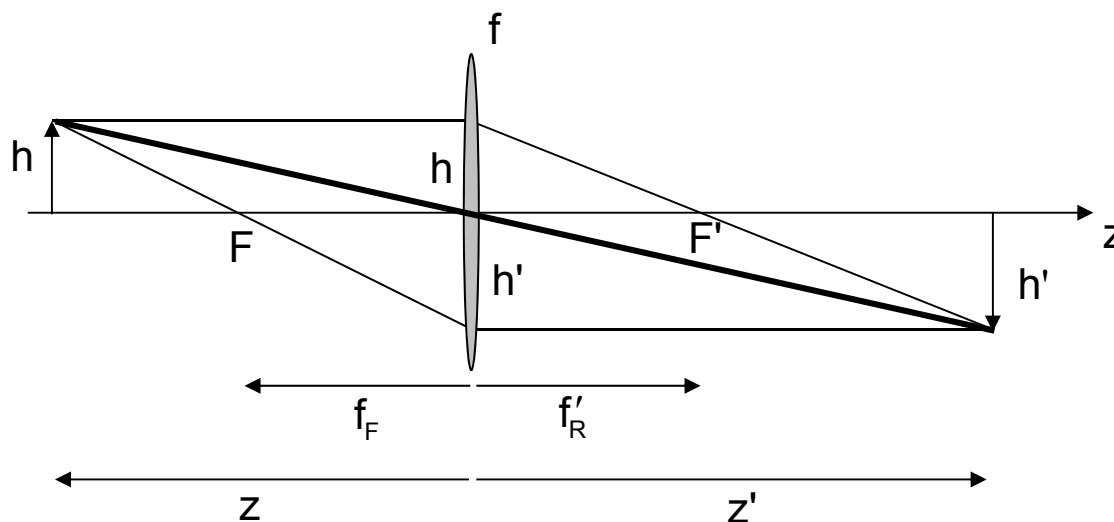


## Thin Lens - Magnification

To determine the magnification as a function of  $z$  and  $z'$  (eliminating  $f$ ), take the ratio of the last two equations:

$$\frac{z'}{z} = \frac{(1-m)f}{\left(\frac{1}{m}-1\right)f} = \frac{1-m}{1-m}m \qquad m = \frac{z'}{z}$$

This last equation implies that a ray drawn from the object point to the image point must pass through the center of the lens.



The center of the lens can be considered to be a very thin plane parallel plate. The ray is not deviated, and its displacement is zero since the plate thickness is zero for an ideal thin lens.



## Thin Lens – Imaging Equation

One final result is to relate the object and image conjugate distances directly with the focal length (eliminate the magnification).

Return to the equation for  $z'$  in terms of  $m$ :

$$\frac{z'}{f} = 1 - m$$

$$\frac{z'}{f} = 1 - \frac{z'}{z}$$

$$\frac{1}{f} = \frac{1}{z'} - \frac{1}{z}$$

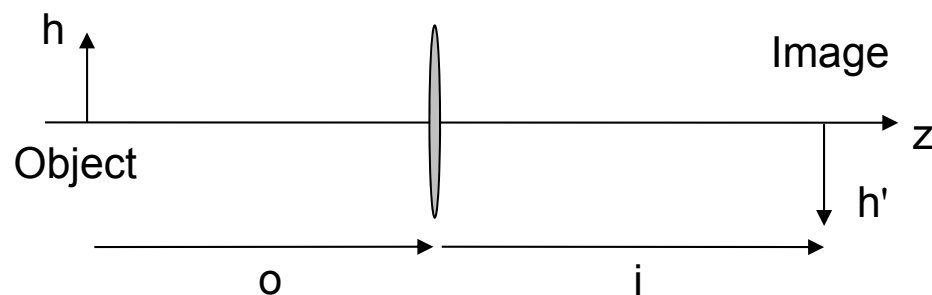
$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

This form of the imaging equation differs from what is usually stated due to sign conventions. We use an object distance measured from the lens (and therefore usually negative.)



## Impact of Sign Conventions – Imaging Equation

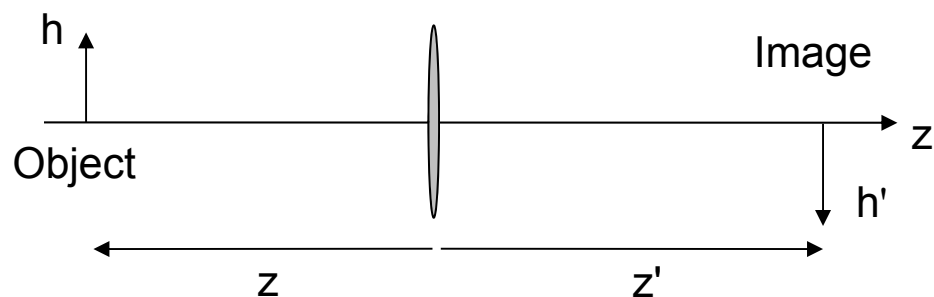
Traditional representation:



$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

There is inconsistency in the defined object and image distances

Sign convention representation:



$$z = -o$$

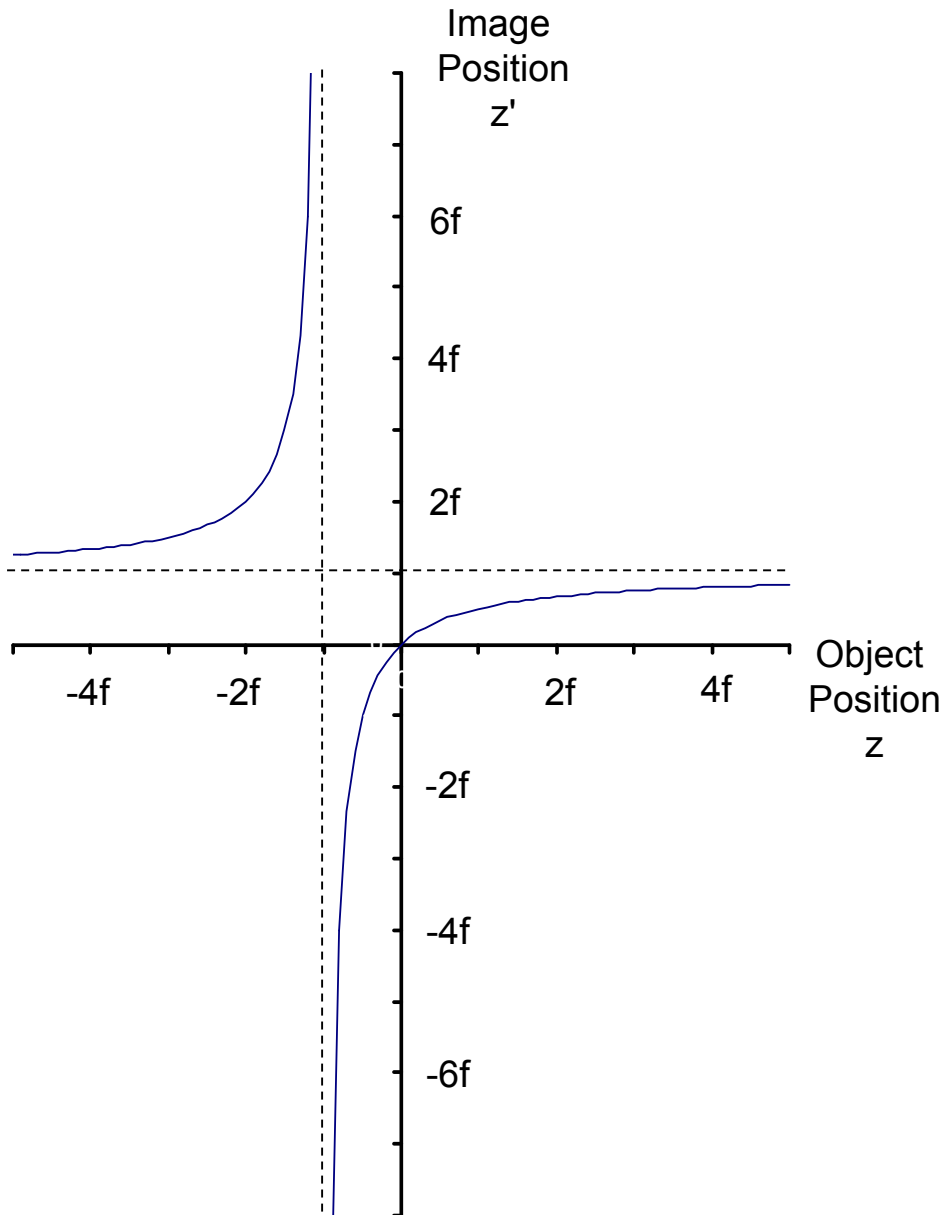
$$z' = i$$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

The use of sign conventions may appear to make easy problems harder, but their use will make hard problems possible.



## Thin Lens – Positive Lens – Conjugates



Positive Lens:

$$f > 0$$

$$f_F < 0$$

$$f'_R > 0$$

$$f = f'_R = -f_F$$



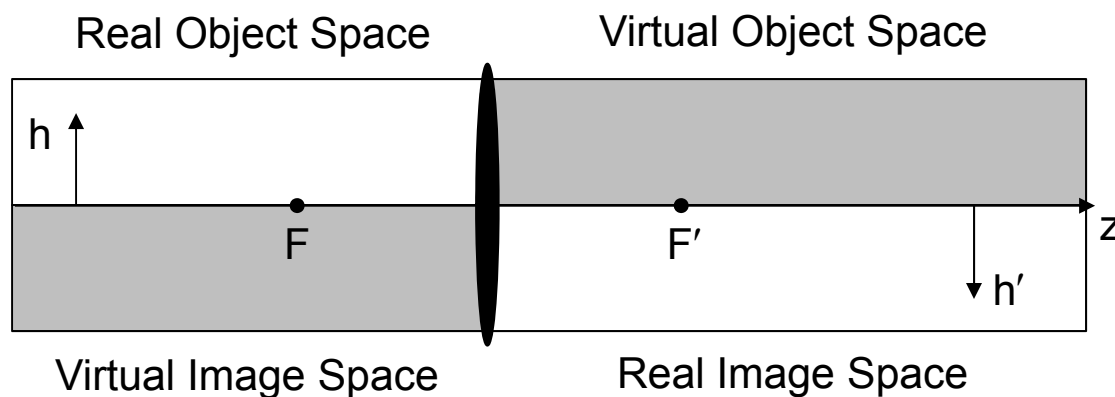
## Optical Spaces

The thin lens creates two optical spaces:

Object Space – Contains the Object and the Front Focal Point  $F$  of the lens.

Image Space – Contains the Image and the Rear Focal Point  $F'$  of the lens.

Both optical spaces extend from  $-\infty$  to  $+\infty$  with real and virtual segments



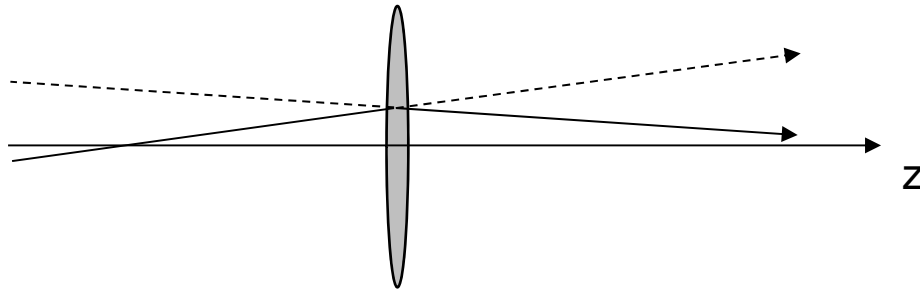
A real object and a real image are shown.





## Optical Spaces and Rays

Rays can be traced from object space to image space. Within both optical spaces, a ray is straight and extends from  $-\infty$  to  $+\infty$  with real and virtual segments. The ray must meet and be continuous at the lens.



A real object is to the left of the lens; a virtual object is to the right of the lens.

A real image is to the right of the lens; a virtual image is to the left of the lens.



## Thin Lens – Real Object and Real Image

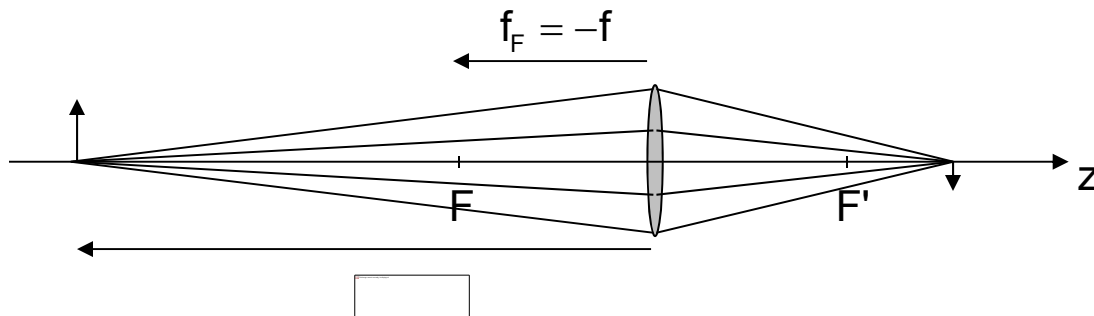
Depending on the object position, the image may be formed either to the right or left of the thin lens. Remember, light always goes from left to right.

Real images are formed to the right of the thin lens ( $z' > 0$ ). These images can be viewed by placing a screen at the image plane.

To obtain a real image with a positive lens:

The real object must be located outside of the front focal point F.

$$z < -f \quad \text{or} \quad z < f_F$$





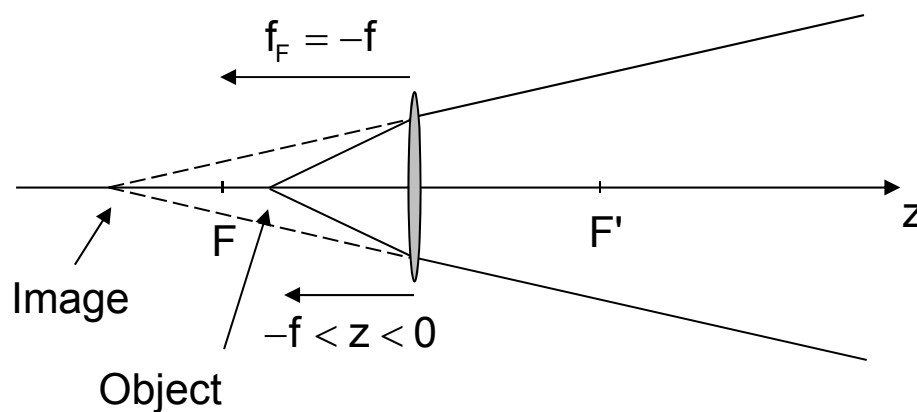
## Thin Lens – Real Object and Virtual Image

Virtual images are formed to the left of the thin lens ( $z' < 0$ ). These images cannot be directly viewed on a screen.

To obtain a virtual image with a positive lens:

The real object is located between front focal point  $F$  and the lens.

$$-f < z < 0 \quad \text{or} \quad f_F < z < 0$$



The image appears to be behind the lens.

This is the situation that occurs with a magnifying glass.

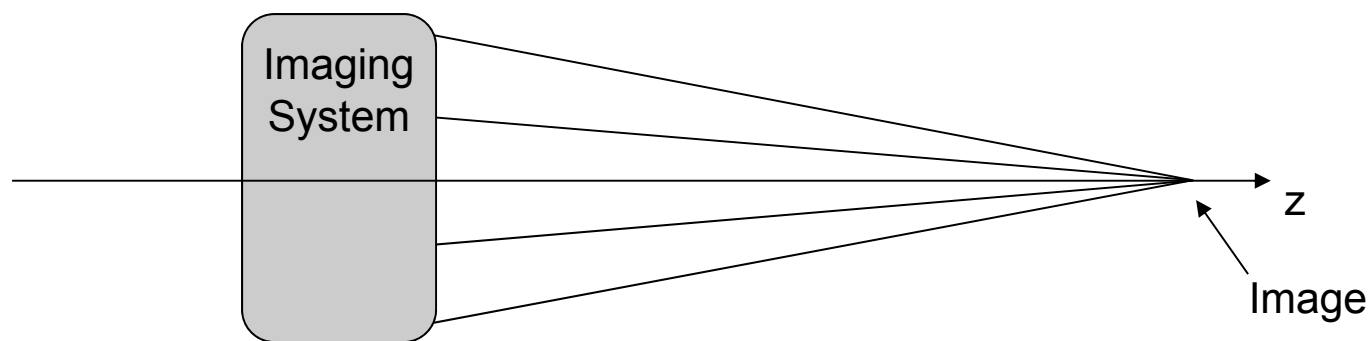
The light diverges from the lens and appears to be coming from the virtual image point.

## Thin Lens – Virtual Object

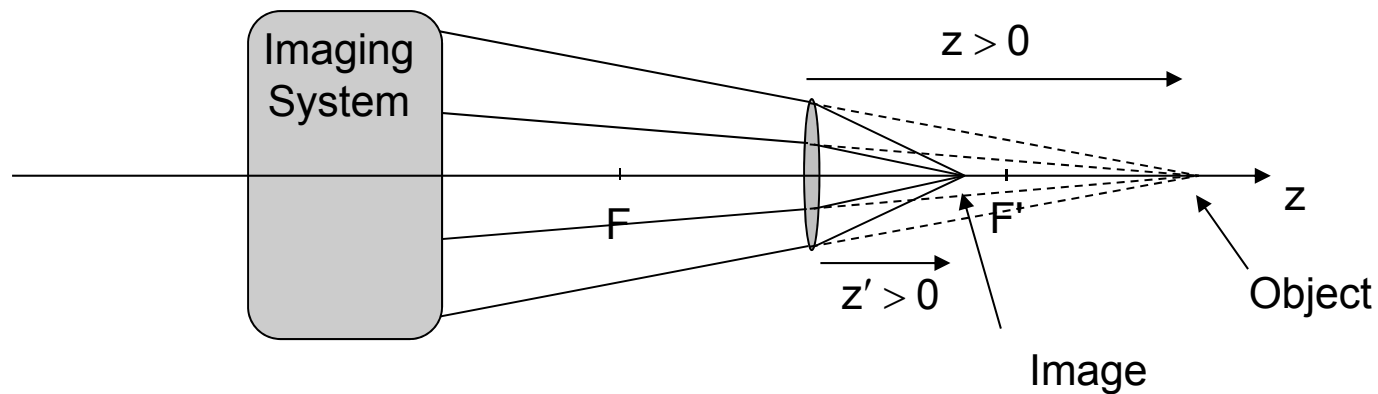
Virtual objects are also possible and occur when an image is projected into the lens by another imaging system. The lens intercepts the rays before they come to focus.

$$z > 0$$

With a positive lens, a real image will be produced.



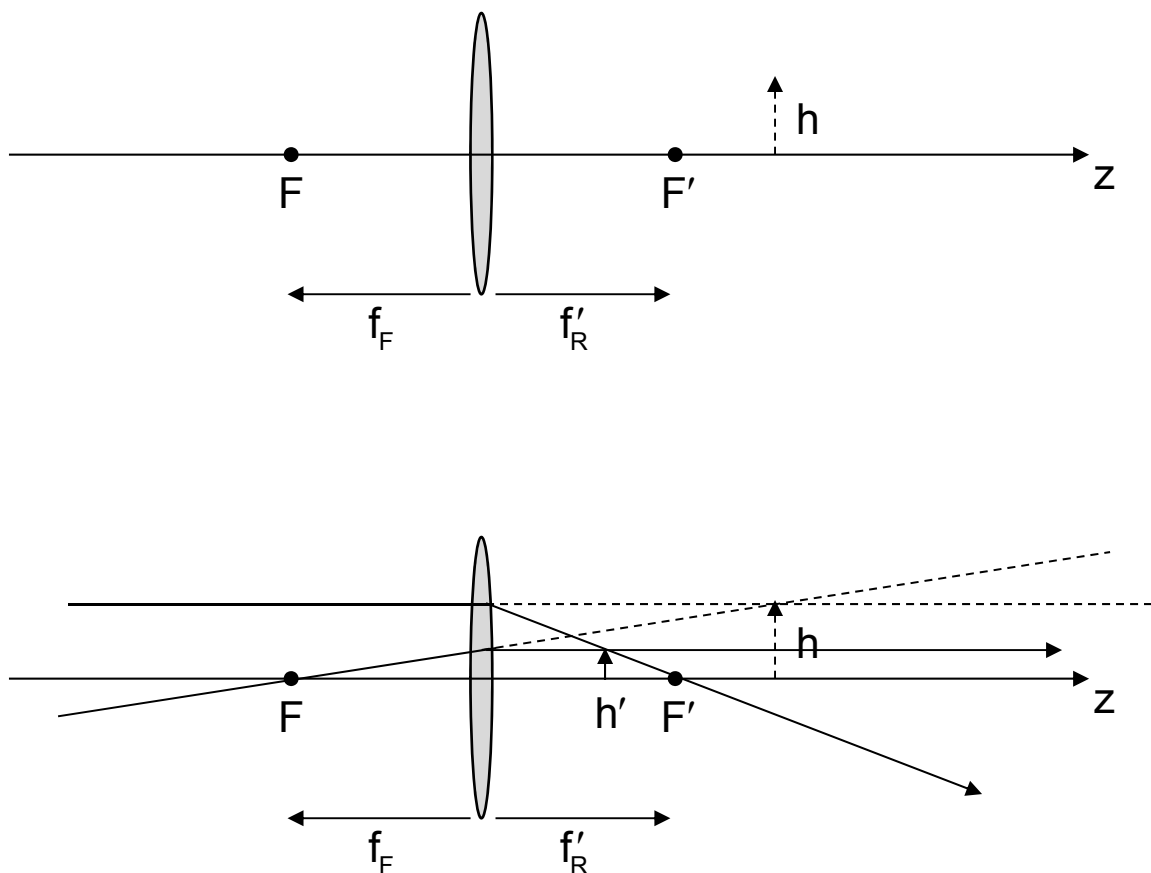
This image serves as the virtual object for the inserted lens.



The imaging equations hold for any of these situations.

## Locating an Image with the Focal Points – Example 3

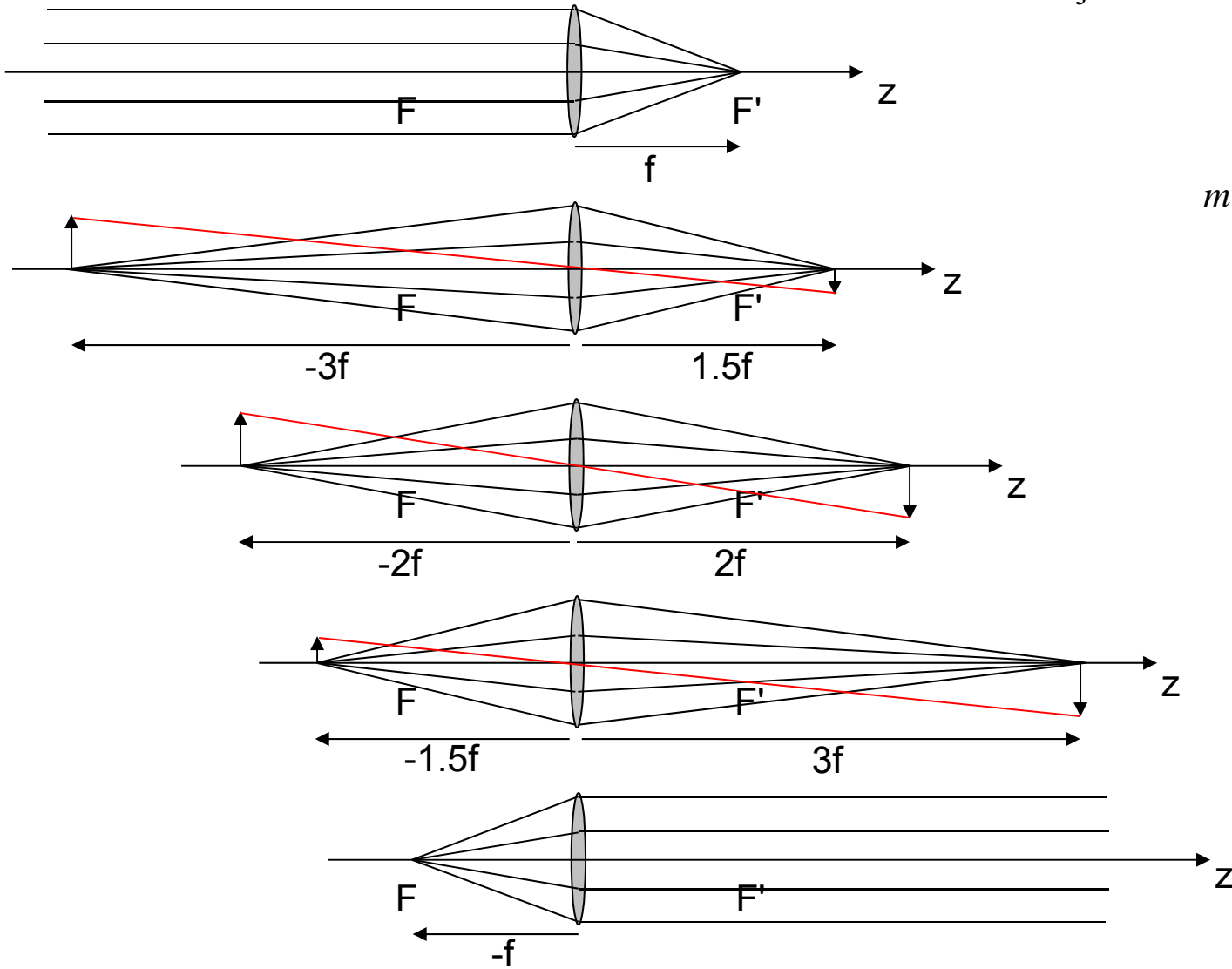
Positive Lens– Virtual object to the right of the lens



Two object rays are constructed that define the virtual object – one parallel to the axis and one through  $F$ . These rays refract to produce the real, minimized image.

Imaging with a Positive Lens

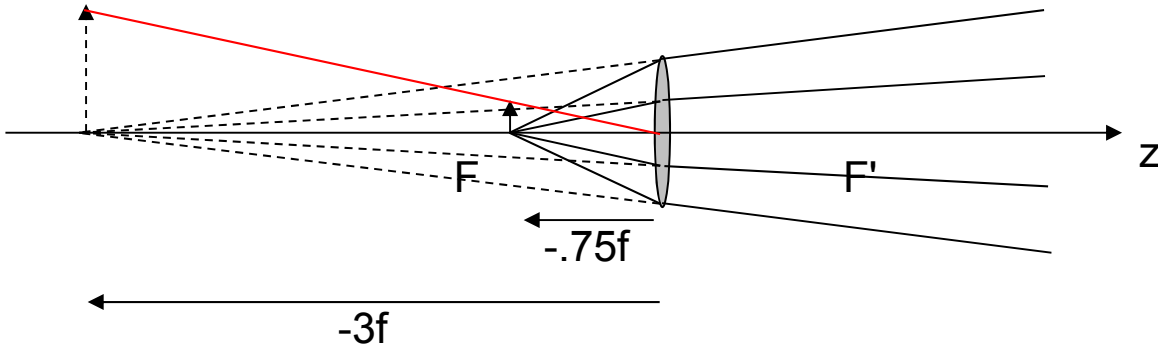
Real Object-Real Image



$$m = \frac{z'}{z}$$

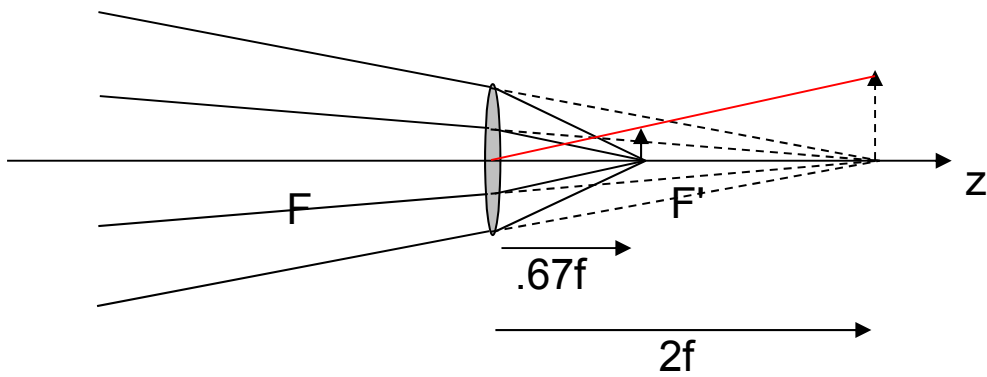
Imaging with a Positive Lens

Real Object-Virtual Image



$$m = \frac{z'}{z}$$

Virtual Object-Real Image

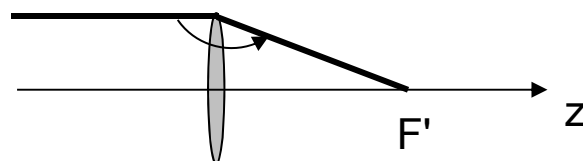


$$m = \frac{z'}{z}$$

## Ray Bending

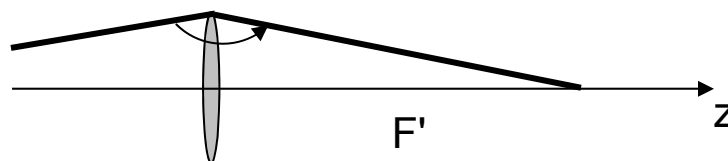
In the ray drawings, note that the ray incident at the top of the lens is bent by the same amount independent of the input ray or object position. There is a fixed relationship between the object ray and the image ray.

First consider the rays for an object at infinity. This defines the ray bending.



This relationship can be a great aid in visualizing the image position for a given object position. As the object position changes, the object ray will change and the ray pair will appear to pivot or rotate about the ray intersection point on the lens with a fixed bending.

For a finite conjugate object outside the front focal point of the lens:



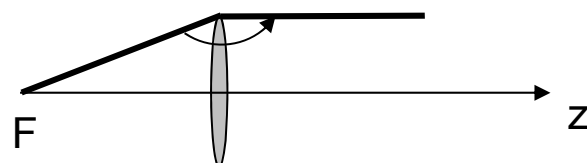
It is easy to see that the image position is outside the rear focal point and the image must be real.



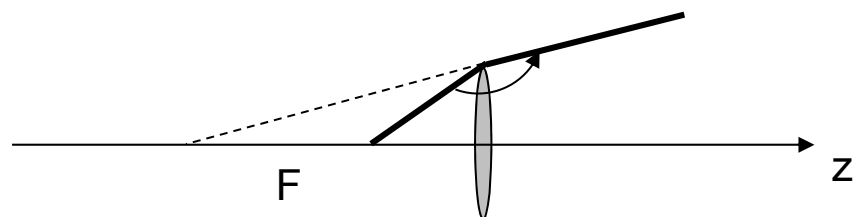


## Ray Bending – Continued

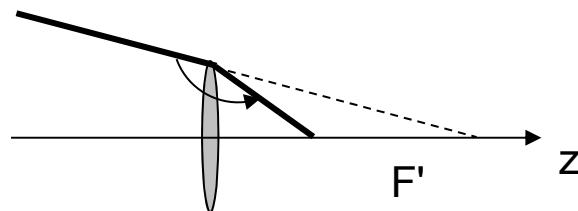
When the object is at the front focal point, the image must be at infinity:



An object between the front focal point and the lens must produce a virtual image. There is not enough ray bending to produce a real image.

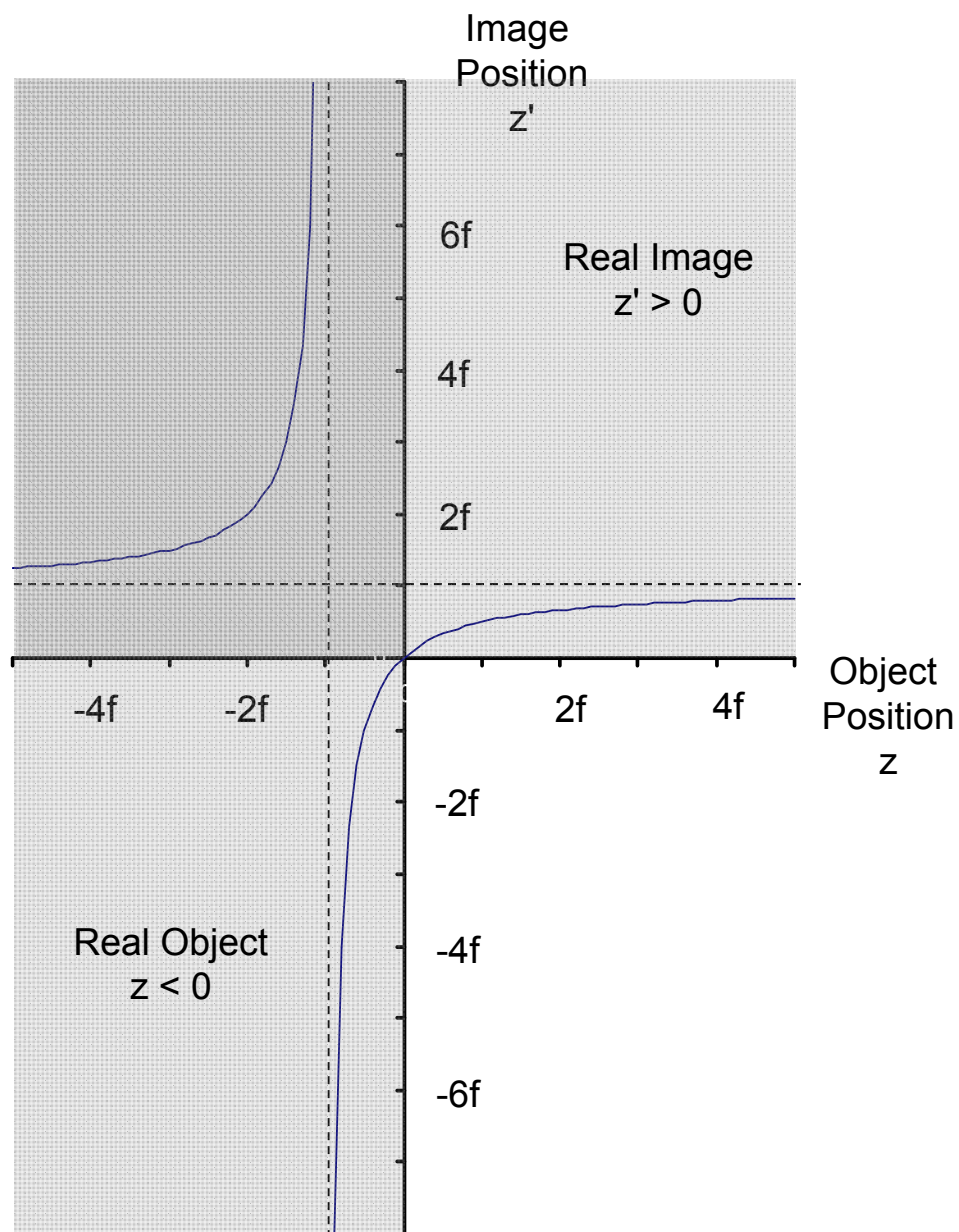


A virtual object must produce a real image located between the lens and the rear focal point. The fixed ray bending forces the image to fall short of the rear focal point.



As will be shown later, the constant ray bending at a certain height on a lens is not a constant change in ray angle, but rather a constant change in ray slope or the tangents of the ray angles.

## Thin Lens – Positive Lens – Real and Virtual Images



Positive Lens:

$$f > 0$$

$$f_F < 0$$

$$f'_R > 0$$

$$f = f'_R = -f_F$$

Real Object  $z < 0$

Real Image  $z' > 0$

Virtual Object  $z > 0$

Virtual Image  $z' < 0$

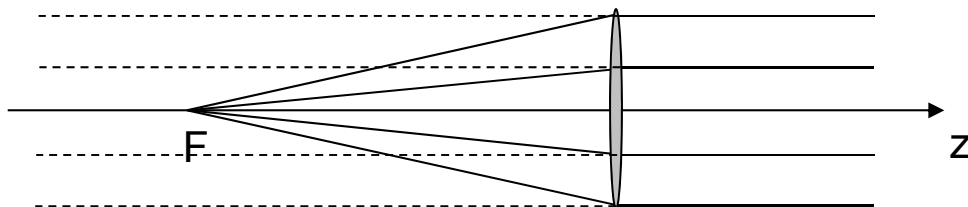


## Object at the Front Focal Point

Note that when the object is at the front focal point, there is an ambiguity as to the image location:

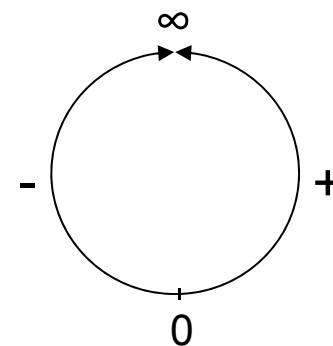
- Positive Infinity
- Negative Infinity

Collimated rays are produced and these could correspond to either a real image at positive infinity or a virtual image at negative infinity.



Positive infinity and negative infinity cannot be distinguished.

There is a mathematical concept that space “wraps” around at infinity:  
“The real projective line”



Applying this to imaging: As a real object moves through the front focal point, the image will move as a real image to positive infinity and then reappear as a virtual image coming in from negative infinity. The transition point is when the object is at the front focal point.

Mini Quiz

An object is 500 mm to the left of a thin lens with a focal length of 100 mm. Where is the image located?

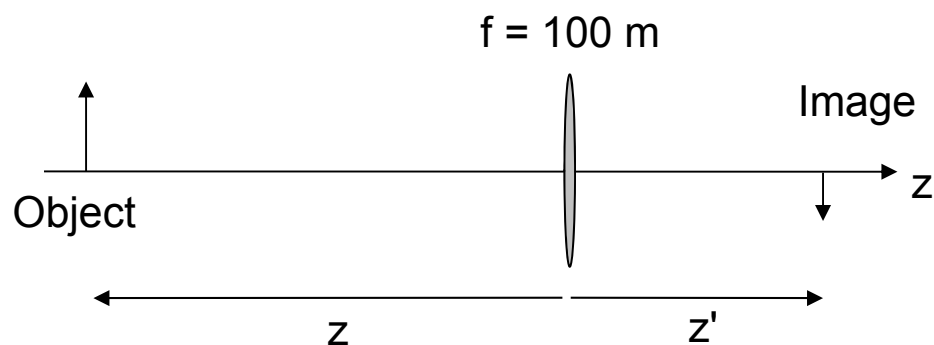
- a. 125 mm to the left of the lens
- b. 125 mm to the right of the lens
- c. 83.3 mm to the right of the lens
- d. 83.3 mm to the left of the lens



Mini Quiz – Solution

An object is 500 mm to the left of a thin lens with a focal length of 100 mm. Where is the image located?

- [ ] a. 125 mm to the left of the lens  
 [X] b. 125 mm to the right of the lens  
 [ ] c. 83.3 mm to the right of the lens  
 [ ] d. 83.3 mm to the left of the lens



$$z = -500 \text{ mm}$$

$$f = 100 \text{ mm}$$

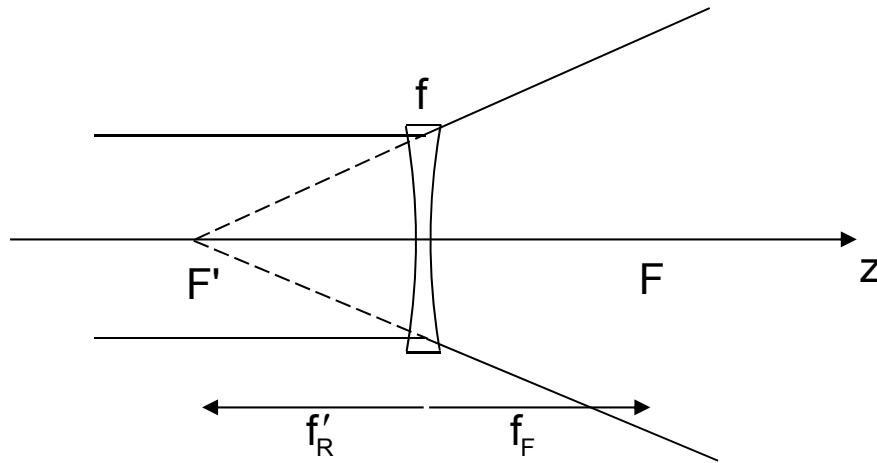
$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$z' = 125 \text{ mm}$$



## Thin Lens – Negative Lens

A negative lens has a negative focal length and will diverge light. The front and rear focal points are reversed and are virtual.



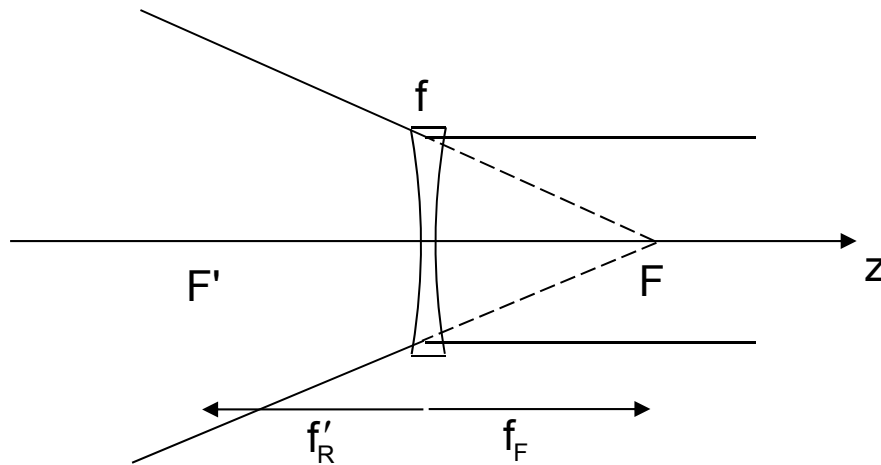
Negative Lens:

$$f < 0$$

$$f'_R < 0$$

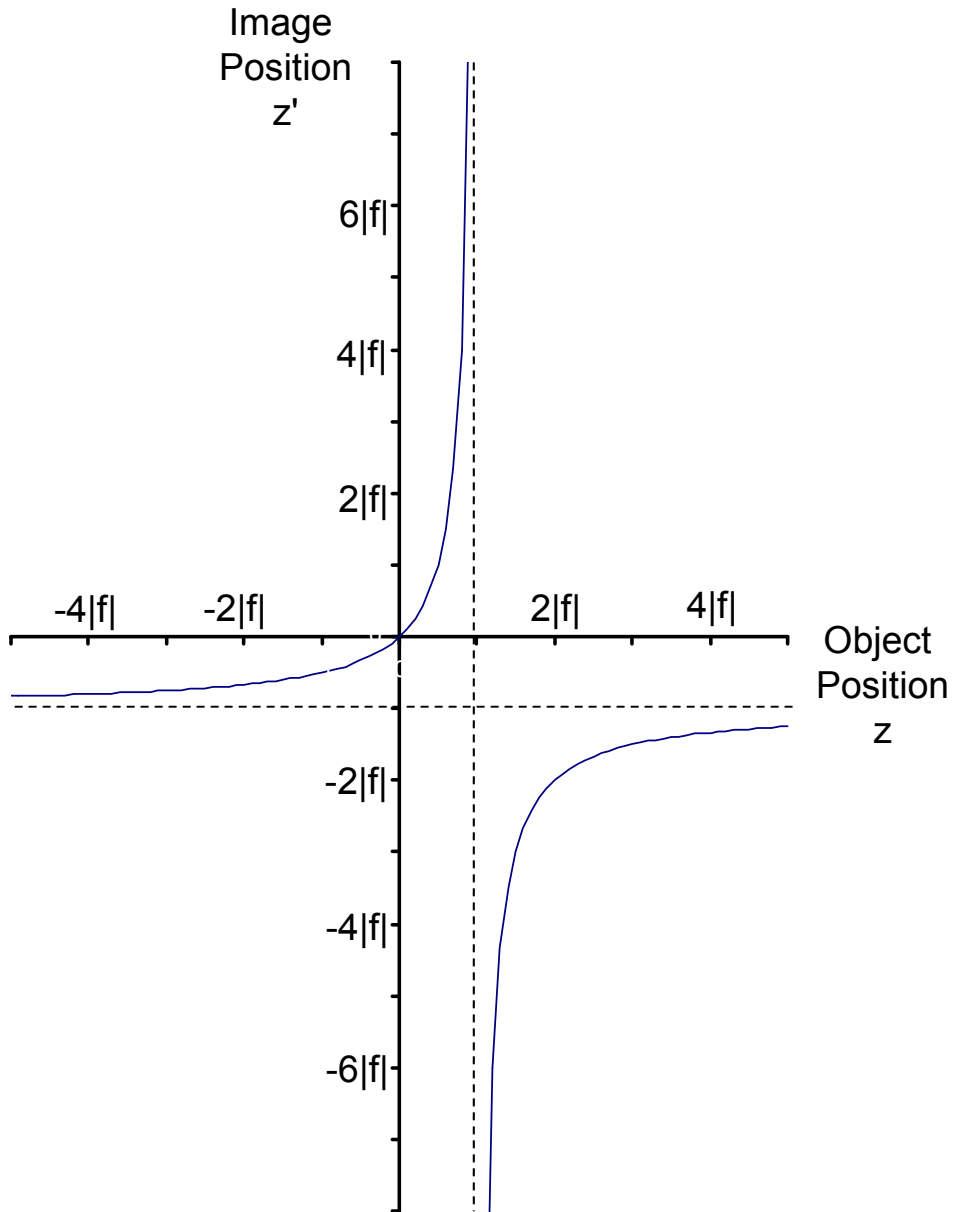
$$f_F > 0$$

$$f = f'_R = -f_F$$



The imaging equations also hold for a negative lens, and a virtual image is produced for a real object ( $z < 0$ ).

# Thin Lens – Negative Lens – Conjugates



Negative Lens:

$$f < 0$$

$$f'_R < 0$$

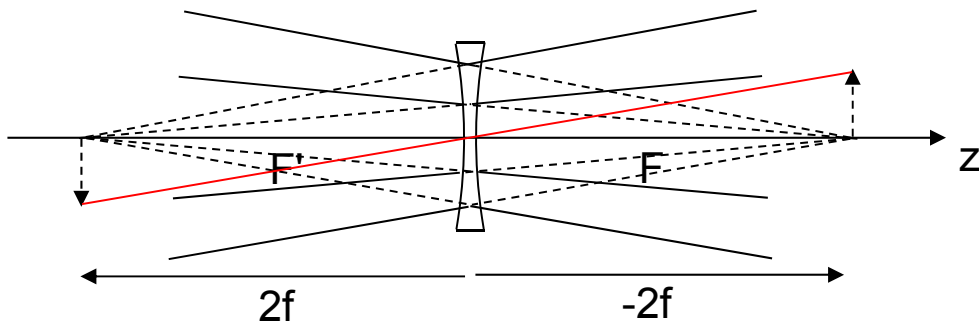
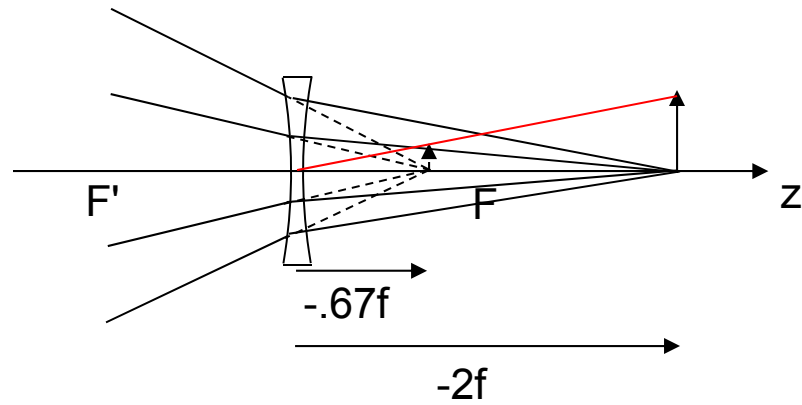
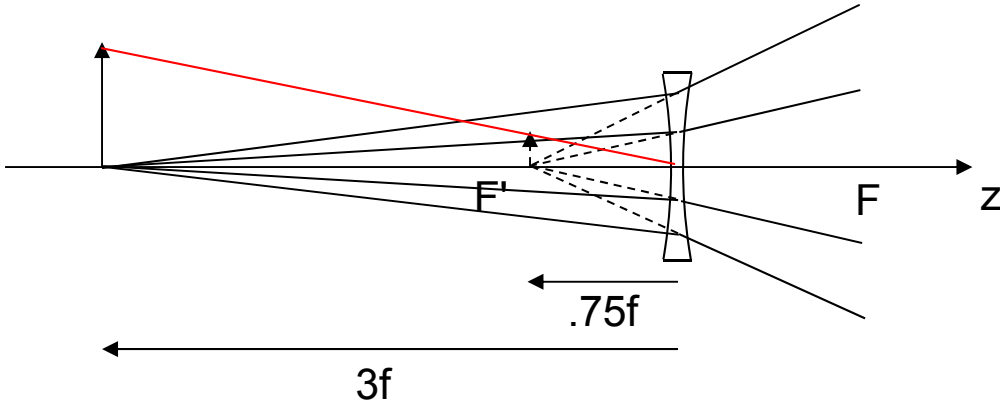
$$f_F > 0$$

$$f = f'_R = -f_F$$

Imaging with a Negative Lens

$$f < 0$$

$$m = \frac{z'}{z}$$

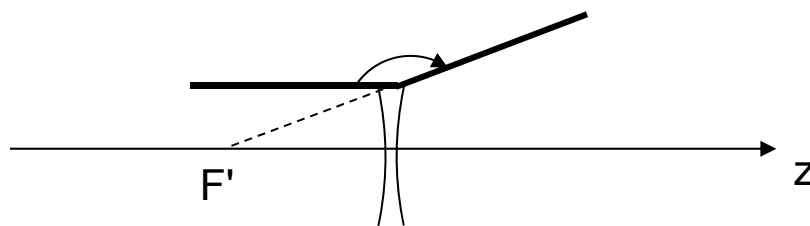




## Ray Bending – Negative Lens

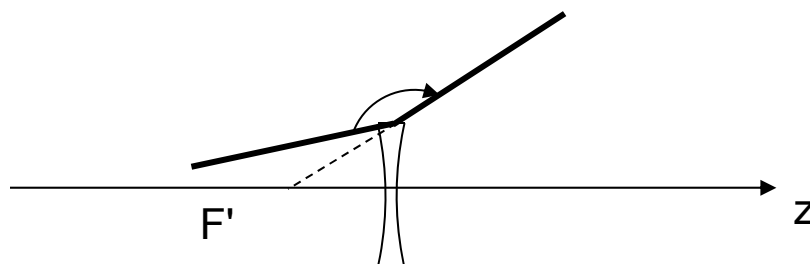
The fixed ray bending concept can also be used with negative lenses to visualize the image position for a given object location. In this case, the object ray is bent outward by the lens:

As before, consider the rays for an object at infinity. This defines the ray bending.



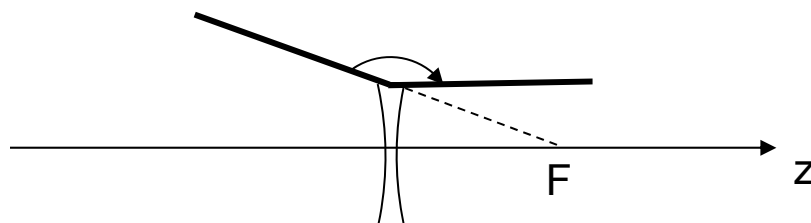
As the object position changes, the object ray will change and the ray pair will appear to pivot or rotate about the ray intersection point on the lens with a fixed bending.

For a real finite conjugate object, a virtual image must be located between the rear focal point and the lens:

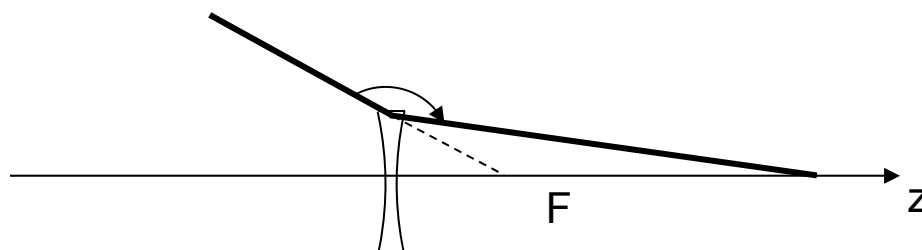


### Ray Bending – Negative Lens – Continued

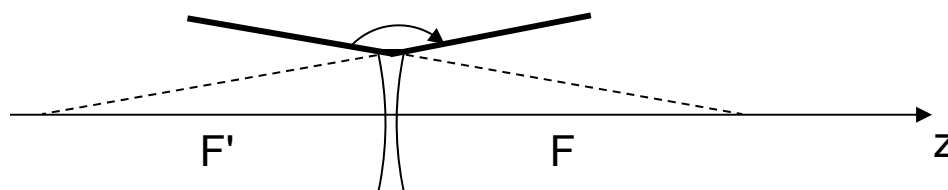
A virtual object at the front focal point of the lens produces an image at infinity:



A virtual object between the lens and the front focal point will produce a real image:



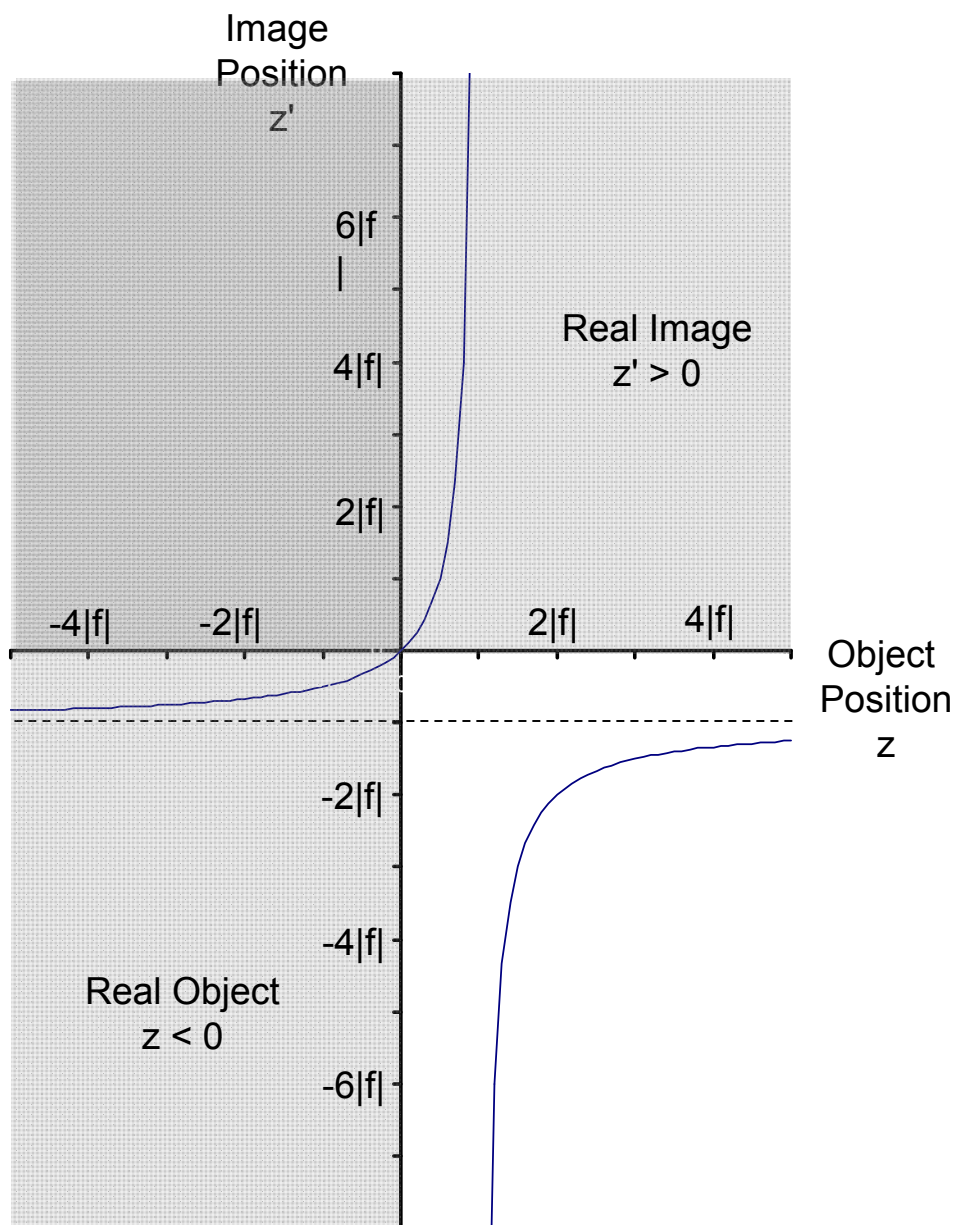
A virtual object to the right or outside the front focal point will produce a virtual image located to the left or outside the rear focal point:



As noted earlier, the constant ray bending at a certain height on a lens is not a constant change in ray angle, but rather a constant change in ray slope or the tangents of the ray angles.



## Thin Lens – Negative Lens – Real and Virtual Images



Negative Lens:

$$f < 0$$

$$f'_R < 0$$

$$f_F > 0$$

$$f = f'_R = -f_F$$

Real Object  $z < 0$

Real Image  $z' > 0$

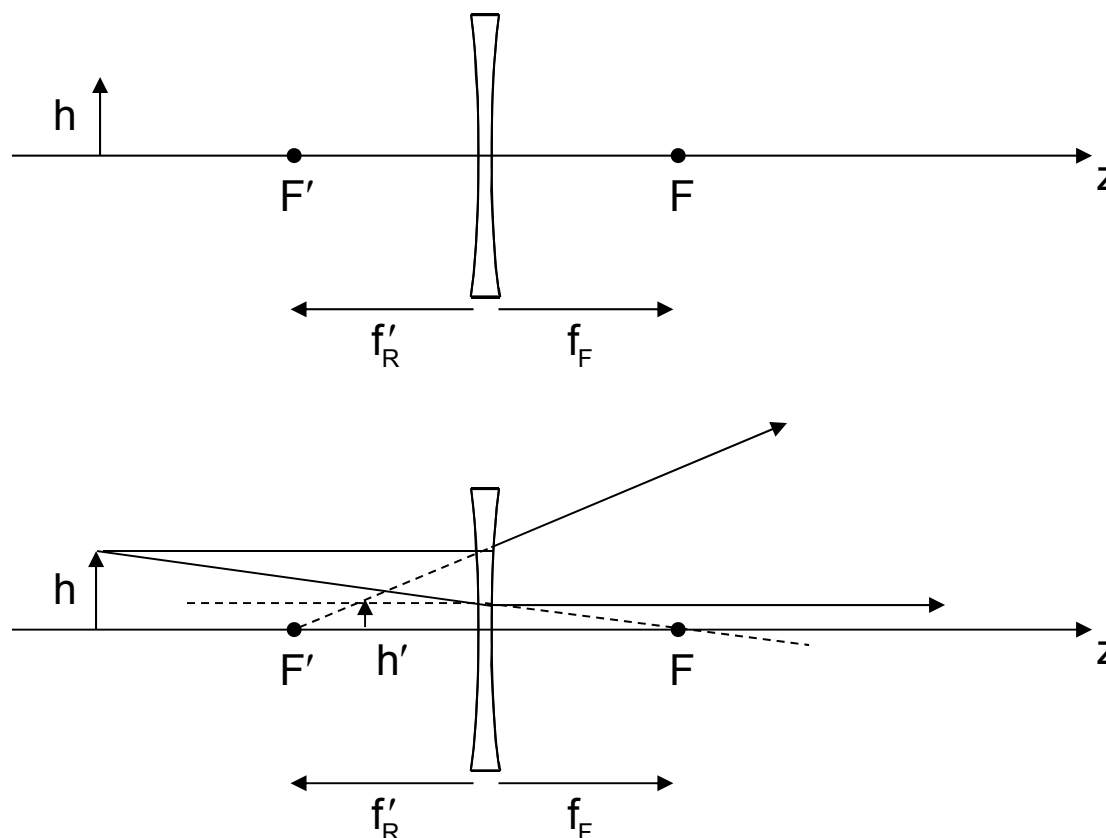
Virtual Object  $z > 0$

Virtual Image  $z' < 0$



### Locating an Image with the Focal Points – Example 4

Negative System – Real object – Note the locations of the Focal Points. The Front Focal Point  $F$  is virtual and in the object space of the lens. Similarly, the Rear Focal Point  $F'$  is also virtual and in the image space of the lens. The same image formation rules apply.

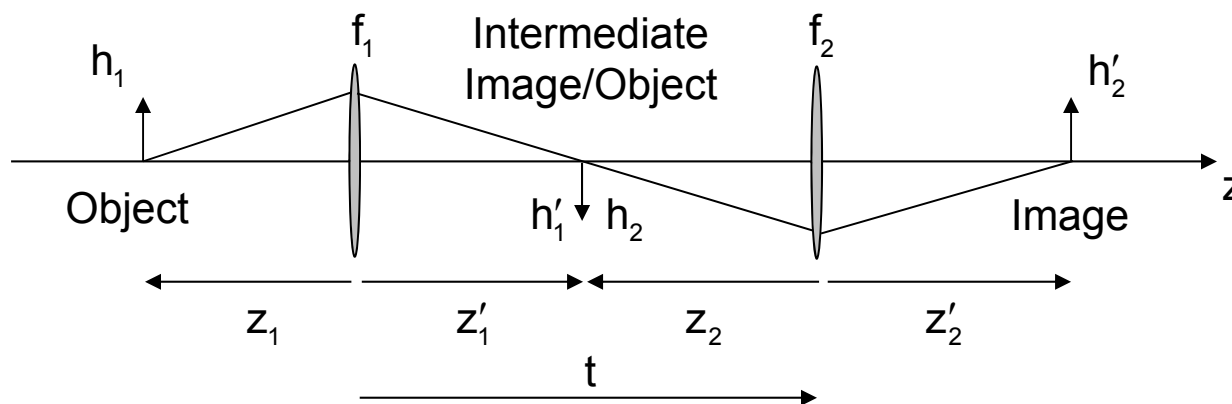


The two image space rays diverge and have a virtual crossing. A minified, erect virtual image is produced. The image is in image space.



## Cascaded Imaging

Cascaded imaging is the process of imaging through a series of lenses by imaging through the lenses one at a time. The image produced by the first lens serves as the object for the second lens, etc.



$$\frac{1}{z'_1} = \frac{1}{z_1} + \frac{1}{f_1}$$

$$t = z'_1 - z_2$$

$$z_2 = z'_1 - t$$

$$\frac{1}{z'_2} = \frac{1}{z_2} + \frac{1}{f_2}$$

The net magnification is the product of the individual magnifications:

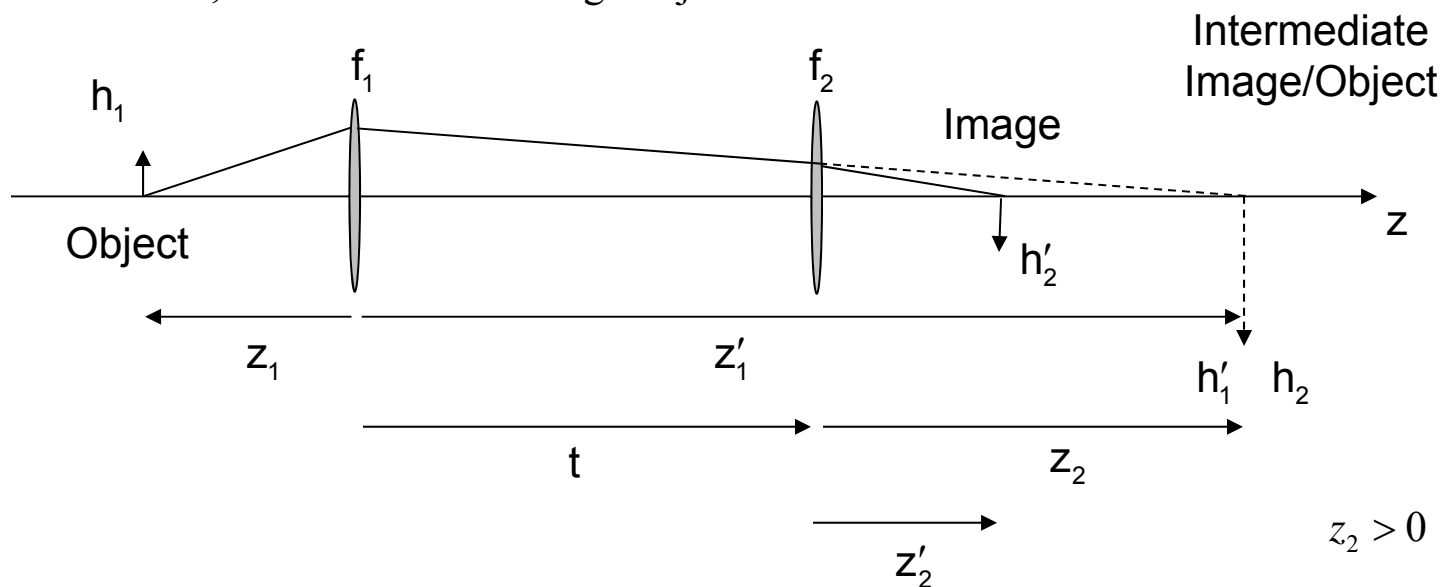
$$m = m_1 m_2 \quad m_1 = \frac{z'_1}{z_1} \quad m_2 = \frac{z'_2}{z_2}$$

The process is repeated for all of the elements in the system.



## Cascaded Imaging

In most cases, the intermediate image/object will be virtual:



All of the same relationships hold, including that for the object distance for the second lens:

$$\frac{1}{z'_1} = \frac{1}{z_1} + \frac{1}{f_1} \quad t = z'_1 - z_2 \quad \frac{1}{z'_2} = \frac{1}{z_2} + \frac{1}{f_2}$$

$$z_2 = z'_1 - t$$

$$m = m_1 m_2 \quad m_1 = \frac{z'_1}{z_1} \quad m_2 = \frac{z'_2}{z_2}$$

The relationships are also valid if the first lens produces a virtual image or  $z'_1 < 0$ .



## Object-Image Approximations

When the magnitude of the object distance  $z$  is more than a few times the magnitude of the system focal length, the image distance  $z'$  is approximately equal to the rear focal length. A positive thin lens in air is assumed ( $n = n' = 1$ ):

$$|z| \gg |f|: \quad z' \approx f \quad L = z' - z \approx f - z \quad m = \frac{z'}{z} \approx \frac{f}{z}$$

$$L \approx -z$$

$L$  is the object-image distance.

For a distant object, the image is located just outside the rear focal point.

The magnification can be approximated by the focal length divided by the object distance.

There are similar approximations for distant images:

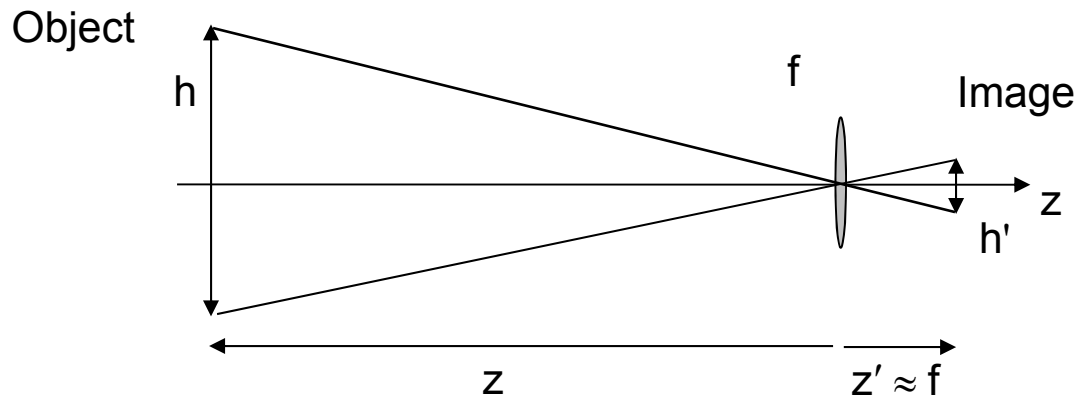
$$|z'| \gg |f|: \quad z \approx -f \quad L = z' - z \approx z' + f \quad m = \frac{z'}{z} \approx -\frac{z'}{f}$$

$$L \approx z'$$



### Another Way of Looking at the Approximation

$$m = \frac{h'}{h} \quad m = \frac{z'}{z} \approx \frac{f}{z} \quad \frac{h'}{f} \approx \frac{h}{z}$$



In many cases, the object/image “size” is given. It is usually considered to be centered on the optical axis with a height of  $\pm h$  or  $\pm h'$ . The height is half the size.

The approximation equations work with either height or size.



## Object-Image Approximations

$$|z| \gg |f|: \quad z' \approx f \quad L = z' - z \approx f - z \approx -z \quad m = \frac{z'}{z} \approx \frac{f}{z}$$

Object Distance	Image Distance	Object-Image Distance	Approx. Object-Image Distance		Magnification	Approx. Magnification
			$L \approx f - z$	$L \approx -z$		
$z$	$z'$	$L = z' - z$	$L \approx f - z$	$L \approx -z$	$m$	$m \approx f/z$
$-f$	Inf	Inf	Inf	Inf	Inf	-1
$-2f$	$2f$	$4f$	$3f$	$2f$	-1	-1/2
$-3f$	$1.5f$	$4.5f$	$4f$	$3f$	-1/2	-1/3
$-4f$	$1.33f$	$5.33f$	$5f$	$4f$	-1/3	-1/4
$-5f$	$1.25f$	$6.25f$	$6f$	$5f$	-1/4	-1/5
$-10f$	$1.11f$	$11.11f$	$11f$	$10f$	-1/9	-1/10
$-20f$	$1.05f$	$21.05f$	$21f$	$20f$	-1/19	-1/20
$-100f$	$1.01f$	$101.01f$	$101f$	$100f$	-1/99	-1/100

The fractional error in these approximations is about  $|f|/|z|$ , so they are very useful when the object distance more than 10-20 times the focal length. Most imaging problems can be solved with little or no computation.



Approximation Examples

A 10 m object at 100 m is imaged onto a 10 mm detector. What is the required focal length of the lens?

$$m = \frac{h'}{h} = \frac{-10\text{mm}}{10,000\text{mm}} = -.001 \quad m = \frac{z'}{z} \approx \frac{f}{z} \quad f \approx mz = (-.001)(-100,000\text{mm}) = 100\text{mm}$$

$$m = 1000 : 1 \quad f \approx \frac{z}{1000} = \frac{100\text{m}}{1000} = \frac{100,000\text{mm}}{1000} = 100\text{mm}$$

A 10 m object at 100 m is imaged with a 50 mm focal length lens. What is the image size (required detector size)?

$$m \approx \frac{f}{z} = \frac{50\text{mm}}{-100,000\text{mm}} = -0.0005 \quad h' = mh = (-0.0005)(10,000\text{mm}) = -5\text{mm}$$

$$m = 2000 : 1 \quad h' = mh = \frac{10\text{m}}{2000} = \frac{10,000\text{mm}}{2000} = 5\text{mm}$$

Inverted and must be smaller



Approximation Examples

A 25 mm focal length lens is used with a 5 mm detector. The object is at 10 m. What is the maximum object size?

$$m \approx \frac{f}{z} = \frac{25\text{mm}}{-10,000\text{mm}} = -0.0025 \quad h = \frac{h'}{m} = \frac{-5\text{mm}}{-0.0025} = 2,000\text{mm} = 2\text{m}$$

$$m = 400 : 1 \quad h = \frac{h'}{m} = (400)5\text{mm} = 2,000\text{mm} = 2\text{m}$$

Inverted and must be larger

A 10 mm x 15 mm LCD display is used in a projector to produce an image on a screen that is 2 m x 3 m is size. The screen is 10 m away from the projector. What focal length lens is required?

$$m = \frac{h'}{h} = \frac{-2,000\text{mm}}{10\text{mm}} = \frac{-3,000\text{mm}}{15\text{mm}} = -200$$

$$m = \frac{z'}{z} \approx \frac{z'}{-f} \quad f \approx -\frac{z'}{m} = -\frac{10,000\text{mm}}{-200} = 50\text{mm}$$

$$m = 200 : 1 \quad f \approx \frac{z'}{200} = \frac{10\text{m}}{200} = \frac{10,000\text{mm}}{200} = 50\text{mm}$$

The image is larger than the object.



Mini Quiz

A 10 m high object is located 50 m away from a camera with a 50 mm focal length lens. Approximately how large is the image on the detector in the camera?

- a. 1 mm
- b. 5 mm
- c. 10 mm
- d. 100 mm



Mini Quiz - Solution

A 10 m high object is located 50 m away from a camera with a 50 mm focal length lens. Approximately how large is the image on the detector in the camera?

- a. 1 mm  
 b. 5 mm  
 c. 10 mm  
 d. 100 mm

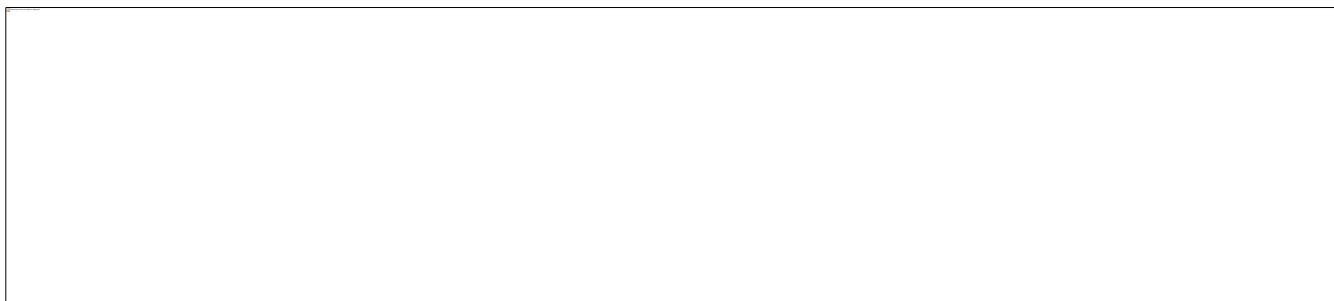


Image is inverted and must be smaller

Exact:



$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$z' = 50.05 \text{ mm}$$

$$m = \frac{z'}{z} = \frac{50.05 \text{ mm}}{-50,000 \text{ mm}} = -0.001001$$

$$h' = mh = -0.001001(10 \text{ m}) = -10.01 \text{ mm}$$

$$\text{Error} = 0.1\% \approx |f|/|z|$$

## Field of View

The half field of view HFOV of an optical system defined by

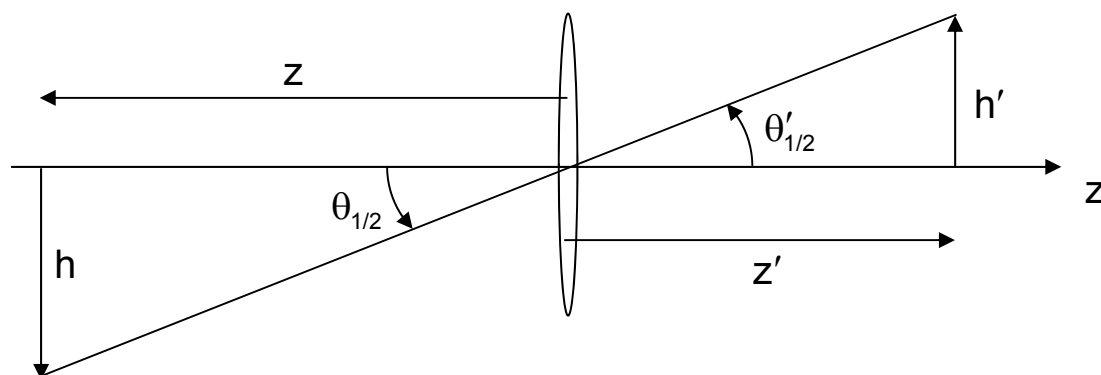
- the maximum object height  $h$
- the maximum image height  $h'$
- the maximum angular size of the object as seen from the optical system  $\theta_{1/2}$
- the maximum angular size of the image as seen from the optical system  $\theta'_{1/2}$

Field of View FOV: the diameter of the object/image

Half field of View HFOV: the radius of the object/image

Full field of View FFOV is sometimes used instead of FOV to emphasize that this is a diameter measure.

For a thin lens:



$$FOV = 2HFOV$$

$$HFOV = \theta_{1/2} = \theta'_{1/2}$$

$$\tan(\theta_{1/2}) = \frac{h}{z} = \frac{h'}{z'}$$



## Field of View and Focal Length

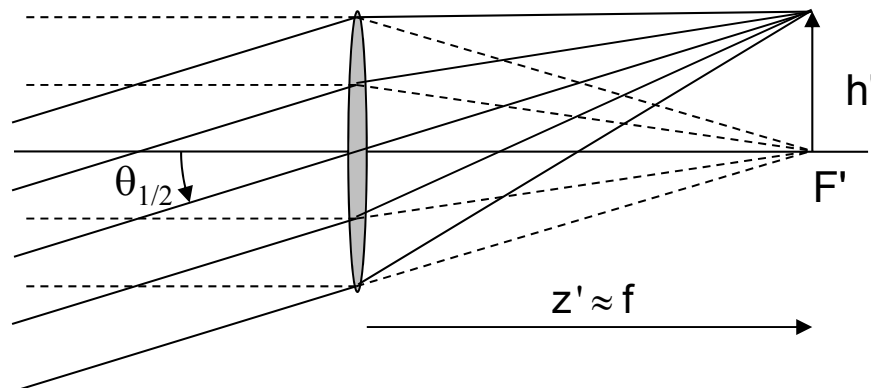
A distant object produces collimated rays at the lens. If the object is not on the optical axis, these parallel rays are tilted with respect to the optical axis.

For distant scenes, the object height is usually expressed as an angular size:

FOV – Field of View – the angular diameter of the object

HFOV – Half Field of View – the angular radius of the object

The image is formed at (or near) the rear focal point of the system:



$$HFOV = \theta_{1/2}$$

$$h' = f \tan(\theta_{1/2})$$

$$\bar{u}' = \tan(\theta_{1/2}) = \frac{h'}{f}$$

In the context of stops and pupils, the central ray drawn in the diagram is called the *Chief Ray*, and  $\bar{u}'$  is the *Chief Ray Angle* (the slope of the ray).



## Field of View and Focal Length

$$HFOV = \theta_{1/2} \quad h' = f \tan(\theta_{1/2}) \quad \bar{u}' = \tan(\theta_{1/2}) = \frac{h'}{f}$$

These relationships determine the image height for the entire scene or the image separation for element in the scene:

- the FOV of the camera is 30 degrees (HFOV = 15 degrees)
- two stars are separated by 10 arc seconds  
1 degree = 60 arc minutes = 3600 arc seconds

In many situations, the FOV is determined by the detector size. The optical system produces a circular image, but the detector only records a rectangular image. Different FOVs result in the horizontal, vertical and diagonal directions:

$$HFOV_H = \tan^{-1}\left(\frac{h'_H}{f}\right) \quad HFOV_V = \tan^{-1}\left(\frac{h'_V}{f}\right) \quad HFOV_D = \tan^{-1}\left(\frac{\sqrt{h'_H{}^2 + h'_V{}^2}}{f}\right)$$

In a general system, the angular FOV is usually measured as the angular size of the object as viewed from the Entrance Pupil of the system.





Field of View - Example35 mm Film – the frame size is 24 x 36 mmUse:  $h' = 18$  mm

(Note this is the Horizontal HFOV; the Vertical or the Diagonal could also be used)

Focal Length (mm)	$\bar{u}'$	HFOV (deg)	FOV (deg)
20	0.9	42.0	84.0
30	0.6	31.0	62.0
40	0.45	24.2	48.4
50	0.36	19.8	39.6
75	0.24	13.5	26.0
100	0.18	10.2	20.4
200	0.09	5.14	10.3
1000	0.018	1.03	2.06

In photographic terms, a normal lens has a focal length of about 40-60 mm. This produces an image perspective and FOV that somewhat matches human vision. The FOV of a normal lens is about 40-50 degrees.

Lenses that produce a larger FOV are called wide angle lenses.

Lenses that produce a smaller FOV are called long focus lenses.



## Field of View - Scaling

### Electronic Sensors – CCD arrays

Example 2/3 inch format – 6.6 x 8.8 mm

Use:  $h' = 4.4$  mm

What are the required focal lengths to get the same FOVs as with 35 mm film?

FOV (deg)	HFOV (deg)	$\bar{u}'$	Focal Length (mm)
84.0	42.0	0.9	4.9
62.0	31.0	0.6	7.3
48.4	24.2	0.45	9.8
39.6	19.8	0.36	12
26.0	13.5	0.24	18
20.4	10.2	0.18	24
10.3	5.14	0.09	49
2.06	1.03	0.018	244

The required focal length scales linearly with the size of the image format for a given angular FOV.

