Name_____SOLUTIONS________

Closed book; closed notes. Time limit: 120 minutes.
Equation sheets are attached and can be removed. A spare raytrace sheet is also attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
As usual, only the magnitude of a magnification or magnifying power may be given.
If a method of solution is specified in the problem, that method must be used.
Raytraces must be done on the raytrace form. Be sure to indicate the initial conditions for your rays.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Provide your answers in a neat and orderly fashion. No credit if it can't be read/followed.
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.
Note: On some quantities, only the magnitude of the quantity is provided. The proper sign convention must be applied.

1) (10 points) A 100 mm focal length thin lens in air is made out of glass KzFSN5. The glass code for this glass is 654396. What is the longitudinal chromatic aberration of this lens? Is this lens a crown or a flint?

$$654396 \quad \rightarrow \quad n_d = 1.654 \quad \nu = 39.6$$

$$\frac{\delta f}{f} = \frac{1}{\nu}$$

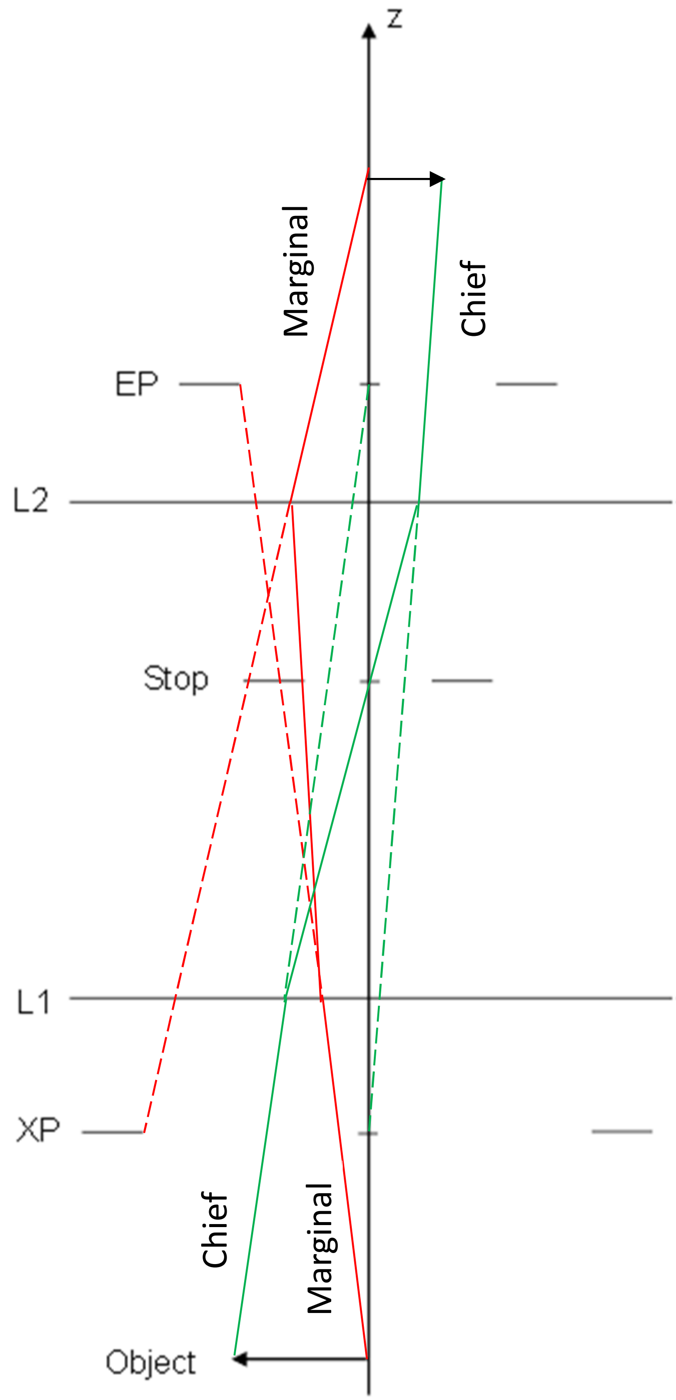
$$\delta f = \frac{f}{\nu} = \frac{100mm}{39.6}$$

$$\delta f = 2.53mm$$

Since the Abbe Number is less than 50, this is a high dispersion glass – a flint.

Crown or Flint? ___Flint__ Longitudinal Chromatic Aberration = __2.53__mm

2) (10 points) This diagram shows an optical system consisting of two thin lenses and an object. Also shown are the locations and sizes of the stop, the entrance pupil (EP) and the exit pupil (XP). Show the paths of the marginal and chief rays through the system along with the location and size of the image. No calculations or equations are required or allowed. Use a ruler as rays go in straight lines!



3) (20 points) In a 3X Galilean telescope, the separation between the objective lens and the eye lens is 100 mm. The objective lens diameter is 30 mm and the eye lens diameter is 12 mm. The telescope is to be used with a human eye that has a 4 mm diameter entrance pupil. The separation between the eye lens and the eye pupil is 20 mm. The object is at infinity.

Where is the system stop?
What is the half-vignetted object space Field of View of this system in degrees?
Which element limits the half-vignetted Field of View?

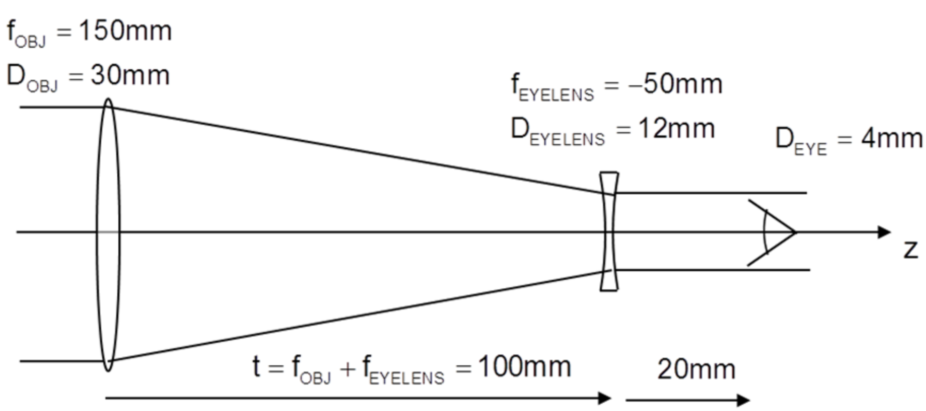
Be sure to provide the reasoning for your answers and a narrative describing your method and calculations.

First design the telescope. A Galilean telescope consists of a positive lens and a negative lens. The MP is positive.

$$MP = 3 = -\frac{f_{OBJ}}{f_{EYELENS}} \qquad t = f_{OBJ} + f_{EYELENS} = 100mm$$

$$f_{OBJ} = -3\, f_{EYELENS} \qquad -2\, f_{EYELENS} = 100mm$$

$$f_{EYELENS} = -50mm \qquad f_{OBJ} = 150mm$$



The next step is to determine the stop location. Trace a potential marginal ray – including the eye. The ray has an initial height of one and a slope of zero.

Continues…

When using the ray trace form, be sure to clearly identify your rays. Do not do calculations in the margins to the sides of the form – the calculations must be included in your narrative.

			OBJ		EYELENS		EYE/Stop						
Surface	0		1		2		3		4		5		6
f			150		-50								
$-\phi$			-.00667		0.02								
t				100		20				-			
Potential Marginal Ray													
$\tilde{y}$	1.0*		1.0		0.333		0.333						
$\tilde{u}$		0		-.00667		0							
Marginal Ray – Scale Factor = 6.0													
y	6.0		6.0		2.0		2.0						
u		0		-0.04		0							
Potential Chief Ray													
$\tilde{\bar{y}}$			-16.0		-2.0		0						
$\tilde{\bar{u}}$		0.0333		0.14		0.1*							
Chief Ray – Scale Factor = 0.9375													
$\bar{y}$			-15.0		1.875		0						
$\bar{u}$		0.0312		0.131		0.09375							

* Arbitrary

Marginal ray scaling: What is the scale factor required to scale this ray to the edge of each aperture?

OBJ: $\frac{a_{OBJ}}{\tilde{y}_{OBJ}} = \frac{15}{1} = 15$ EYELENS: $\frac{a_{EYELENS}}{\tilde{y}_{EYELENS}} = \frac{6}{0.333} = 18$ EYE: $\frac{a_{EYE}}{\tilde{y}_{EYE}} = \frac{2}{.333} = 6$

The smallest ratio is at the eye. The eye is the system stop. While not required here, the marginal ray is found by scaling the potential marginal ray by 6.

Continues…

Since the eye is the stop, the FOV can be limited by either the Objective Lens or the Eye Lens of the telescope. Trace a potential marginal ray starting at the eye. Evaluate the vignetting condition at each lens using a scaled version of this ray. The lens which accommodates the smallest FOV will limit the FOV and determine the half-vignetted FOV:

Potential Chief Ray: $\tilde{\bar{y}}_{EYE} = 0$ $\tilde{\bar{u}}_{EYE} = 0.1 \quad (arbitrary)$

Half Vignetting Condition: $a = |\bar{y}| = A|\tilde{\bar{y}}|$ $A = \dfrac{a}{|\tilde{\bar{y}}|}$

Objective Lens: $\tilde{\bar{y}}_{OBJ} = -16.0mm$ $a_{OBJ} = 15.0mm$ $A_{OBJ} = \dfrac{15mm}{16mm} = 0.9375$

Eye Lens: $\tilde{\bar{y}}_{EYELENS} = -2.0mm$ $a_{EYELENS} = 6mm$ $A_{EYELENS} = \dfrac{6.0mm}{2.0mm} = 3.0$

The smaller scale factor is at the objective, and the objective limits the half-vignetted Field of View. Scale the potential Chief Ray by A_{OBJ} to obtain the Chief Ray.

The resulting Object Space Chief Ray slope:

$$\bar{u} = 0.0312 \qquad HFOV = tan^{-1}(\bar{u}) = \pm 1.79\,deg$$

The stop of the system is __the EYE___

Half-vignetted Field of View = +/-__1.79_ degrees in object space

FOV Limited by ___Objective Lens___

4) (30 points) The telephoto ratio is defined as the ratio of the overall length of a lens system (as measured from the first element to the image plane) to the focal length of the system:

$$\text{Telephoto Ratio} = \frac{\text{System Length}}{\text{System Focal Length}} = \frac{L}{f}$$

The object is assumed to be at infinity. In other words, the telephoto ratio describes how much shorter an optical system is when compared to an equivalent single thin lens system.

An f/5.6 lens system is constructed with two thin lenses in air.

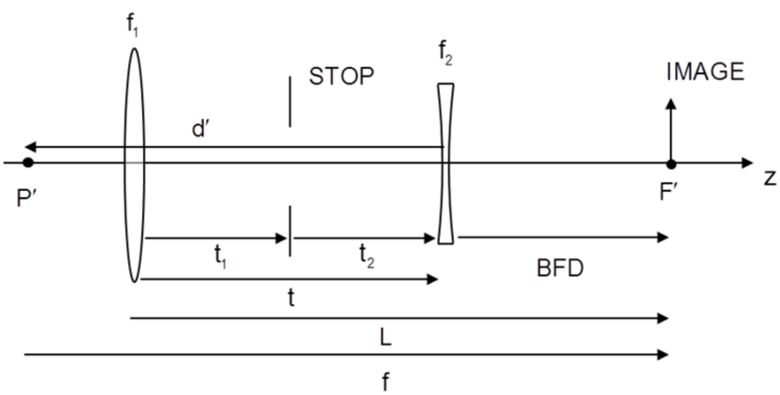
The system has a telephoto ratio of 0.75.
The system has a focal length of 200 mm.
The system provides a back focal distance of 70 mm.
The system stop is located halfway between the two thin lenses.
The object is at infinity.

Determine the focal lengths of the two thin lenses, the stop diameter, the entrance pupil location, and the element spacings.

As always, provide a narrative of your solution that includes your calculations.

Use Gaussian methods only – no raytrace analysis is permitted.

First design the lens system:



$$f = 200mm$$

$$T.R. = \frac{L}{f} = 0.75$$

$$L = 150mm$$

$$BFD = 70mm$$

$$L = t + BFD$$

$$t = 80mm$$

$$t = t_1 + t_2 = 80mm$$

$$t_1 = t_2 = 40mm$$

Continues…

$$BFD = f + d'$$

$$d' = -130mm \qquad \phi = \frac{1}{f} = 0.005mm^{-1}$$

$$d' = -\frac{\phi_1}{\phi} t$$

$$\phi_1 = 0.008125mm^{-1} \qquad f_1 = 123.1mm$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t$$

$$\phi_2 = \frac{\phi - \phi_1}{1 - \phi_1 t}$$

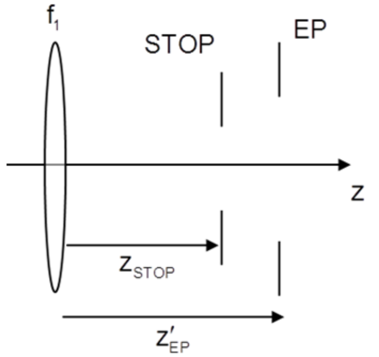
$$\phi_2 = -0.008928mm^{-1} \qquad f_2 = -112.0mm$$

The system operates at f/5.6:

$$f/5.6 \quad \rightarrow \quad \frac{f}{D_{EP}} = 5.6 \qquad D_{EP} = 35.71mm$$

However, the EP location is not known. Image the stop through the first lens to determine the EP location and the magnification of the EP relative to the stop.

The light from the stop goes from Right to Left.



$$n = n' = -1 \qquad z_{STOP} = t_1 = 40mm$$

$$\frac{n'}{z'_{EP}} = \frac{n}{z_{STOP}} + \frac{1}{f_1}$$

$$\frac{-1}{z'_{EP}} = \frac{-1}{40mm} + \frac{1}{123.1mm}$$

$$z'_{EP} = 59.25mm$$

Continues…

The EP is to the right of the first lens element.

We can now determine the EP magnification:

$$m_{EP} = \frac{z'_{EP} / n'}{z_{STOP} / n} = \frac{59.25mm}{40mm} = 1.48$$

$$D_{EP} = m_{EP} D_{STOP} \qquad D_{EP} = 35.71mm$$

$$D_{STOP} = 24.1mm$$

Lens 1: Focal Length = __123.1___mm

Lens 2: Focal Length = __-112.0___mm

Separation between Lens 1 and Lens 2 = ___80____mm

Stop Diameter = ___24.1___mm

EP: Located __59.25_mm to the _R__ of the first lens element

5) (15 points) The image in the eye is formed in an index of refraction of 1.336. The rear focal length of the eye is 22.4 mm. An object is 1 meter in front of the eye (in air) and has a height of +/- 20 mm. What is the height of the image formed in the eye (on the retina)? **Use Gaussian methods only – no raytrace analysis is permitted.**

Assume that the eye changes length to keep the image in focus.

$$f'_R = 22.4mm \qquad n' = 1.336$$

$$f = \frac{f'_R}{n'} = 16.77mm$$

$$\phi = 0.0596mm^{-1}$$

Imaging:

$$z = -1m = -1000mm \qquad n = 1$$

$$\frac{n'}{z'} = \frac{n}{z} + \phi$$

$$\frac{n'}{z'} = 0.0586mm^{-1} \qquad z' = 22.8mm$$

The eye has elongated slightly for this near object.

Image size:

$$m = \frac{z'/n'}{z/n} = -0.0171$$

$$h' = mh \qquad h = \pm 20mm$$

$$h' = \mp 0.341mm \qquad Inverted$$

Image Height = +/- __0.341__ mm

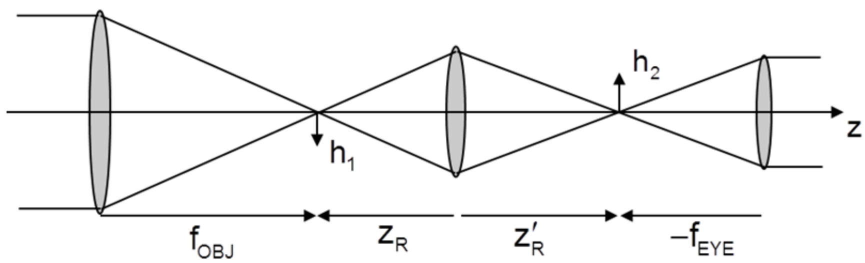
6) (15 points) A 20X relayed Keplerian telescope is constructed out of three thin lenses in air. The relay lens of the telescope operates with a magnification of 2.0. The focal length of the objective lens is 200 mm, and the overall telescope length is 370 mm.

a) Determine the design of the telescope.

b) Assuming that the system stop is located at the objective lens, determine the eye relief of the telescope (distance from the eye lens to the XP).

NOTE: Only Gaussian imaging methods may be used for this problem. No raytrace analysis is permitted.

The relay lens will invert and magnify the first intermediate image:



$$MP = m_R MP_K \qquad MP_K = -\frac{f_{OBJ}}{f_{EYE}}$$

$$m_R = -2.0$$

$$MP_K = -10 \qquad f_{OBJ} = 200mm$$

$$f_{EYE} = 20mm$$

$$m_R = \frac{z'_R}{z_R} = -2.0$$

$$z'_R = -2.0 z_R$$

Continues…

Length of the telescope and the relay lens:

$$L = f_{OBJ} + f_{EYE} - z_R + z'_R = 370mm$$

$$-z_R + z'_R = 150mm$$

$$-z_R - 2z_R = 150mm$$

$$z_R = -50mm$$

$$z'_R = 100mm$$

$$\frac{1}{z'_R} = \frac{1}{z_R} + \frac{1}{f_{RELAY}}$$

$$f_{RELAY} = 33.33mm$$

Objective-Relay Separation = 250mm
Relay- Eye Lens Separation = 120mm

Total Length = 370mm

To determine the eye relief, first image the stop of objective through the relay lens to produce an intermediate pupil:

$$\frac{1}{z'_{IP}} = \frac{1}{z_{STOP}} + \frac{1}{f_{RELAY}} \qquad z_{STOP} = -250mm$$

$z'_{IP} = 38.46mm$ to the Right of the Relay Lens

Now image this intermediate pupil through the eye lens to determine the XP location and the Eye Relief:

$$z_{IP} = -\left(z'_R + f_{EYE} - z'_{IP}\right) = -81.54mm$$

$$\frac{1}{ER} = \frac{1}{z_{IP}} + \frac{1}{f_{EYE}}$$

$ER = 26.50mm$ to the Right of the Eye Lens

f_{RELAY} = _33.33__mm f_{EYE} = __20.0__mm

Objective Lens-Relay Lens Separation = ___250____ mm

Relay Lens-Eye Lens Separation = ____120___ mm

Eye Relief = __26.50___ mm

7) (5 points) A field lens is added to a Keplerian telescope telescope at the intermediate image plane. What is the effect of the field lens on each of the following?

a) MP – No Change

b) Eye Relief – Reduced

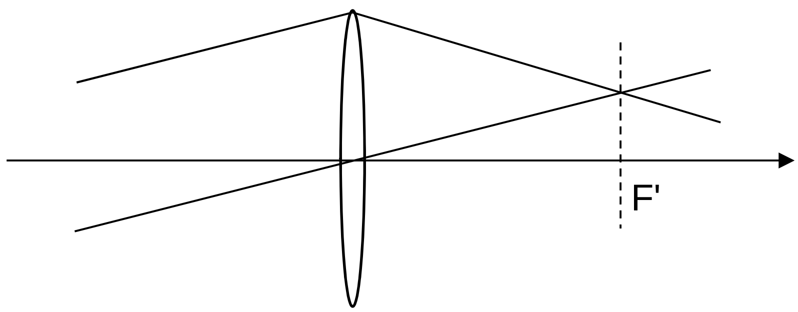
c) Exit Pupil Diameter – No Change

d) Field of View – Increases

e) Telescope Length – No Change

f) Image Orientation – No Change

Bonus (10 points) The figure shows a ray path through a thin lens. Draw a single line on the figure to locate the rear focal plane of the lens. Explain your result.



Parallel rays in object space produce conjugate rays that cross in the rear focal plane.