# Section 21

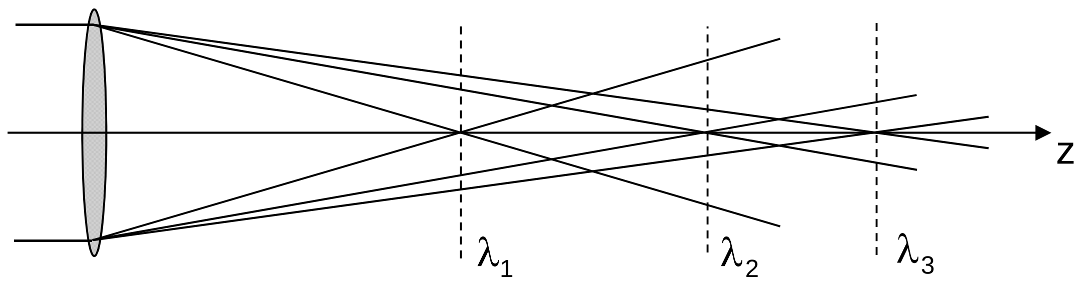
# Chromatic Effects

Chromatic Aberration

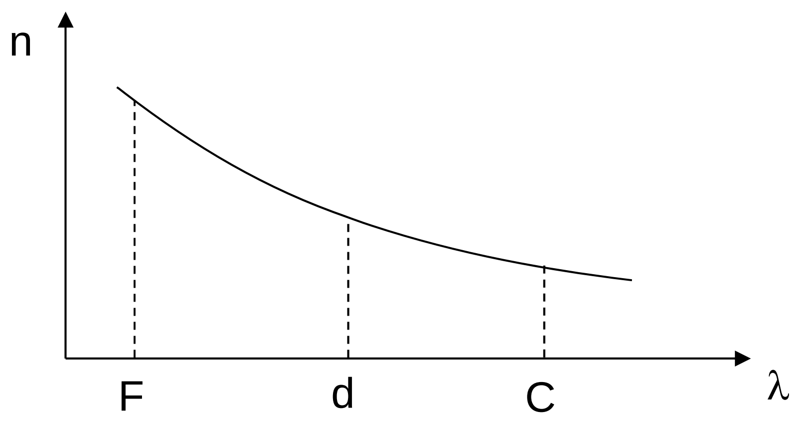
For a thin lens:

$$\phi \equiv \frac{1}{f} = (n-1)(C_1 - C_2)$$

Since the index changes with wavelength, so will the focal length.




Where do Red, Green (or Yellow) and Blue focus?

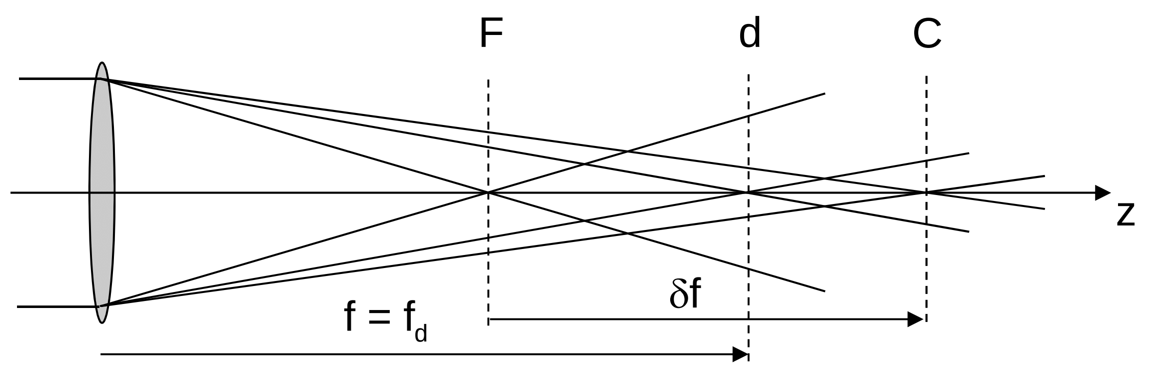



Because of the higher index for Blue or F light, Blue light is bent more and the Blue focus is closest to the lens.

The foci corresponding to the F, d and C wavelengths are not evenly spaced due to the shape of the dispersion curve.

## Axial or Longitudinal Chromatic Aberration

Axial chromatic aberration or axial color is a variation of the system focal length with wavelength.




$$\delta\phi = \phi_F - \phi_C = (n_F - 1)(C_1 - C_2) - (n_C - 1)(C_1 - C_2)$$

$$\delta\phi = (n_F - n_C)(C_1 - C_2)$$

$$\delta\phi = \frac{(n_F - n_C)}{(n_d - 1)}(n_d - 1)(C_1 - C_2)$$

$$\nu = \frac{n_d - 1}{n_F - n_C} \qquad \phi_d = (n_d - 1)(C_1 - C_2)$$

$$\delta\phi = \phi_F - \phi_C = \frac{\phi_d}{\nu}$$

$$\delta f = f_C - f_F = \frac{1}{\phi_C} - \frac{1}{\phi_F} = \frac{\phi_F - \phi_C}{\phi_C \phi_F}$$

$$\delta f = f_C - f_F = \frac{\delta\phi}{\phi_C \phi_F} \approx \frac{\delta\phi}{\phi_d^2} \qquad \phi_C \phi_F \approx \phi_d^{\,2}$$

$$\delta f = f_C - f_F = \frac{\phi_d}{\nu}\frac{1}{\phi_d^2} = \frac{1}{\nu\phi_d} = \frac{f_d}{\nu}$$

## Axial Chromatic Aberration - Continued

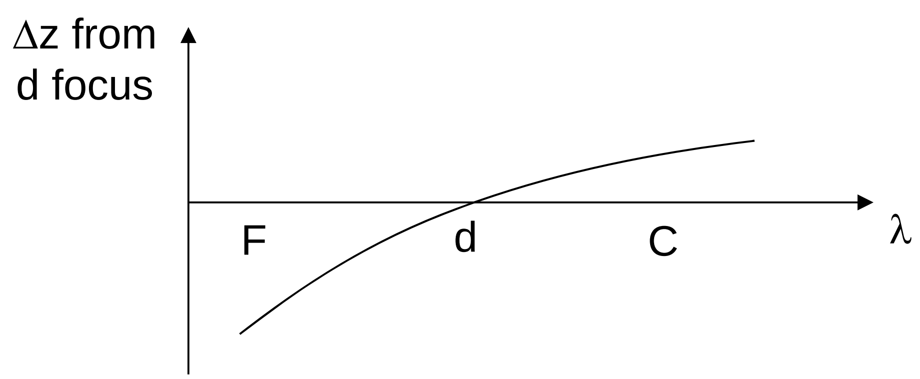
$$\delta\phi = \phi_F - \phi_c = \delta\phi_{FC} \qquad \delta\phi = \delta\phi_{FC} = \frac{\phi_d}{\nu} \qquad \frac{\delta\phi_{FC}}{\phi_d} = \frac{\delta f_{CF}}{f_d} = \frac{1}{\nu}$$

$$\delta f = f_C - f_F = \delta f_{CF} \qquad \delta f = \delta f_{CF} = \frac{f_d}{\nu} \qquad \frac{\delta\phi}{\phi} = \frac{\delta f}{f} = \frac{1}{\nu}$$

Since Abbe numbers are typically 30-70, the longitudinal chromatic aberration of a singlet is 1.5-3% of the focal length.

The relative order of the foci is reversed for a negative lens. F focuses closest to the lens for both a positive and a negative lens.

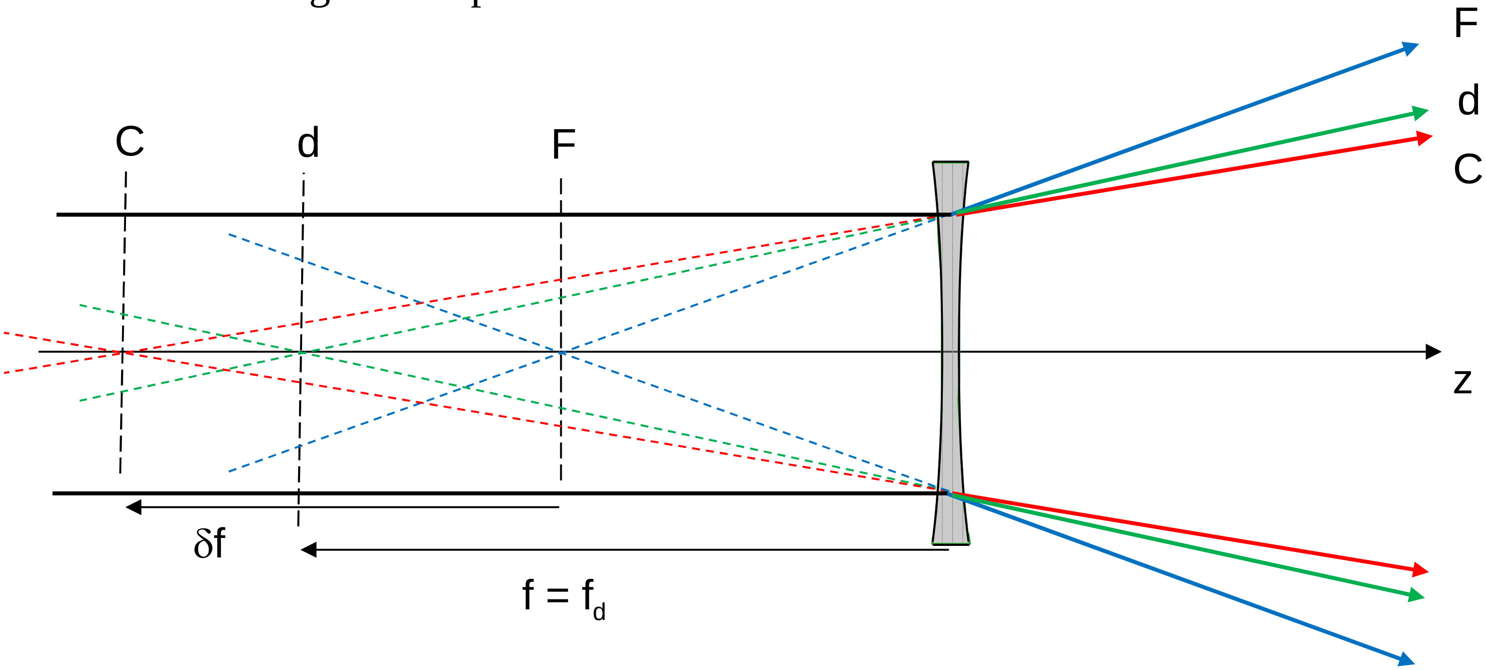
Longitudinal focus position:




Because of the flattening of the dispersion curve, the d-C separation tends to be less than the F-d separation.

## Axial Chromatic Aberration of a Negative Lens

For a negative lens, the Blue or F focus remains closest to the lens as Blue light has the largest ray bending. Of course all of the foci are virtual, but the order relative to the lens is the same for a negative or positive lens.




The same relationships hold but now the quantities are negative:

$$\delta\phi = \phi_F - \phi_c = \delta\phi_{FC}$$

$$\delta f = f_C - f_F = \delta f_{CF}$$

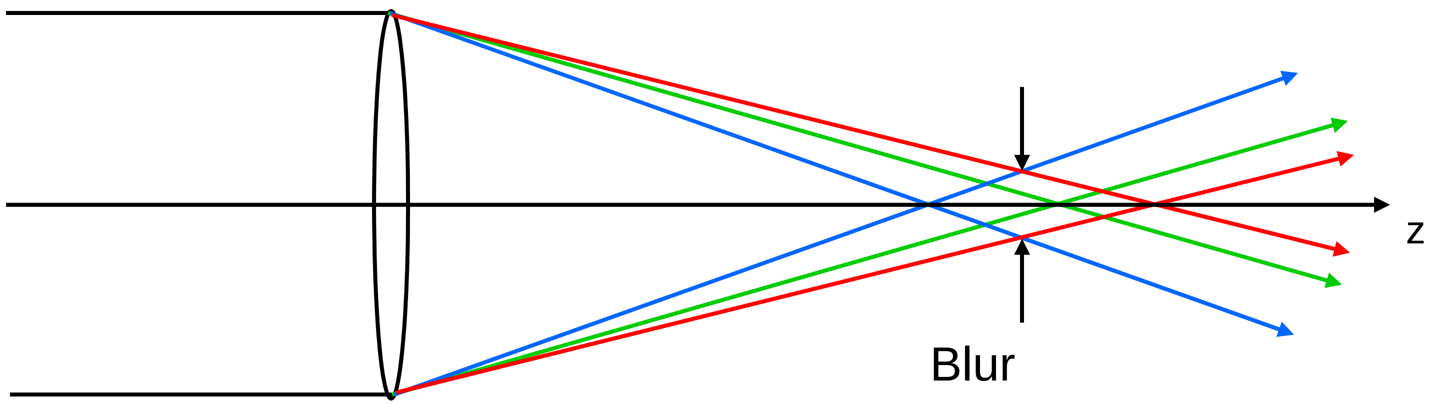
$$\frac{\delta\phi_{FC}}{\phi_d} = \frac{\delta f_{CF}}{f_d} = \frac{1}{\nu}$$

$$\frac{\delta\phi}{\phi} = \frac{\delta f}{f} = \frac{1}{\nu}$$

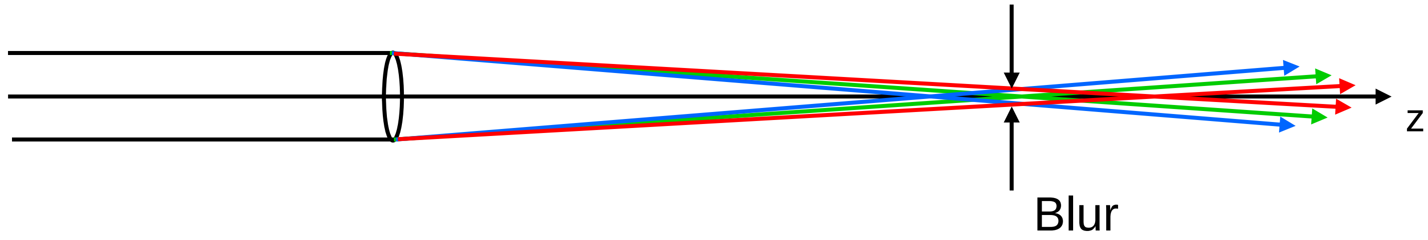
$$f_d = f < 0 \qquad \phi_d = \phi < 0 \qquad \delta f_{CF} = \delta f < 0 \qquad \delta\phi_{FC} = \delta\phi < 0$$

## Axial or Longitudinal Chromatic Aberration

The blur associated with the chromatic aberration of the objective lens limits the performance of an objective.

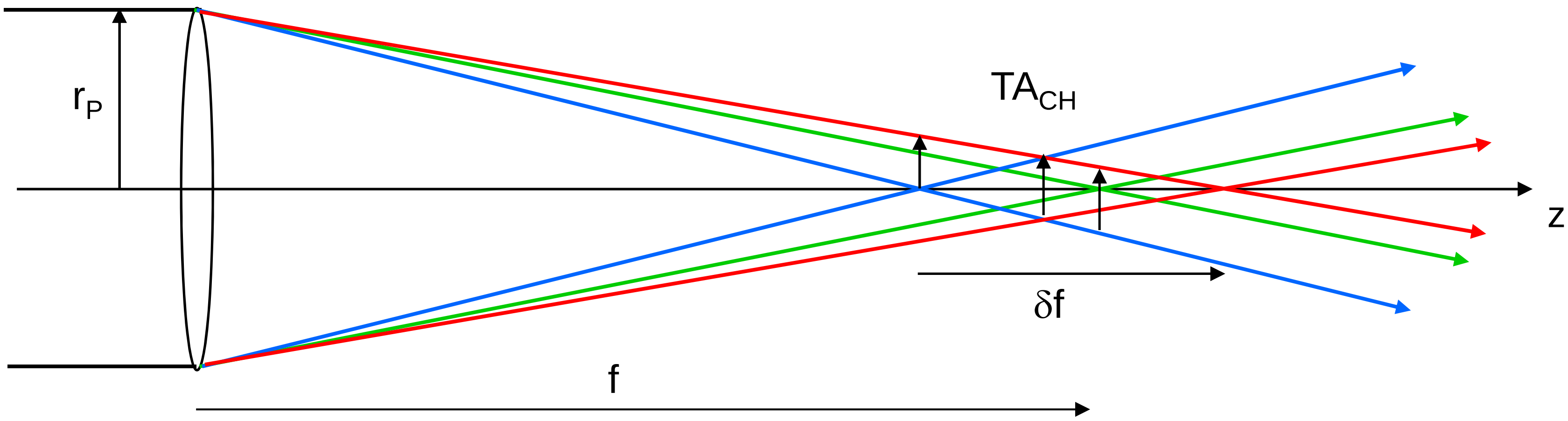



To reduce the blur, a small diameter objective lens is required. Since the longitudinal aberration δf is constant for a given f, the blur is proportional to the lens diameter.



## Transverse Axial Chromatic Aberration

Transverse axial chromatic aberration measures the image blur size due to axial chromatic aberration.



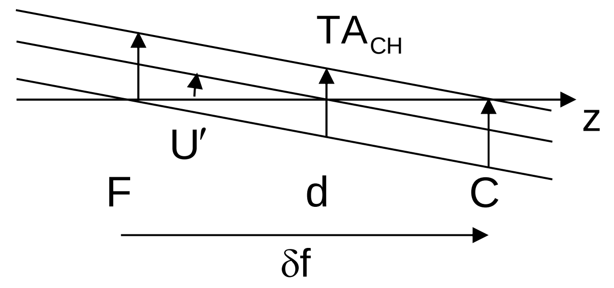

$$f >> \delta f$$

The rays from the edge of the pupil are approximately parallel in the vicinity of the focus. Remember that the axial chromatic is 1.5-3% of the focal length and that the diagram is greatly exaggerated.

# Transverse Axial Chromatic Aberration

Transverse axial chromatic aberration measures the image blur size due to axial chromatic aberration.

Because $f >> \delta f$, the three marginal rays (F, d and C) are approximately parallel.




$$\tan U' = -\frac{TA_{CH}}{\delta f}$$

$$\tan U' = -\frac{r_P}{f}$$

$$\frac{TA_{CH}}{\delta f} = \frac{r_P}{f}$$

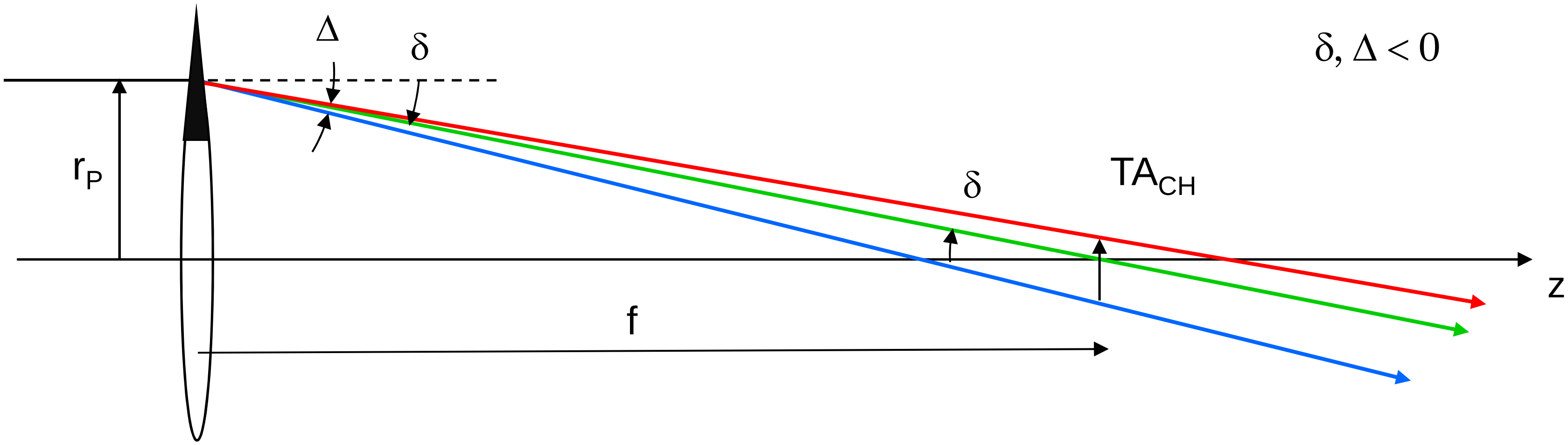
$$\frac{TA_{CH}}{r_P} = \frac{\delta f}{f} = \frac{1}{\nu}$$

$$TA_{CH} = \frac{r_P}{\nu}$$

$TA_{CH}$ depends only on the glass and the pupil radius $r_P$ (assumes that the stop is at the lens).

# Transverse Axial Chromatic Aberration – Alternate Derrivation

Consider the edge of the lens to be a thin prism of Abbe Number ν with a deviation δ and a dispersion Δ.




$$Blur = TA_{CH} = |\Delta| f$$

$$\Delta = \frac{\delta}{\nu} = -\frac{r_P}{\nu f}$$

$$\delta = -\frac{r_P}{f}$$

$$Blur = TA_{CH} = \frac{r_P}{\nu f} f$$

$$TA_{CH} = \frac{r_P}{\nu}$$

The blur is the product of the dispersion and the focal length.

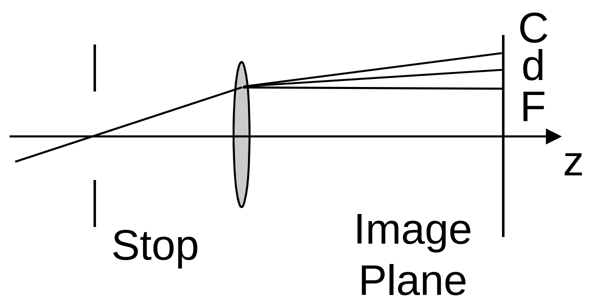
The ray deviation and dispersion grow as the pupil radius normalized by the focal length.

The net result is independent of the focal length.

## Lateral Chromatic Aberration

Longitudinal chromatic aberration is chromatic aberration of the marginal ray of the system.

Lateral chromatic aberration or lateral color is caused by dispersion of the chief ray.

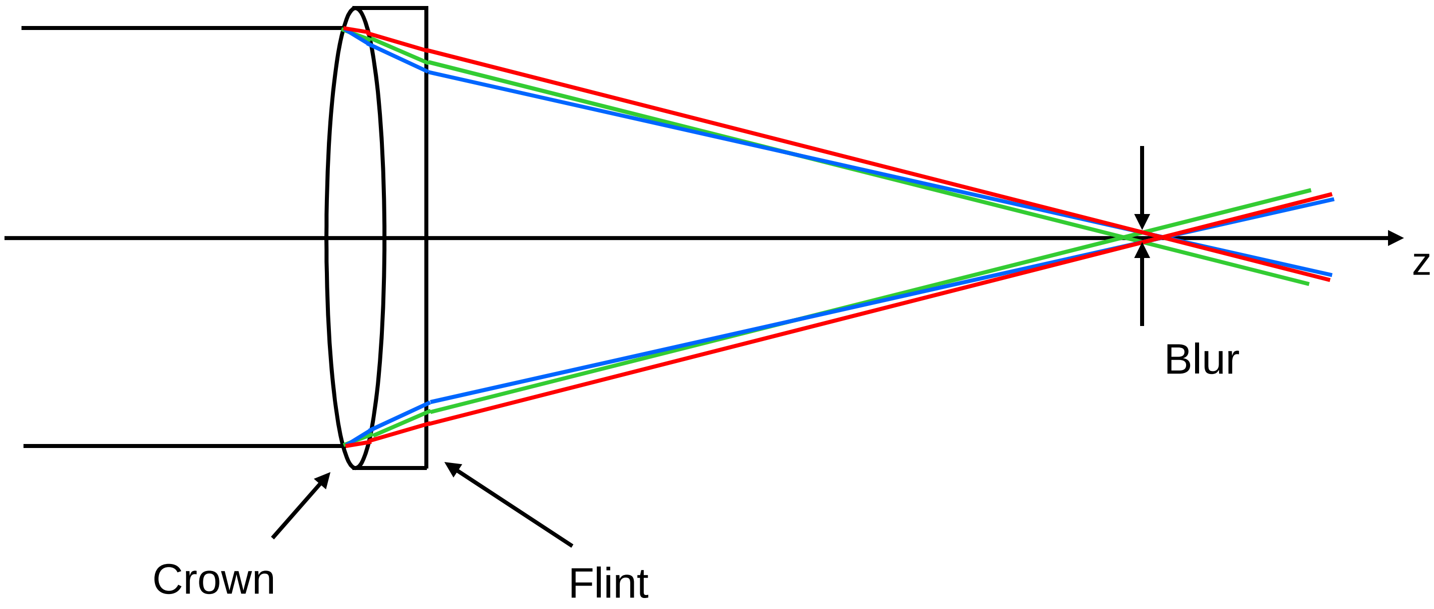



The edge of the lens behaves like a prism. Off-axis image points will exhibit a radial color smear. The blur length increases linearly with image height. Each color has a different lateral magnification.

## Achromatic Objective or Doublet

Two lens elements with different dispersive properties are combined into a single objective lens.

Red and blue light are made to focus at the same location and a greatly reduced image blur results even with large diameter lenses.



Achromatic Doublet

The thin lens achromatic doublet corrects longitudinal chromatic aberration by combining a positive thin lens and a negative thin lens. Two different glasses ( $\nu_1$, $P_1$ and $\nu_2$, $P_2$ ) are used. The nominal powers and focal lengths are for d light.

$$\phi = \phi_1 + \phi_2$$

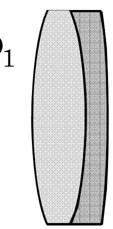
$\phi_1, \phi_2$ at d

$$\delta\phi = \delta\phi_{FC} = \delta\phi_{FC1} + \delta\phi_{FC2}$$

For each lens:

$$\delta\phi_{FCi} = \frac{\phi_{di}}{\nu_i} = \frac{\phi_i}{\nu_i}$$

$$\delta\phi_{FC} = \frac{\phi_1}{\nu_1} + \frac{\phi_2}{\nu_2}$$




Achromat: $\delta\phi_{FC} = \phi_F - \phi_C = 0$

$$\frac{\phi_1}{\nu_1} = -\frac{\phi_2}{\nu_2}$$

$$\phi_2 = -\frac{\nu_2}{\nu_1}\phi_1$$

$$\phi = \phi_1 + \phi_2 = \phi_1 - \frac{\nu_2}{\nu_1}\phi_1$$

$$\frac{\phi_1}{\phi} = \frac{\nu_1}{\nu_1 - \nu_2}$$

$$\frac{\phi_2}{\phi} = -\frac{\nu_2}{\nu_1 - \nu_2}$$

$\phi, \phi_1, \phi_2$ at d

## Secondary Chromatic Aberration

The solution for the thin lens achromatic doublet result forces the same axial focus for F and C light (zero primary chromatic aberration), but d light can focus at a different location. This residual is the secondary chromatic aberration or secondary color of the doublet.

$$\delta\phi_{dC} = \phi_d - \phi_C \qquad \delta f_{Cd} = f_C - f_d$$

$$\delta\phi_{dC} = \delta\phi_{dC1} + \delta\phi_{dC2}$$

$$\delta\phi_{dC1} = (n_{d1} - 1)(C_1 - C_2) - (n_{C1} - 1)(C_1 - C_2)$$

$$\delta\phi_{dC1} = (n_{d1} - n_{C1})(C_1 - C_2)$$

$$\delta\phi_{dC1} = \frac{(n_{d1} - n_{C1})}{(n_{F1} - n_{C1})}(n_{F1} - n_{C1})(C_1 - C_2)$$

$$P_1 = \frac{(n_{d1} - n_{C1})}{(n_{F1} - n_{C1})} \qquad \delta\phi_{FC1} = (n_{F1} - n_{C1})(C_1 - C_2)$$

$$\delta\phi_{dC1} = P_1\,\delta\phi_{FC1} \qquad \delta\phi_{FC1} = \frac{\phi_1}{\nu_1}$$

and

$$\delta\phi_{dC2} = P_2\,\delta\phi_{FC2} \qquad \delta\phi_{FC2} = \frac{\phi_2}{\nu_2}$$

$$\delta\phi_{dC} = P_1\,\delta\phi_{FC1} + P_2\,\delta\phi_{FC2}$$

$$\delta\phi_{dC} = P_1\frac{\phi_1}{\nu_1} + P_2\frac{\phi_2}{\nu_2}$$

For an achromat: $\dfrac{\phi_1}{\nu_1} = -\dfrac{\phi_2}{\nu_2}$

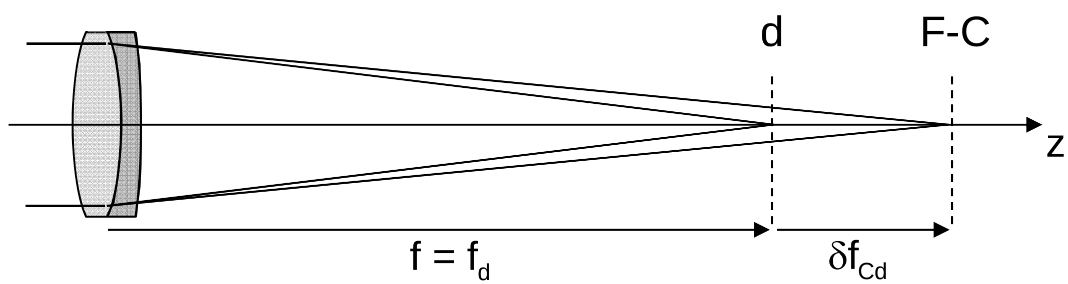
$$\delta\phi_{dC} = (P_1 - P_2)\frac{\phi_1}{\nu_1}$$

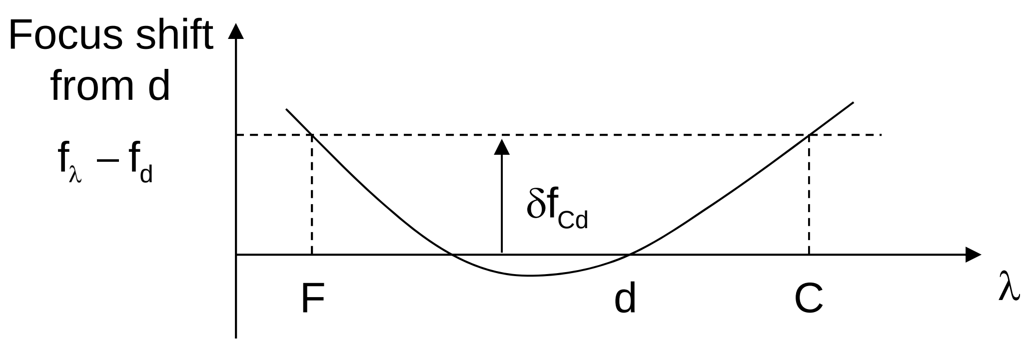
$$\frac{\phi_1}{\phi} = \frac{\nu_1}{\nu_1 - \nu_2} \quad \Rightarrow \quad \frac{\phi_1}{\nu_1} = \frac{\phi}{\nu_1 - \nu_2}$$

$$\delta\phi_{dC} = \frac{(P_1 - P_2)}{(\nu_1 - \nu_2)}\phi = \frac{\Delta P}{\Delta\nu}\phi$$

## Secondary Chromatic Aberration or Secondary Color

$$\frac{\delta\phi_{dC}}{\phi} = \frac{\delta f_{Cd}}{f} = \frac{(P_2 - P_1)}{(\nu_2 - \nu_1)} = \frac{\Delta P}{\Delta \nu} \qquad \delta\phi_{dC} = \phi_d - \phi_C \qquad \delta f_{Cd} = \delta f_2 = f_C - f_d$$

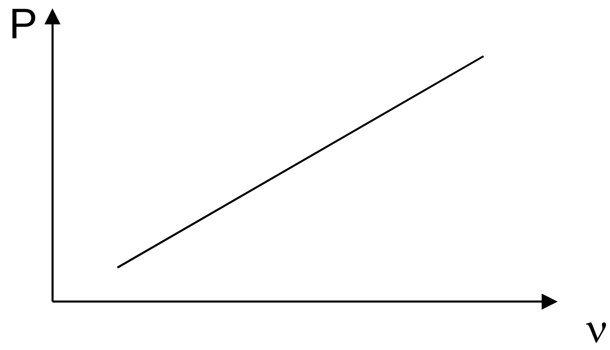



The d focus is probably not the maximum focus shift from F and C.

Partial Dispersion Ratio and the Abbe Number

On a plot of $P$ versus $\nu$, most glasses lie on a straight line.




$$P = P_{d,C} = \frac{n_d - n_C}{n_F - n_C}$$

$$\nu = \frac{n_d - 1}{n_F - n_C}$$

The slope of this line is approximately: $\frac{\Delta P}{\Delta \nu} \approx 0.00045 = \frac{1}{2200}$

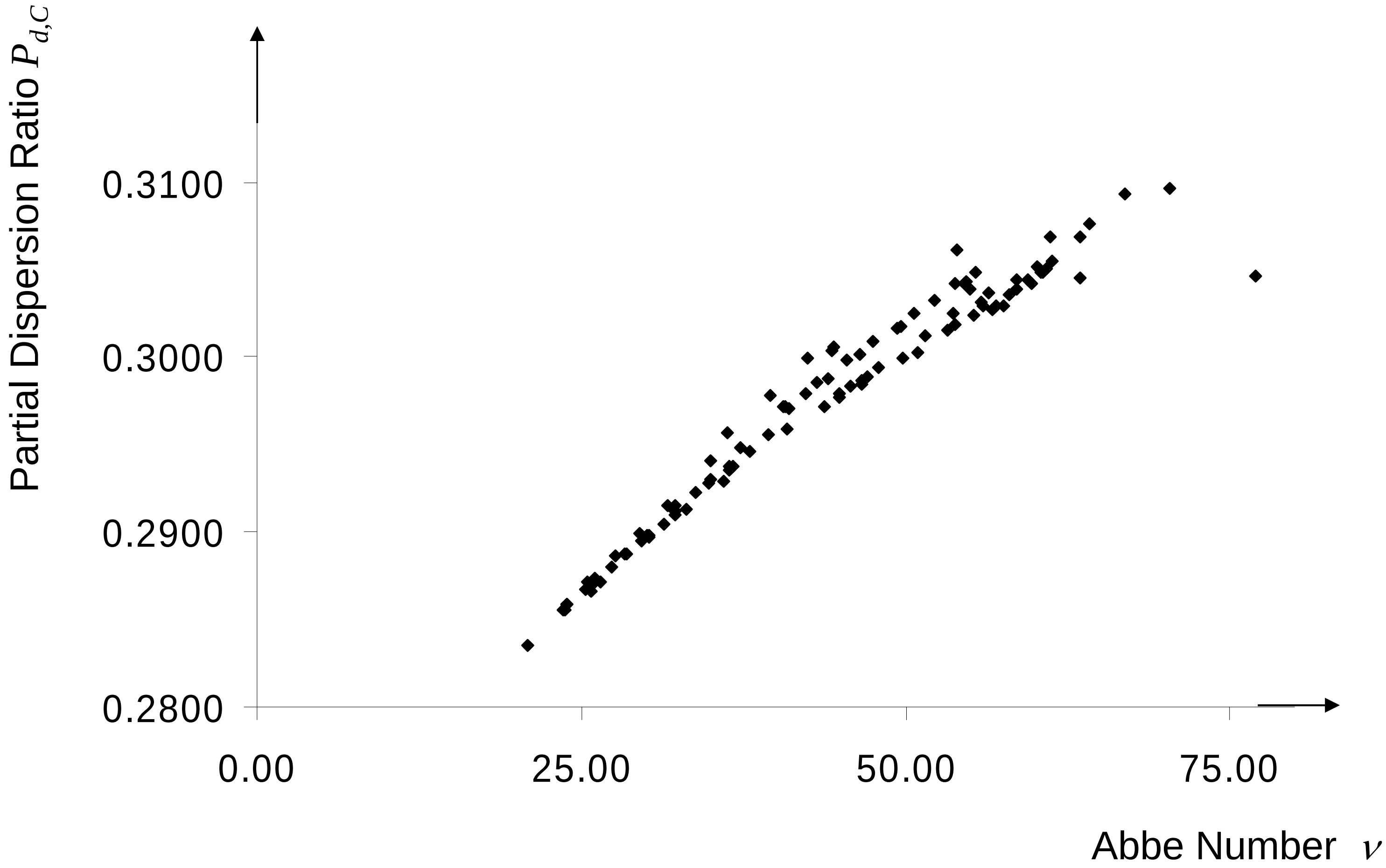
Example: BK7 $\nu = 64.17$
$P = .3075$

F2 $\nu = 36.37$
$P = .2937$

$$\frac{\Delta P}{\Delta \nu} = \frac{-.0138}{-27.80} = \frac{1}{2014} = .0005$$

P versus ν – Real Glass Data

Data for all of the glasses in the Schott Glass Catalog:



## Singlet versus Doublet Performance

Singlet:

$$\delta f = \delta f_{CF} = f_C - f_F \qquad \frac{\delta f_{CF}}{f_d} = \frac{1}{\nu}$$

$$\nu \approx 30 - 70$$

$$\delta f_{CF} \approx \frac{f}{50}$$

Primary chromatic aberration of a singlet.

Doublet:

$$\delta f_{Cd} = f_C - f_d \qquad \frac{\delta f_{Cd}}{f} = \frac{\Delta P}{\Delta \nu}$$

$$\frac{\Delta P}{\Delta \nu} \approx 0.00045 = \frac{1}{2200}$$

$$\delta f_{Cd} = \frac{f}{2200}$$

Secondary chromatic aberration of a doublet.

The use of the achromatic doublet reduces chromatic focal length variation by a factor of about 40 over the same focal length singlet.

Excess Power

The doublet design places excess power in the positive element that is cancelled by the negative element.

Both elements contribute equal, but opposite, amounts of longitudinal chromatic aberration.

Just as with the achromatic thin prism, the longitudinal chromatic aberration grows slower with the low dispersion glass (positive element) than the high dispersion glass (negative element).

Large differences in the Abbe numbers minimize the excess power and provide better performance.

Large $\Delta\nu$ Small $\Delta\nu$

## Zero Secondary Color

In order to obtain zero secondary chromatic aberration with a doublet, the partial dispersion ratios of the two glasses must be zero.

$$\Delta P = 0 \quad \Rightarrow \quad \delta f_{Cd} = \frac{\Delta P}{\Delta \nu} f = 0$$

There are some special glass pairs which have different Abbe numbers but the same partial dispersion ratio. Unfortunately, the difference in Abbe number is usually small, resulting in an achromatic doublet with significant excess power.

To correct chromatic aberration at additional wavelengths more than two glasses are used:

Apochromat – 3 glasses with correction at three wavelengths.

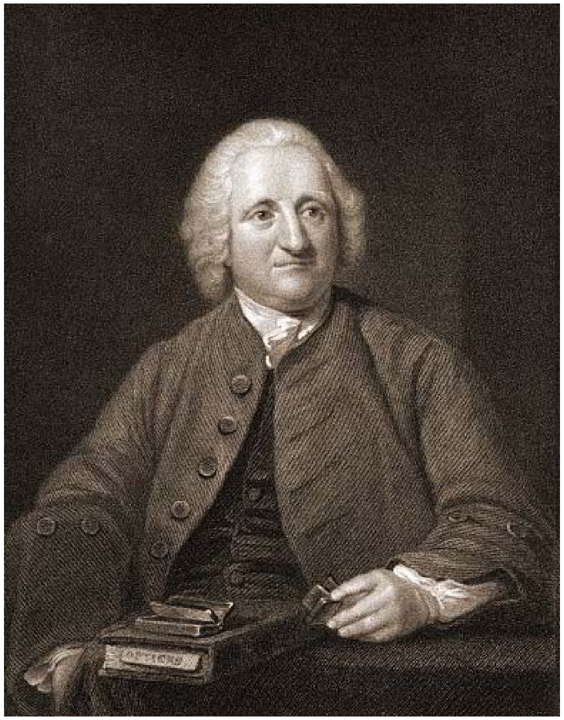
Super Apochromat – 4 or more glasses and wavelengths.

## Chromatic Aberration Correction

In the early 1700s, it was believed that chromatic aberration was fundamental and could not be corrected.

Even Isaac Newton mistakenly held this belief!

In the mid-1700s, the work of Chester Moor Hall and John Dollond led to the development of the achromatic objective.



John Dollond
1706-1761, English

## The Story of the Achromatic Doublet

The original inventor is Chester Moor Hall, a Barrister in London. In 1733, he commissioned two different opticians, Edward Scarlett and James Mann, to each make one of the lens elements.

By chance, both opticians subcontracted the work to the same man, George Bass. Chester Moor Hall then continued to keep his invention secret.

Chester Moor Hall
1703-1771, English

The Story of the Achromatic Doublet

Around 1750, George Bass told John Dollond about the achromatic lens he had made, or at least the fact that different glasses have different dispersing powers. Dollond then began a series of experiments using different types of glass.

Dollond's son, Peter, saw the commercial advantages and once they had made test lenses, patented the invention in 1758.

Peter Dollond
1731-1821, English

## Achromatic Doublet Patent

Chester Moor Hall twice attempted to challenge the patent.

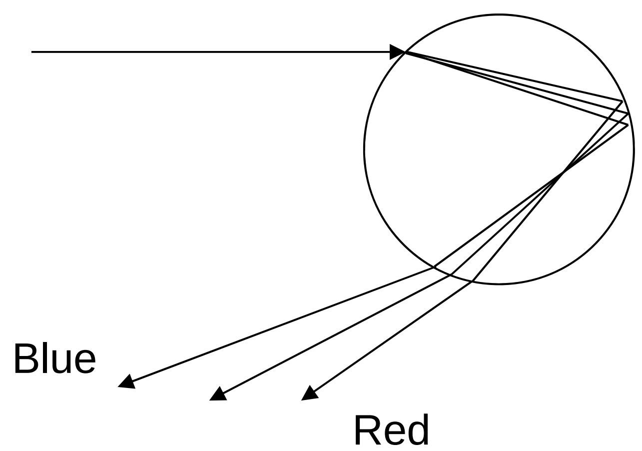
He lost his case on the grounds that the person who should profit by the invention is the one who benefits the public by it, not one who keeps it locked in his desk drawer.

This was a landmark decision in patent law that remains in place to today.

Dollond went on to become the dominant manufacturer of telescopes in the late 1700s and early 1800s. The name "Dollond" became a synonym for a telescope.

## Rainbows – Primary Rainbow

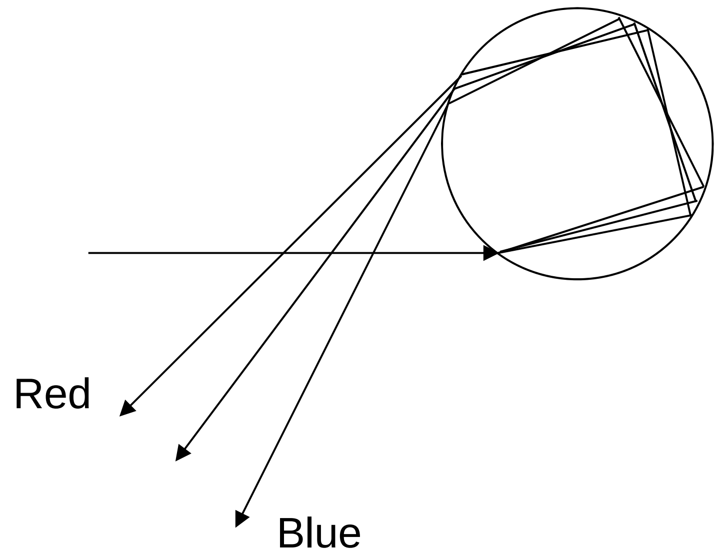
Rainbows result from the combination of refraction, reflection and dispersion from a raindrop. The entering ray is refracted and dispersed twice. For the primary rainbow, there is single internal Fresnel reflection. Blue light is deviated more than red light.




Raindrops have an index of refraction of water, $n$ = 1.33.

## Rainbows – Secondary Rainbow

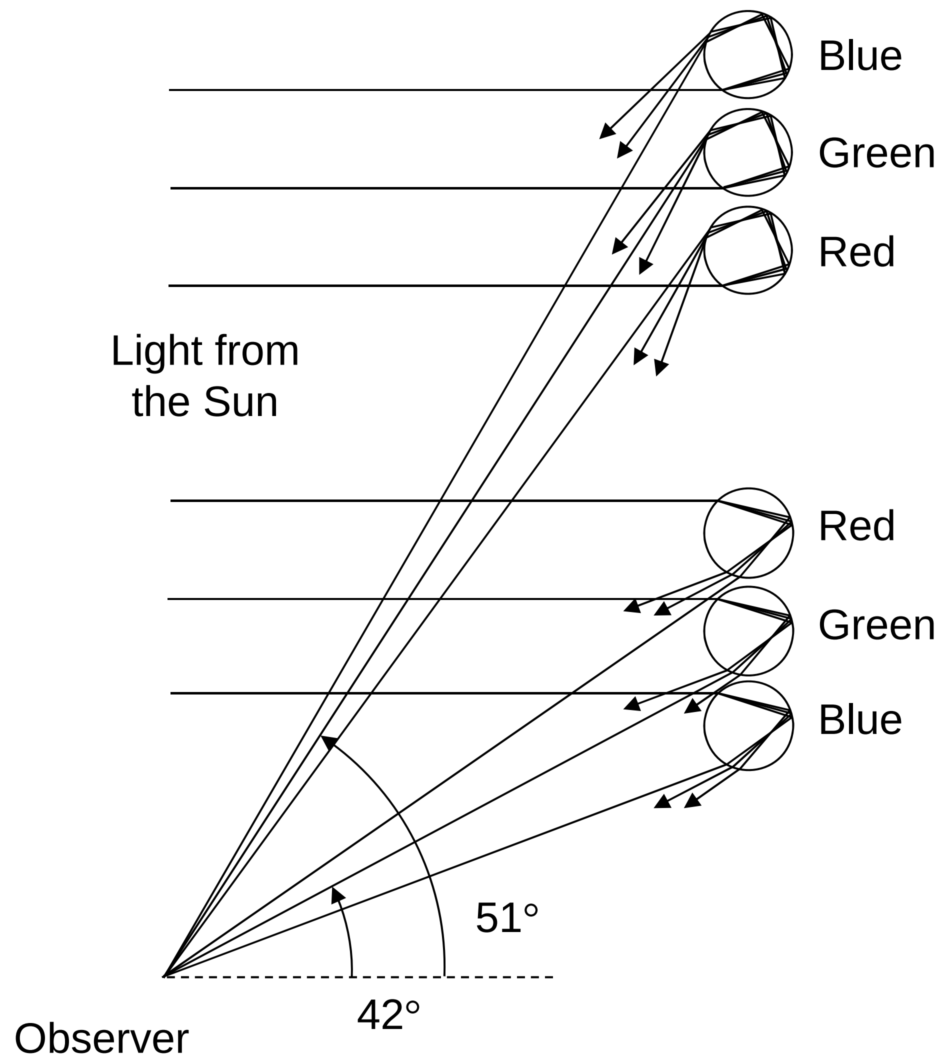
Under good viewing conditions, a second dimmer rainbow can be seen outside the primary bow. The secondary rainbow is created when there are two reflections inside the drop. The direction of propagation within the drop is reversed.



Observing Rainbows

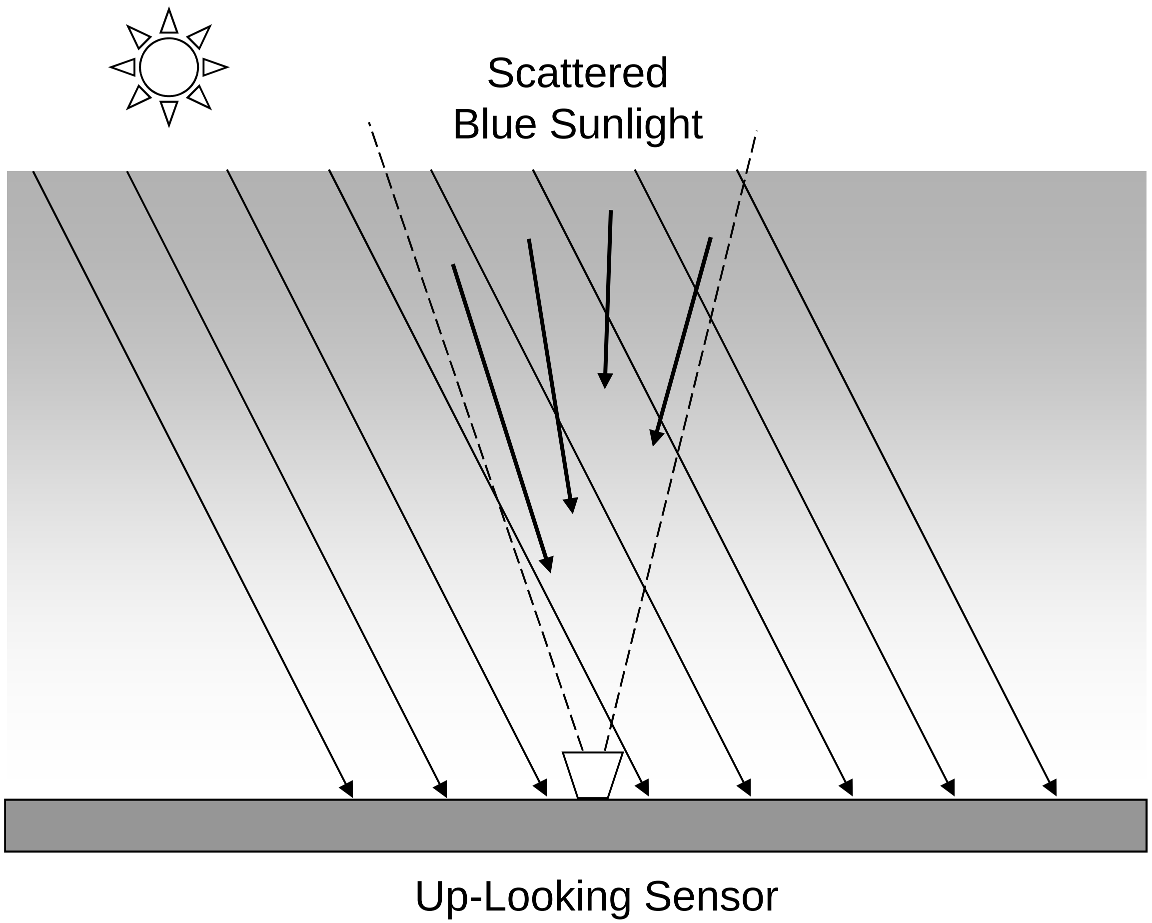
In the primary rainbow, the droplets directing the red light to the observer are above those that direct the blue light. Because the angle of rotation is opposite, the colors of the secondary rainbow are reversed. The primary rainbow is at an angle of about 42°, and the secondary rainbow is at 51°.

Each observer uses a different set of raindrops to view their individual rainbow.



Why is the Sky Blue?

Molecules in the atmosphere act as scattering centers for the incident sunlight. The primary scattering mechanism is Rayleigh scattering which has a $1/\lambda^4$ dependence. As a result, blue light is preferentially scattered, and the sky appears blue.



Why are Sunsets Red?

Sunsets are red for the same reason that the sky is blue. The long path length through the atmosphere at sunset depletes the blue and green content of the direct sunlight, leaving reds and oranges.