Elementary Optical Systems

Section 13

Magnifiers and Telescopes

Elementary Optical Systems

Many optical systems can be understood when treated as combinations of thin lenses. Mirror equivalents exist for many. The goal of this approach is to examine the paraxial properties (image size and location; entrance and exit pupils; etc.) of a variety of systems.

The types of systems examined includes:

Objectives
Collimators
Magnifiers
Field lenses
Telescopes
Eyepieces
Microscopes
Telecentric systems
Relays
Illumination systems
Scanners

Visual Magnification

All optical systems that are used with the eye are characterized by a visual magnification or a visual magnifying power.

While the details of the definitions of this quantity differ from instrument to instrument and for different applications, the underlying principle remains the same:

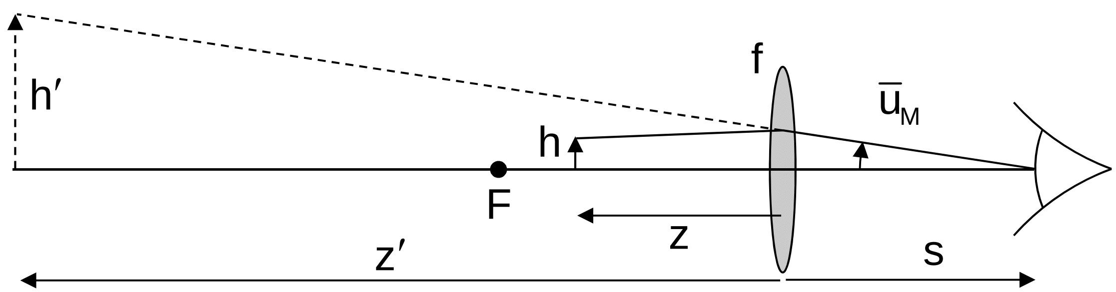
How much bigger does an object appear to be when viewed through the instrument?

The size change is measured as the change in angular subtense of the image produced by the instrument compared to the angular subtense of the object.

The angular subtense of the object is measured when the object is placed at the optimum viewing condition.

Magnifiers

As an object is brought closer to the eye, the size of the image on the retina increases and the object appears larger. The largest image magnification possible with the unaided eye occurs when the object is placed at the near point of the eye, by convention 250 mm or 10 in from the eye. A magnifier is a single lens that provides an enlarged erect virtual image of a nearby object for visual observation. The object must be placed inside the front focal point of the magnifier.



The magnifying power MP is defined as (stop at the eye):

$$MP = \frac{\text{Angular size of the image (with lens)}}{\text{Angular size of the object at the near point}}$$

$$MP = \frac{\bar{u}_M}{\bar{u}_U} \qquad d_{NP} = -250\ \text{mm}$$

Magnifiers – Magnifying Power

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \qquad\qquad z = \frac{f z'}{f - z'}$$

$$\frac{h'}{h} = m = \frac{z'}{z} \qquad\qquad h' = h\frac{z'}{z}$$

$$h' = \frac{h(f - z')}{f}$$

With magnifier: $\bar{u}_M = \dfrac{h'}{z' - s} = \dfrac{h(f - z')}{f(z' - s)}$

Unaided eye: $\bar{u}_U = \dfrac{h}{d_{NP}}$

$$MP = \frac{\bar{u}_M}{\bar{u}_U} = \frac{d_{NP}(f - z')}{f(z' - s)} \qquad\qquad d_{NP} = -250 \text{ mm}$$

Note that as the eye-lens distance decreases, the MP increases. A common assumption is that the lens is located at the eye (s = 0):

$$MP = \frac{d_{NP}}{z'} - \frac{d_{NP}}{f} = \frac{250\,mm}{f} - \frac{250\,mm}{z'}$$

Magnifiers – Magnifying Power – Continued

$$MP = \frac{d_{NP}}{z'} - \frac{d_{NP}}{f} = \frac{250\,mm}{f} - \frac{250\,mm}{z'}$$

The magnification is a function of both f and the image location. The most common definition of the MP of a magnifier assumes that the lens is close to the eye and that the image is presented to a relaxed eye ($z' = \infty$).

$$MP = -\frac{d_{NP}}{f} = \frac{250\,mm}{f} = \frac{10"}{f}$$

The maximum MP occurs if the image is presented at the near point of the eye:

$$MP = 1 - \frac{d_{NP}}{f} = \frac{250\,mm}{f} + 1 = \frac{10"}{f} + 1 \qquad z' = d_{NP}$$

Magnifiers – Required MP

The angular subtense of the image at the eye:

$$\theta \approx -\bar{u}_M = -MP\,\bar{u}_U$$

$$\bar{u}_U = \frac{h}{d_{NP}}$$

$$\theta \approx -\frac{MP\,h}{d_{NP}} = \frac{MP\,h}{250\,mm}$$

The human eye has a resolution of about 1 arc minute. A small object must be enlarged to 1 arc minute to be seen. This determines the required MP.

$$MP = -\frac{\theta\, d_{NP}}{h}$$

$$d_{NP} = -250\ mm = -10''$$

$$\theta = 1 \text{ arc min} = .0003 \text{ rad}$$

$$MP \geq .075\,mm / h$$

or

$$MP \geq .003''/ h$$

Magnifiers up to about 25X are practical; 10X is common.

Telescopes

Telescopes are afocal or nearly afocal systems used to change the apparent angular size of an object. The image through the telescope subtends an angle θ' different from the angle subtended by the object θ. The magnifying power MP of a telescope is

$$MP = \frac{\theta'}{\theta}$$

$|MP| > 1$ Telescope magnifies

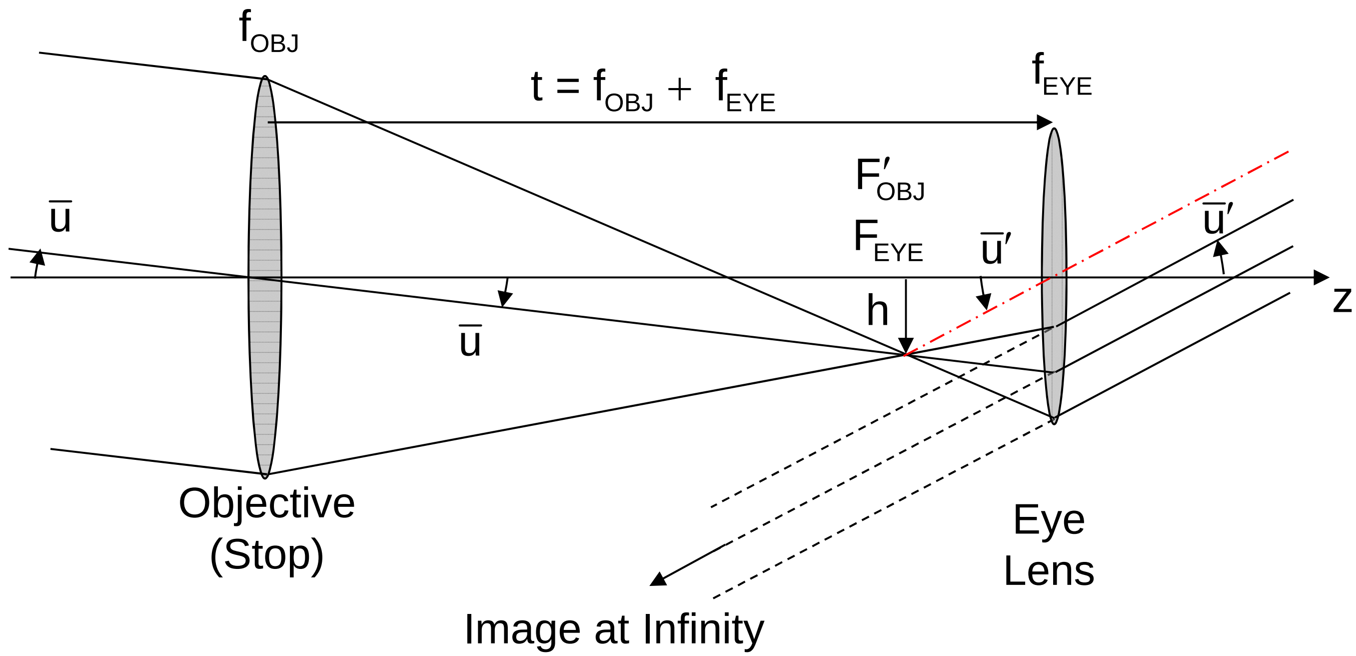
$|MP| < 1$ Telescope minifies

The angles θ and θ' are often considered to be paraxial angles.

$$MP = \frac{\bar{u}'}{\bar{u}} = \text{ Angular Magnification}$$

Keplerian Telescope

A Keplerian telescope or astronomical telescope consists of two positive lenses separated by the sum of the focal lengths. The system stop is usually at or near the objective lens.



This telescope can be considered to be a combination of an objective plus a magnifier. The objective creates an aerial image (a real image in the air) at the common focal point that is magnified by the eye lens and presented to the relaxed eye at infinity.

$$h = \bar{u} f_{OBJ} \qquad h = -\bar{u}' f_{EYE} \qquad MP = \frac{\bar{u}'}{\bar{u}} = -\frac{f_{OBJ}}{f_{EYE}}$$

The image presented to the eye is inverted and reverted (rotated by 180° or "upside down"). The MP of a Keplerian telescope is negative.

Telescopes – Magnifying Power

The telescope is also afocal: $m = -\dfrac{f_2}{f_1} = -\dfrac{f_{EYE}}{f_{OBJ}}$ $\qquad MP = \dfrac{1}{m} = -\dfrac{f_{OBJ}}{f_{EYE}}$

A conundrum – to make the scene appear larger, the magnitude of the MP must be greater than one. This implies that the magnitude of the magnification is less than one. The image is smaller than the object. So how do telescopes work?

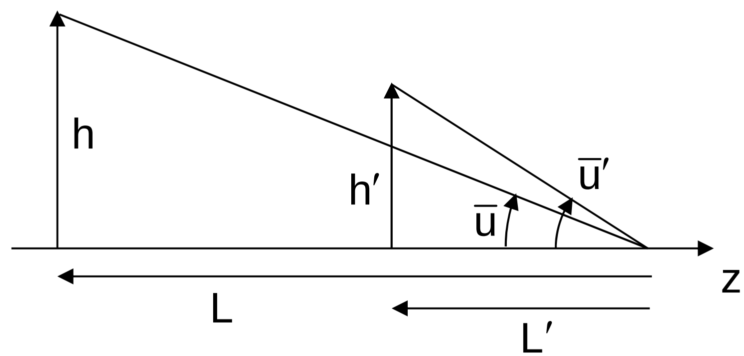
$$|MP| > 1 \quad \rightarrow \quad |m| < 1$$

Don't forget about the longitudinal magnification. What is important is the angular subtense of the image compared to the object: the angular magnification.

$$\bar{m} = m^2 \quad \text{(in air)}$$

$$|\bar{u}| = \frac{h}{L} \qquad |\bar{u}'| = \frac{h'}{L'} = \frac{|m|h}{\bar{m}L} = \frac{|m|h}{m^2 L} = \frac{|\bar{u}|}{|m|}$$

$$|MP| = \left|\frac{\bar{u}'}{\bar{u}}\right| = \frac{1}{|m|} > 1$$



The image gets closer faster than it gets smaller, and the angular subtense increases. The scene appears foreshortened.

Telescopes will list only the magnitude of the MP (10X, etc).

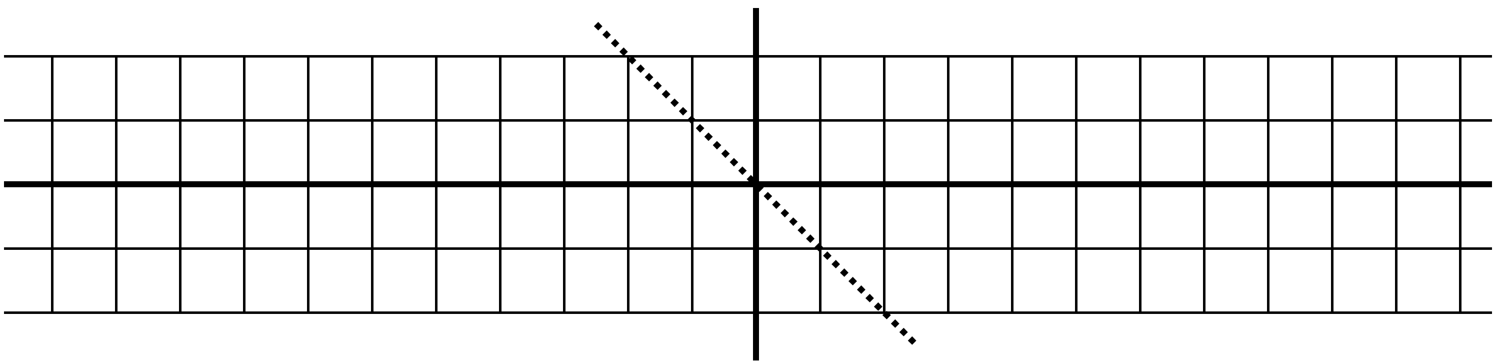
Angular Magnification – Afocal System

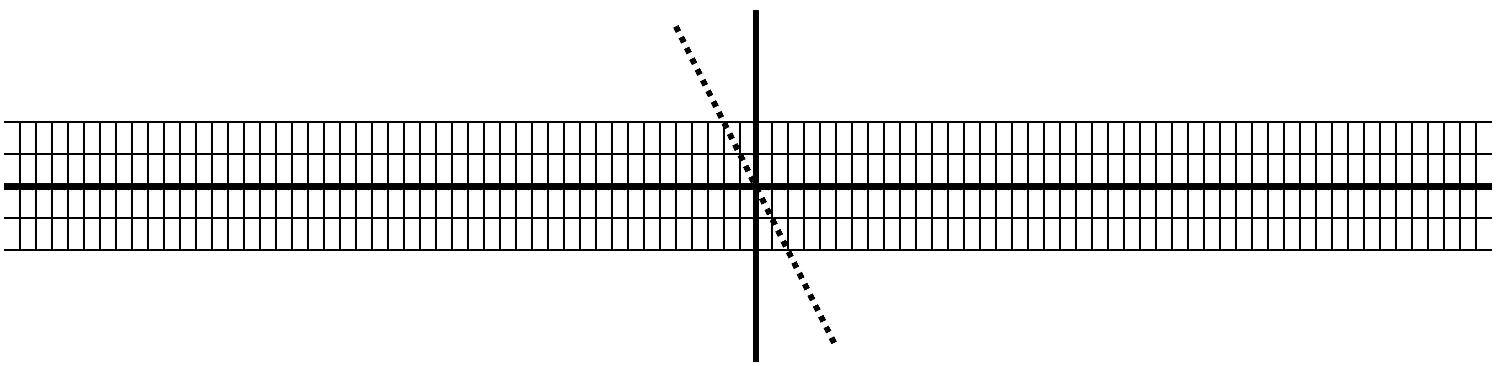
$$MP = \frac{m}{\bar{m}} = \frac{1}{m} = \text{ Angular Magnification}$$

$m = 1/2$

$\bar{m} = 1/4$

Angular Mag = 2





Afocal Systems Do Not Have Cardinal Points

In an afocal system, rays parallel to the optical axis emerge from the system also parallel to the optical axis so the system has a power $\phi = 0$ or infinite focal length. This also follows from Gaussian reduction:

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t = \frac{1}{f_1} + \frac{1}{f_2} - \frac{t}{f_1 f_2} \qquad t = f_1 + f_2$$

$$\phi = \frac{f_1 + f_2}{f_1 f_2} - \frac{f_1 + f_2}{f_1 f_2} = 0$$

The Gaussian imaging equations do not apply and the cardinal points are not defined.

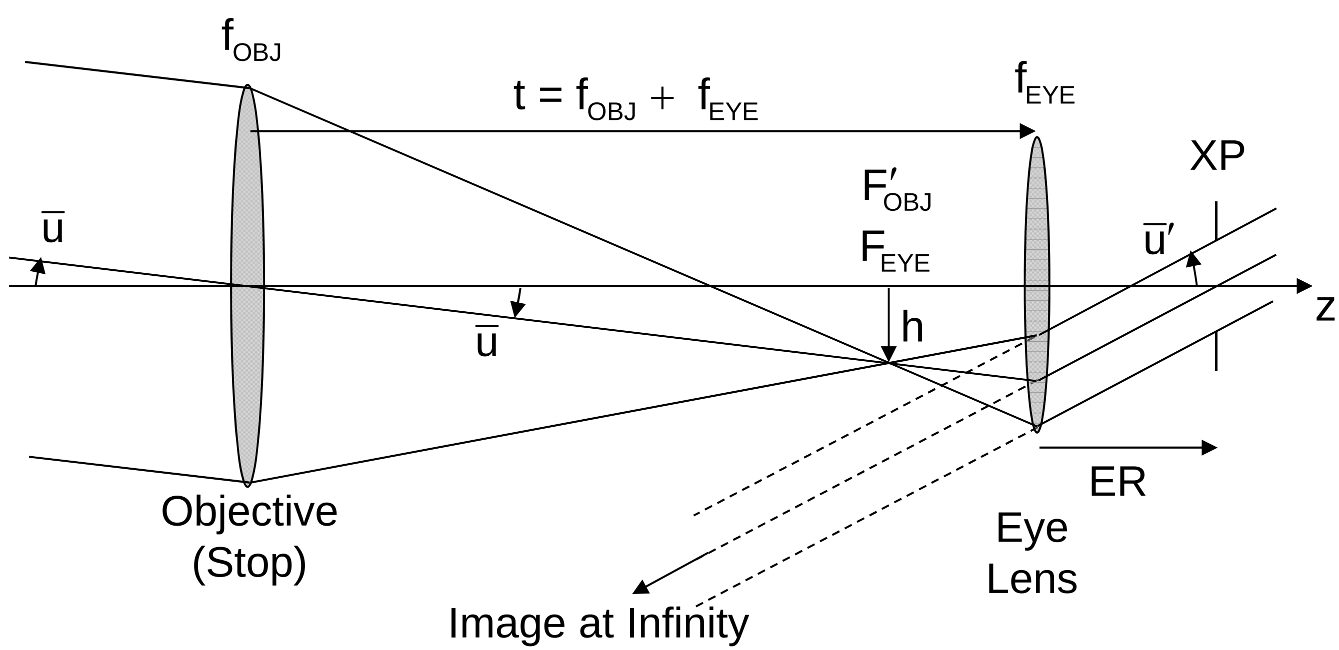
For an afocal system, the lateral magnification is constant. Unless m = 1, there are no planes of unit magnification. Even if m = 1, then all planes are planes of unit magnification. In either case, the principal planes are not defined.

Similarly, the angular magnification of an afocal system is constant. The nodal points cannot be defined.

While it is acceptable to state that the focal points of an afocal system are at infinity, it is better to never talk about the focal points of an afocal system. Since rays parallel to the axis in one optical space never come to focus in the other optical space, there really are no focal points.

Stops and Pupils

In most telescopes, the stop is at the objective lens to minimize the size and cost of this largest element. The objective lens also serves as the entrance pupil. The exit pupil is the image of the stop produced by the eye lens. The distance between the eye lens and the XP is called the eye relief (ER).



Exit Pupil

Exit pupil location:

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_{EYE}} \qquad z = -\left(f_{OBJ} + f_{EYE}\right)$$

$$z' = ER \qquad \frac{1}{z'} = \frac{-1}{f_{OBJ} + f_{EYE}} + \frac{1}{f_{EYE}} = \frac{f_{OBJ}}{\left(f_{OBJ} + f_{EYE}\right) f_{EYE}}$$

$$z' = \frac{\left(f_{OBJ} + f_{EYE}\right) f_{EYE}}{f_{OBJ}} = \left(1 + \frac{f_{EYE}}{f_{OBJ}}\right) f_{EYE}$$

$$ER = z' = \left(1 - m\right) f_{EYE}$$

$$m = \frac{z'}{z} = -\frac{f_{EYE}}{f_{OBJ}}$$

$$z' = \frac{f_{EYE}}{f_{OBJ}}\left(f_{OBJ} + f_{EYE}\right)$$

$$ER = z' = \left(1 - m\right) f_{EYE}$$

Exit pupil size:

$$D_{XP} = |m| D_{EP} = \frac{D_{EP}}{|MP|} = \frac{f_{EYE}}{f_{OBJ}} D_{EP}$$

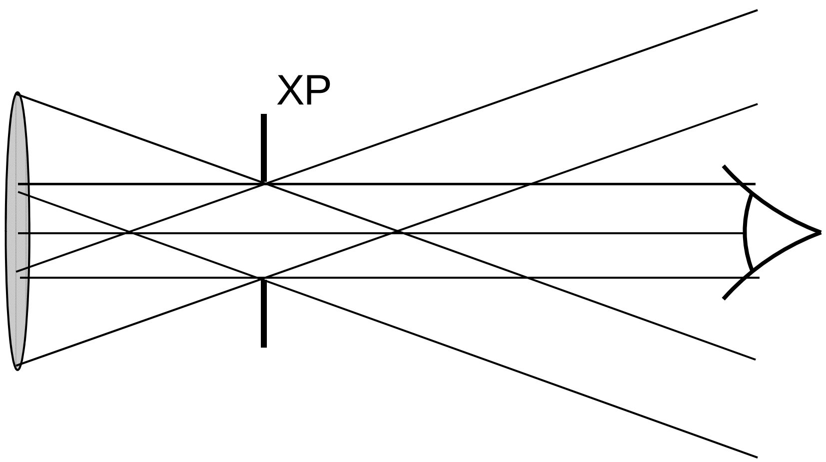
A Keplerian telescope produces a real XP to the right of the eye lens. The XP of a visual instrument is also known as the eye circle or the Ramsden circle.

Measuring the diameters of the EP and XP allows a simple way of determining the MP of the telescope system:

$$|MP| = D_{EP} / D_{XP}$$

Exit Pupil and The Eye

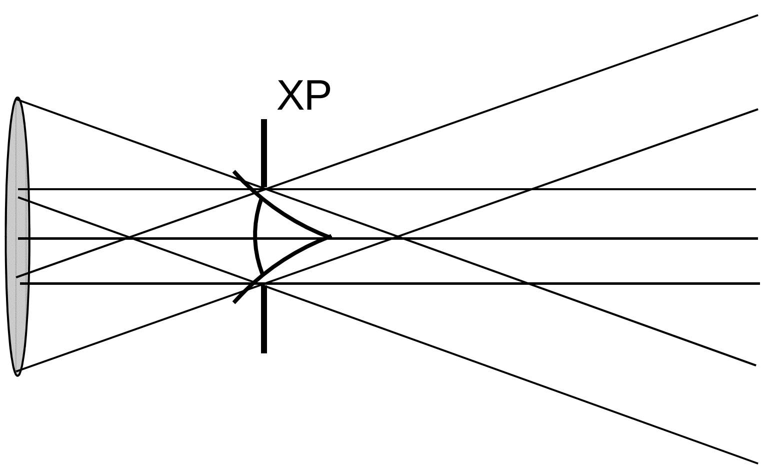
The EP of the eye should be placed at the XP of the telescope to properly couple the two optical systems. If the eye is not at the XP, vignetting can result:



Ray bundles are shown for different FOVs.

The eye will see only on-axis (or near-axis) object points.

If the eye is displaced laterally, portions of the off-axis field are seen.



When the eye is in the XP, the entire FOV of the telescope is seen.

The eye can rotate to look around within the FOV.

Exit Pupil and Eye Relief

The XP should be made larger or smaller than the pupil of the eye so that vignetting does not occur with head or eye motion. This compensates for eye rotation as the rotation point of the eye is not at the EP of the eye. The pupil translates with eye rotation.

A close match of the instrument XP and the eye EP requires precise alignment of the two pupils. Small displacements will change the light level in the image. This is true even if the eye is at the XP.

The human eye pupil diameter varies from 2-8 mm, with a diameter of about 4 mm under ordinary lighting conditions.

When the XP of the instrument overfills the EP of the eye, the eye becomes the system stop. Larger instruments tend to have large XPs, while compact instruments may have small XP diameters (1-1.5 mm).

Sufficient eye relief should be provided to allow the eye to access the XP. Hand-held instruments should have 15-20 mm of eye relief. Microscopes may have as little as 2-3 mm of eye relief. Other systems should have a very long eye relief. For example, a riflescope needs a large ER to avoid problems due to kickback.

Telescope and Binocular Specification

Binoculars are a pair of parallel telescopes; one for each eye.

The specification provided on telescopes and binoculars is of the form *AXB* (for example 7X35).

$$A = |MP| \qquad B = \text{Objective Diameter in mm}$$

$$D_{XP} = \frac{D_{EP}}{|MP|}$$

Mini Quiz

A 5X Keplerian telescope is constructed out of two thin lenses. The separation between the two lenses is 120 mm, and the diameter of the objective lens is 25 mm. The system stop is at the objective. Determine the eye relief and the diameter of the exit pupil for this telescope.

[ ] a. ER = 20 mm and DXP = 10 mm
[ ] b. ER = 24 mm and DXP = 10 mm
[ ] c. ER = 20 mm and DXP = 5 mm
[ ] d. ER = 24 mm and DXP = 5 mm

Mini Quiz – Solution

A 5X Keplerian telescope is constructed out of two thin lenses. The separation between the two lenses is 120 mm, and the diameter of the objective lens is 25 mm. The system stop is at the objective. Determine the eye relief and the diameter of the exit pupil for this telescope.

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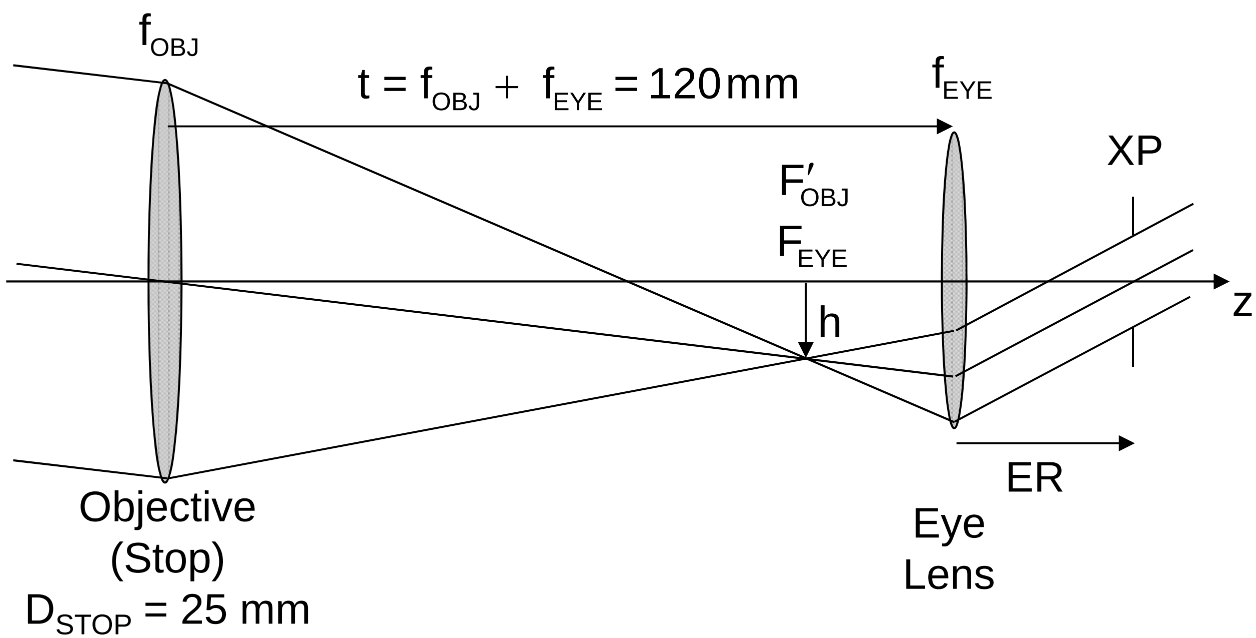


Mini Quiz – Solution – Page 2

Telescope Design 5X Keplerian:

$$MP = -5 = -\frac{f_{OBJ}}{f_{EYE}} \qquad f_{OBJ} = 5 f_{EYE} \qquad t = f_{OBJ} + f_{EYE} = 6 f_{EYE} = 120\,mm$$

$$f_{EYE} = 20\,mm \qquad f_{OBJ} = 100\,mm$$

Eye Relief – Image Stop Through the Eye Lens:

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f_{EYE}} \qquad z = -\left(f_{OBJ} + f_{EYE}\right) = -t = -120\,mm$$

$$z' = ER = 24\,mm$$

Exit Pupil Diameter: $D_{EP} = D_{STOP} = 25\,mm$

$$m_{XP} = \frac{z'}{z} = \frac{24\,mm}{-120\,mm} = -0.2$$

$$D_{XP} = |m_{XP}| D_{STOP} = 0.2\left(25\,mm\right) = 5\,mm$$

or

$$D_{XP} = \frac{D_{EP}}{|MP|} = \frac{25\,mm}{5} = 5\,mm$$

Required Resolution

The resolution of the eye is about 1 arc min:

$$\Delta\theta' = 1 \text{ arc min}$$

$$\Delta\theta = \frac{\Delta\theta'}{|MP|} = \frac{1 \text{ arc min}}{|MP|}$$

The visual resolution has been minified into object space.

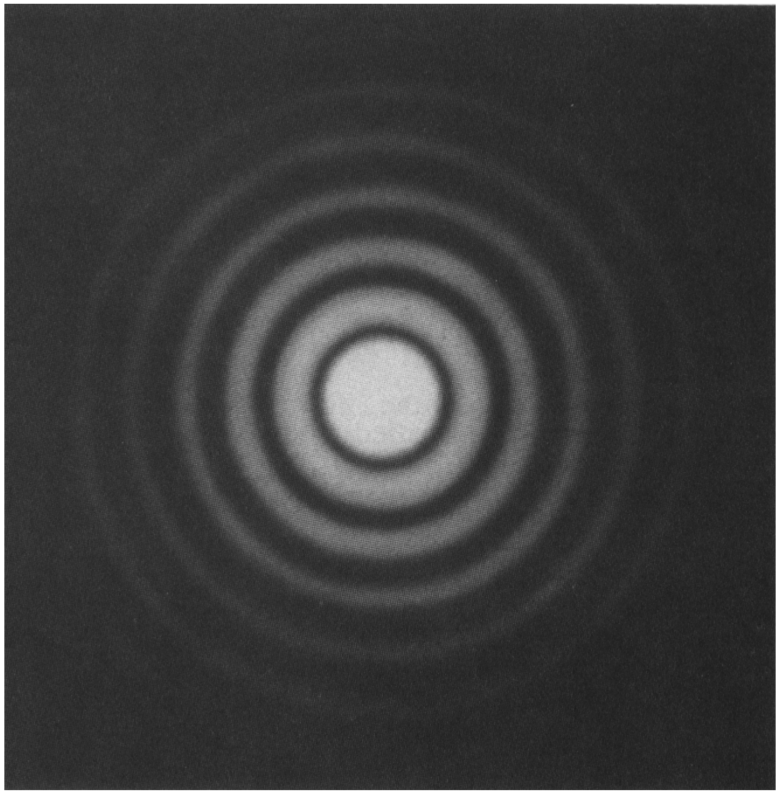
If two objects are separated by Δθ, the minimum MP to visually resolve them is:

$$|MP|_{MIN} = \frac{1 \text{ arc min}}{\Delta\theta}$$

For critical work, MPs larger than this value are often used to minimize visual fatigue. There is often no need to work at the visual resolution limit.

Diffraction

Because light is a wave, it does not focus to a perfect point image. Diffraction from the aperture limits the size of the image spot. For an aberration free system, an Airy Disk pattern is produced:



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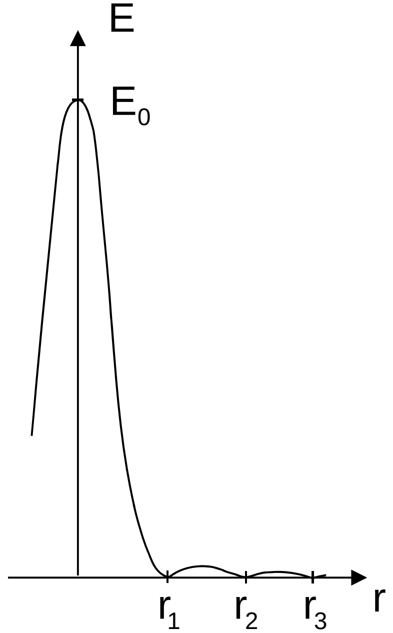
The pattern has a bright central core surrounded by rings.

Airy Disk



$$E = E_0 \left[\frac{2J_1\left(\pi r / \lambda f/\#\right)}{\pi r / \lambda f/\#} \right]^2$$

where r is the radial coordinate, J_1 is a Bessel function, and $f/\#_W$ is the image space f-number.

	Radius r	Peak E	% Energy in Ring
Central Maximum	0	1.0 E_0	83.9
First Zero	1.22 λ f/#	0.0	
First Ring	1.64 λ f/#	0.017 E_0	7.1
Second Zero	2.24 λ f/#	0.0	
Second Ring	2.66 λ f/#	0.0041 E_0	2.8
Third Zero	3.24 λ f/#	0.0	
Third Ring	3.70 λ f/#	0.0016 E_0	1.5
Fourth Zero	4.24 λ f/#	0.0	

Airy Disk Diameter

The diameter of the Airy Disk is

$$D = 2.44\, \lambda\, f/\#$$

In visible light, λ is approximately 0.5 μm, and

$$D \approx f/\# \text{ in microns}$$

This is a very useful approximation for determining the best possible resolution for a given f-number.

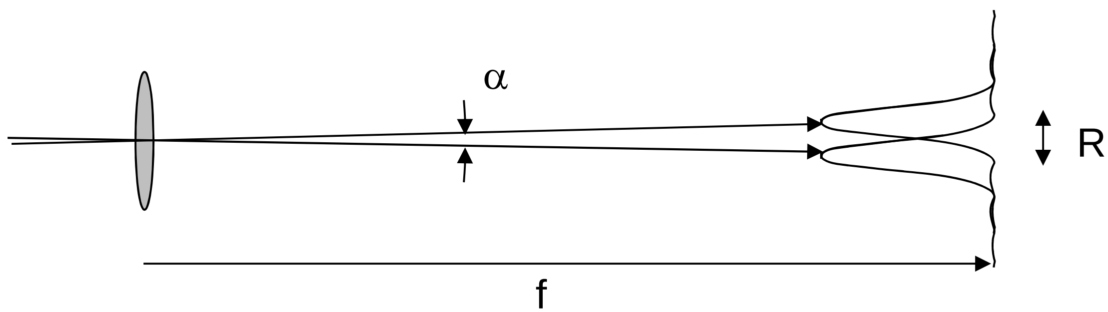
Rayleigh Resolution

The Rayleigh resolution criterion states that the images of two point objects can be resolved if the peak of one image falls on the first zero of the other image. The separation equals the radius of the Airy disk:

$$\text{Resolution} = 1.22\lambda f/\# \qquad\qquad f/\# = \frac{f}{D_{EP}}$$

The angular resolution is found by dividing by the focal length (or image distance):

$$\text{Angular Resolution} = \alpha = 1.22\lambda / D_{EP}$$



$$R = \alpha f = 1.22\lambda f/\# = 1.22\lambda \frac{f}{D_{EP}}$$

$$\alpha = 1.22\lambda / D_{EP}$$

Diffraction-Based Resolution

The angular resolution based on the Rayleigh criterion is

$$\text{Angular Resolution} = \alpha = 1.22\,\lambda / D_{EP}$$

This is the angular separation of two object points or the angular separation of two intermediate image points from the perspective of the objective lens.
Assuming that $\lambda = 0.55\ \mu\text{m}$, and the D_{EP} is in mm:

$$\alpha = \frac{1.22\lambda}{D_{EP}} = \frac{1.22(0.55\,\mu\text{m})}{D_{EP}} \frac{1\,\text{mm}}{1000\,\mu\text{m}} \frac{180^\circ}{\pi\,\text{rad}} \frac{3600\,\text{arc sec}}{1^\circ} = \frac{138\,\text{arc sec}}{D_{EP}}$$

In the telescope image space or eye space, this angle is magnified by the MP:

$$\alpha' = |MP|\,\alpha = \frac{138|MP|}{D_{EP}}\text{arc sec}$$

When this Rayleigh resolution equals the eye resolution, the maximum useful MP is obtained:

$$\alpha' = 1\text{ arc min} = 60\text{ arc sec} \geq \frac{138|MP|}{D_{EP}}\text{arc sec}$$

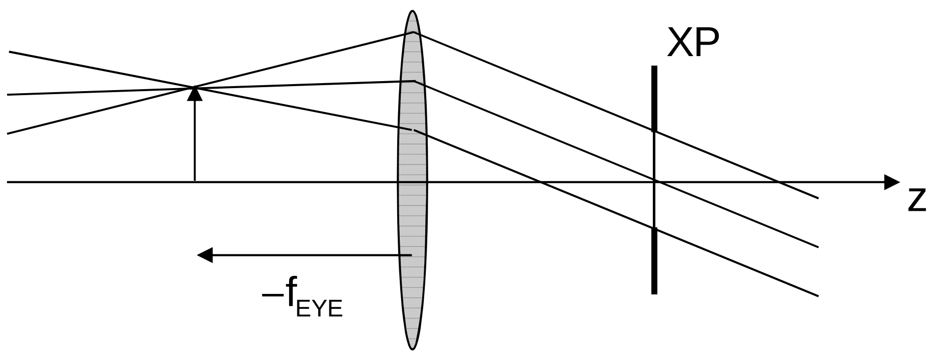
$$|MP| \leq 0.43 D_{EP} \qquad D_{EP}\ \text{in mm}$$

Beyond this MP, no image improvement results as the Airy discs are just being magnified.

Magnifications several times this limit are used to minimize visual effort. It is easier to view when not at the visual resolution limit. This extra MP is termed Empty Magnification.

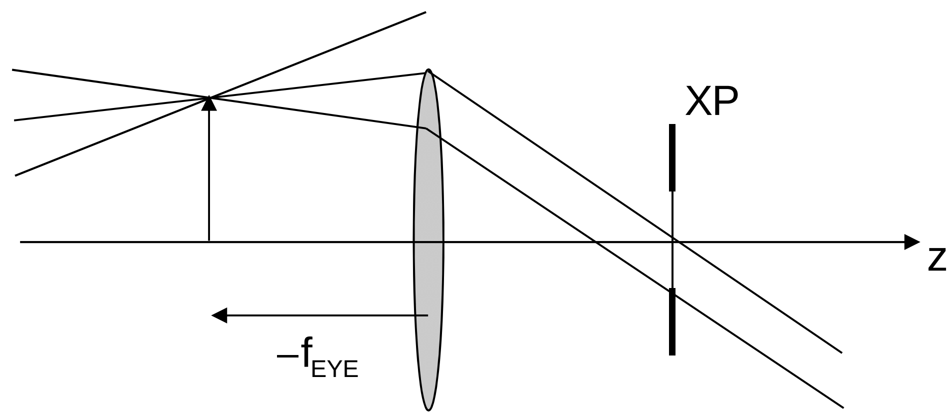
Telescopes – Field of View

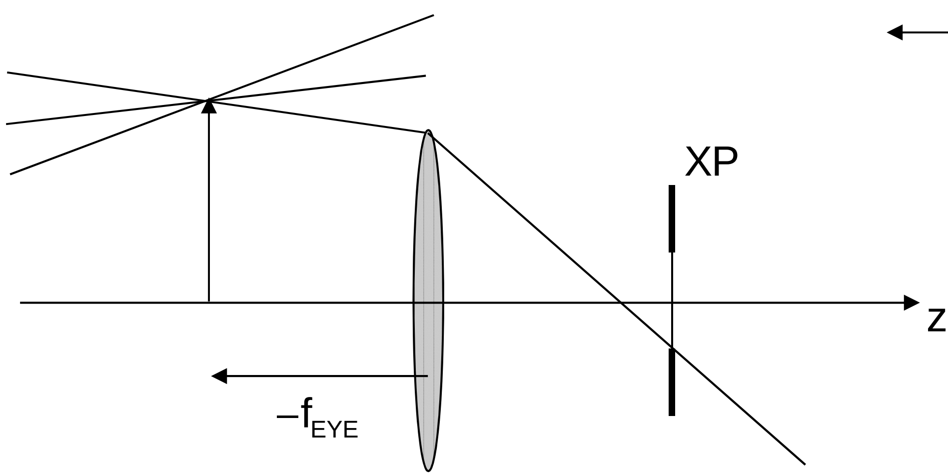
The FOV of the Keplerian telescope is limited by vignetting at the eye lens. As the FOV or intermediate image height increases, the ray bundle is clipped by the eye lens.



Unvignetted

50% Vignetted



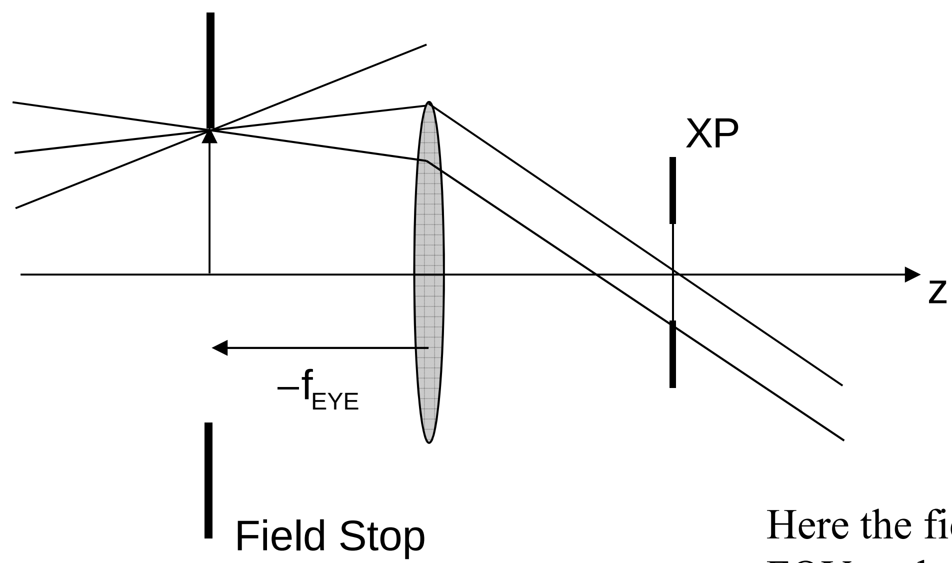


Fully Vignetted (the limit when one ray passes through the XP)

Field Stop

A field stop is a physical aperture placed at an intermediate image plane to restrict or limit the system FOV. This aperture serves to limit the field to a well-corrected or non-vignetted region. This is often a cosmetic consideration. In a simple Keplerian telescope, the angular field of view of a telescope is the angular size of the field stop as seen by the objective lens.

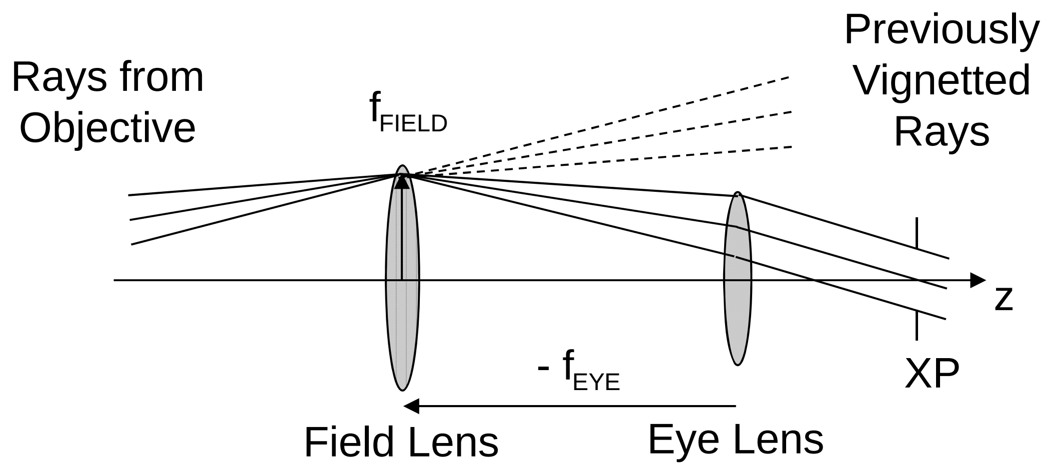
The field stop defines the maximum chief ray angle for the system. A chief ray corresponding to a larger FOV is blocked by the field stop.



Here the field stop limits the FOV to the 50% vignetted FOV.

Field Lenses

The field of view of the instrument can be increased by the addition of a field lens. This lens is placed at the intermediate image plane, and it bends the chief ray and its bundle of rays back towards the axis and into the aperture of the eye lens.



Field Lens – Eye Lens Combination

The combination of the field lens and the eye lens is called an eyepiece. Gaussian reduction easily gives the power of the eyepiece. Assume thin lenses with a separation equal to the focal length of the eye lens:

$$\phi = \phi_{EYE} + \phi_{FIELD} - \phi_{EYE}\phi_{FIELD}t \qquad t = f_{EYE} = 1/\phi_{EYE}$$

$$\phi = \phi_{EYE} \qquad f = f_{EYE}$$

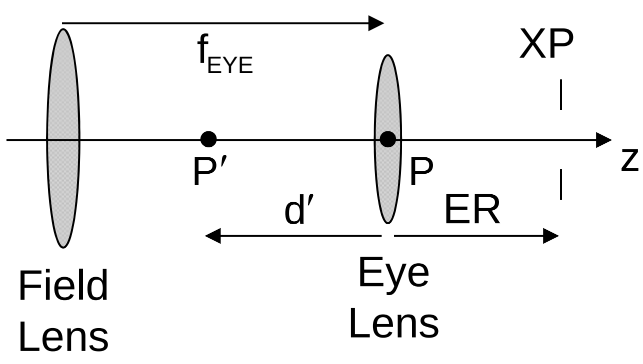
The eyepiece power equals the eye lens power. The MP of the telescope is unchanged.

$$d = \frac{\phi_{EYE}}{\phi}t = \frac{f_{EYE}}{f_{EYE}}f_{EYE} = f_{EYE}$$

The front principal plane of the eyepiece remains at the eye lens. As a result the magnification of the stop is unchanged. The XP has the same size as without the field lens.

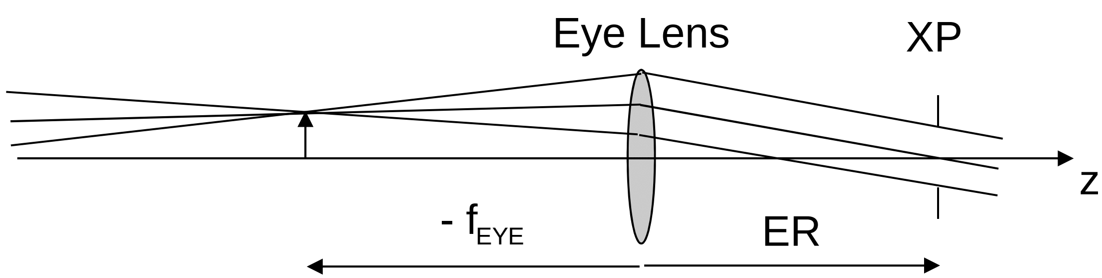
$$d' = -\frac{\phi_{FIELD}}{\phi}t = -\frac{f_{EYE}}{f_{FIELD}}f_{EYE} = -\frac{f^2{}_{EYE}}{f_{FIELD}}$$

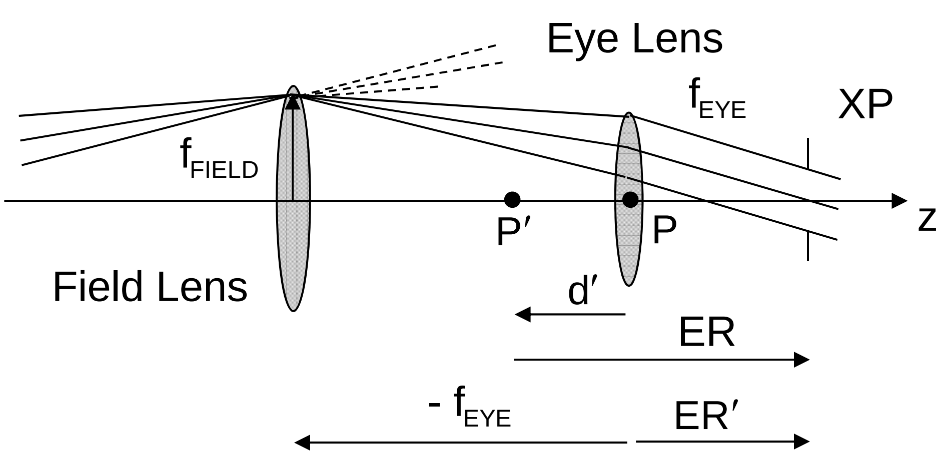
The field lens shifts the rear principal plane to reduce the original eye relief by d'.



Field Lenses – Summary

The field lens does not change the MP of the telescope or the size of the XP. The XP moves closer to the eye lens reducing the eye relief. Maintaining a usable ER limits the strength of the field lens and the FOV increase possible for a given eye lens diameter. Since the field lens is located at an image plane, dirt and imperfections on it become part of the image. In practice, the field lens is often displaced from the image plane to minimize these effects through defocus.





$$MP = -\frac{f_{OBJ}}{f_{EYE}}$$

$$D_{XP} = \frac{D_{EP}}{|MP|}$$

$$d' = -\frac{f^2_{EYE}}{f_{FIELD}}$$

$$ER' = ER + d'$$

Eyepieces

An eyepiece or ocular is the combination of the field lens and the eye lens. A simple eyepiece does not have a field lens. A compound eyepiece has both an eye lens and a field lens.

The properties of eyepieces are applicable to other optical instruments such as microscopes.

The eyepiece can contain a field stop at the intermediate image plane to restrict the system FOV. The aperture of a field lens located at an intermediate image plane serves the function of a field stop.

Reticles and graticles provide alignment and measurement fiducial marks, and they are placed in the intermediate image plane to be superimposed on the image. Since both the reticle and the image are in focus, reticles must be clean and defect free.

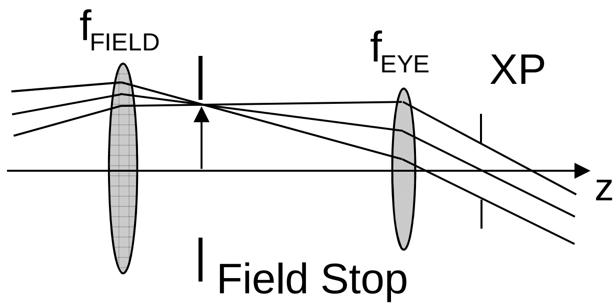
The MP of an eyepiece is defined the same as that of a magnifier:

$$MP_{EYEPIECE} = \frac{250\ mm}{f_{EYEPIECE}}$$

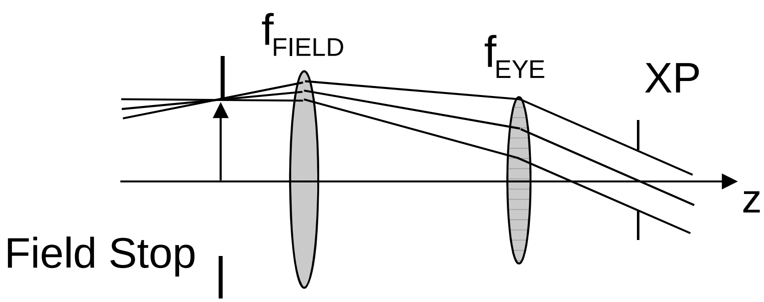
Compound Eyepieces

It is good practice to displace the field lens from the intermediate image plane. The two general classifications of compound eyepieces are the Huygens eyepiece and the Ramsden eyepiece. A great number of specific and historical design variations exist. In both of these configurations, it is also common to place a field stop at the intermediate image plane.

The intermediate image plane for a Huygens eyepiece falls between the two elements.



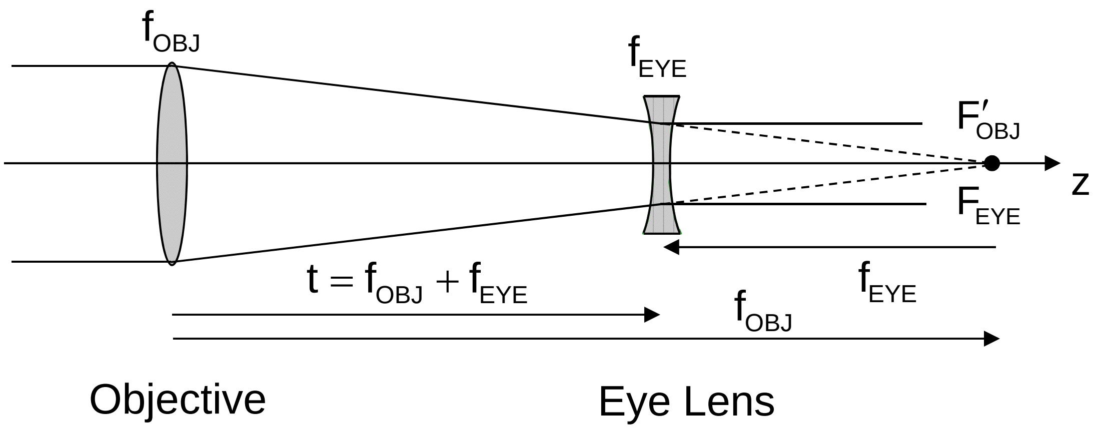
The Ramsden eyepiece places the field lens behind the intermediate image. It is a good choice to use with reticles as the eyepiece does not change the magnification or size of the intermediate image. This eyepiece has about 50% more eye relief than the Huygens eyepiece.



A Kellner eyepiece replaces the singlet eye lens of the Ramsden eyepiece with a doublet for color correction.

Galilean Telescope

The Galilean telescope uses a positive lens and a negative lens to obtain an erect image and a positive MP (MP > 1).



$$m = -\frac{f_{EYE}}{f_{OBJ}} \qquad 0 < m < 1$$

$$ER = z' = (1 - m) f_{EYE} < 0$$

$$MP = -\frac{f_{OBJ}}{f_{EYE}} \qquad MP > 1$$

The XP is internal or virtual and not accessible to the eye. There is poor coupling between the telescope and the eye, and the FOV of the system is small. There is no intermediate image plane, so it cannot be used with reticles.

The Galilean telescope is used for inexpensive systems such as opera glasses.

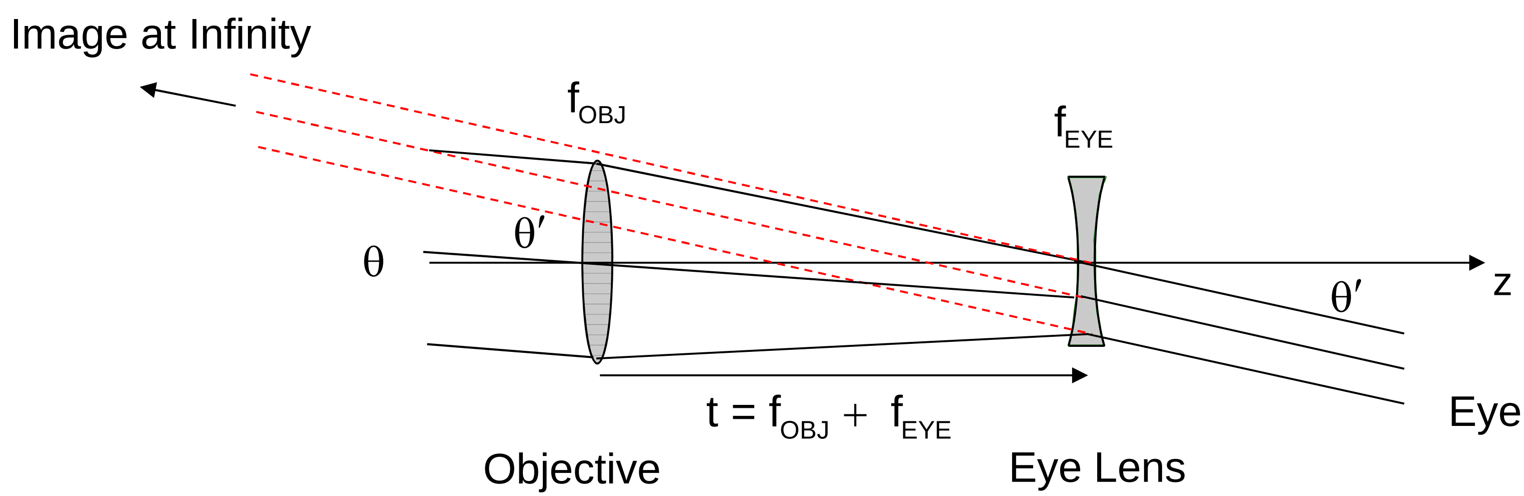
For a Galilean telescope to be constructed, the negative lens must be stronger than the positive lens.

$$|f_{EYE}| < f_{OBJ}$$

Galilean Telescope

Rays from an off-axis object enter the telescope at θ and emerge at θ′.

$$MP = \frac{\theta'}{\theta} = -\frac{f_{OBJ}}{f_{EYE}}$$



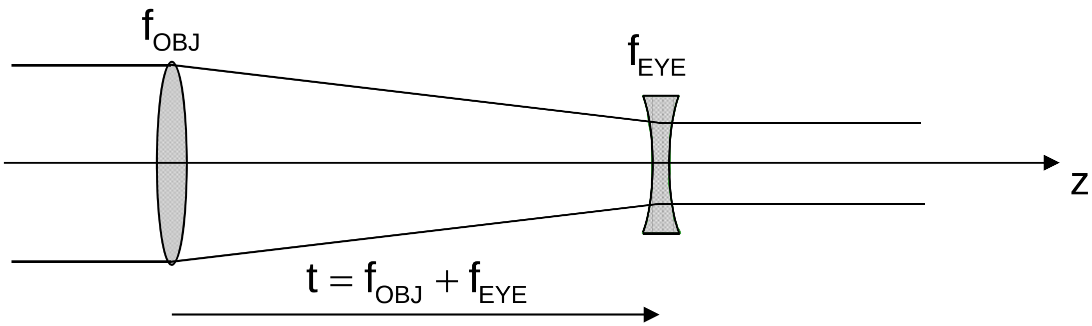
Collimated input light comes out of the telescope at a larger angle. The image appears bigger to the eye.

The image in this configuration is erect (right-side up).

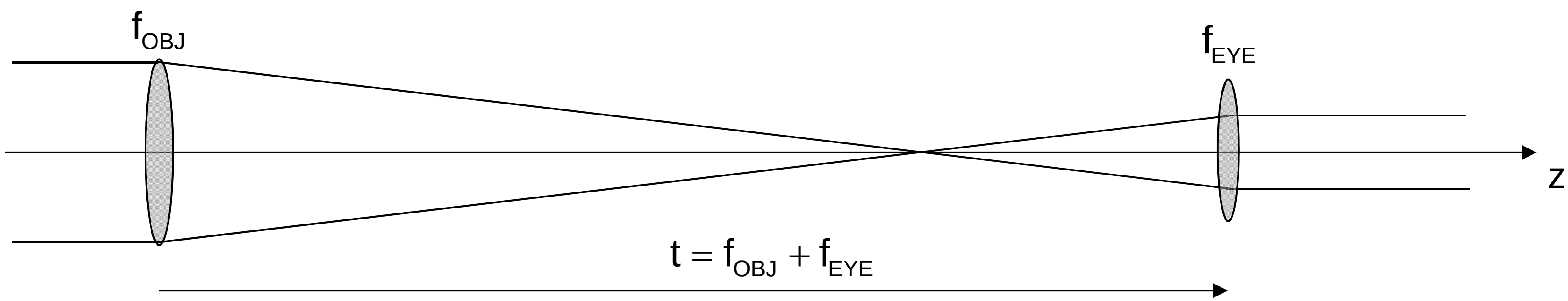
Comparison of Galilean and Keplerian Telescopes

For a given $|MP|$, the Galilean telescope is shorter than the corresponding Keplerian telescope. Its FOV is also smaller.

Galilean Telescope:



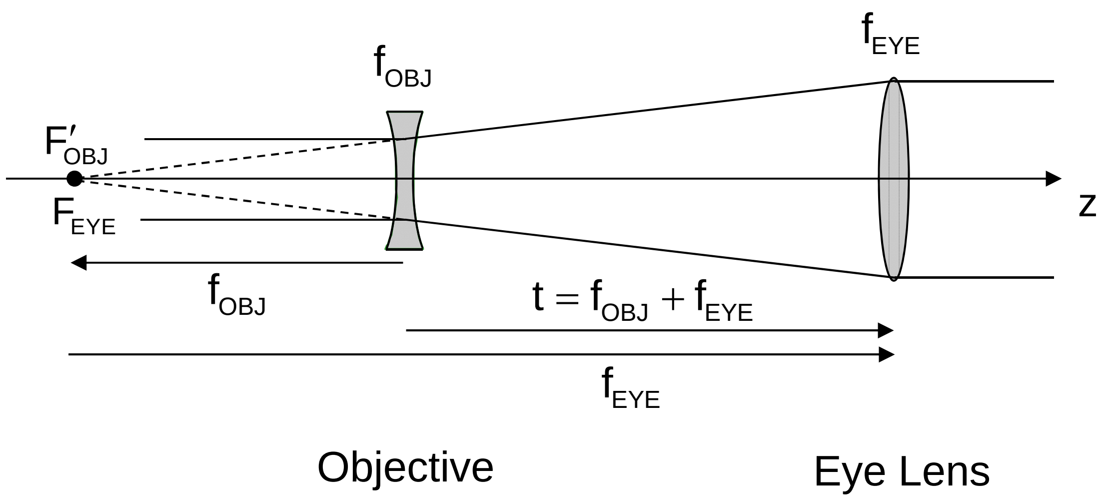
Equivalent Keplerian Telescope:



The image in the Keplerian telescope is upside-down and must be corrected with image erecting prisms or a relay lens in order to obtain an erect image.

Reversed Galilean Telescope

A reversed Galilean telescope provides a minified erect image (0 < MP < 1). This configuration is used in door peepholes and many camera viewfinders. In these systems, the eye is often the system stop.



$$m > 1$$

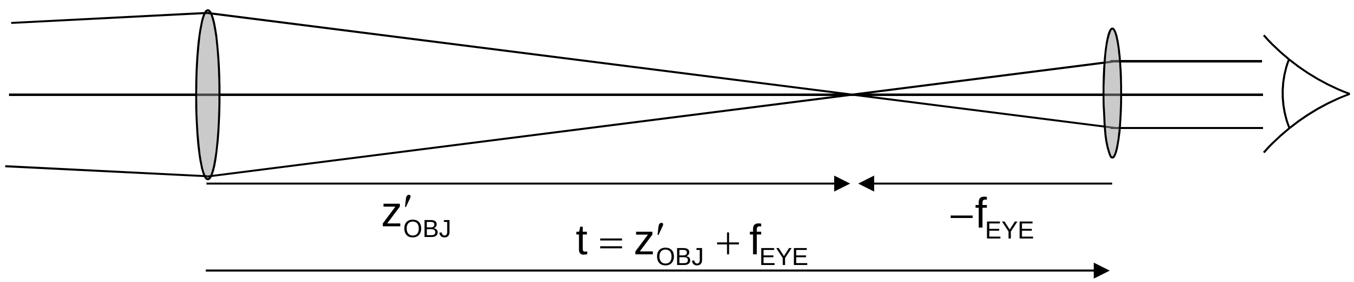
$$0 < MP < 1$$

Erect Image

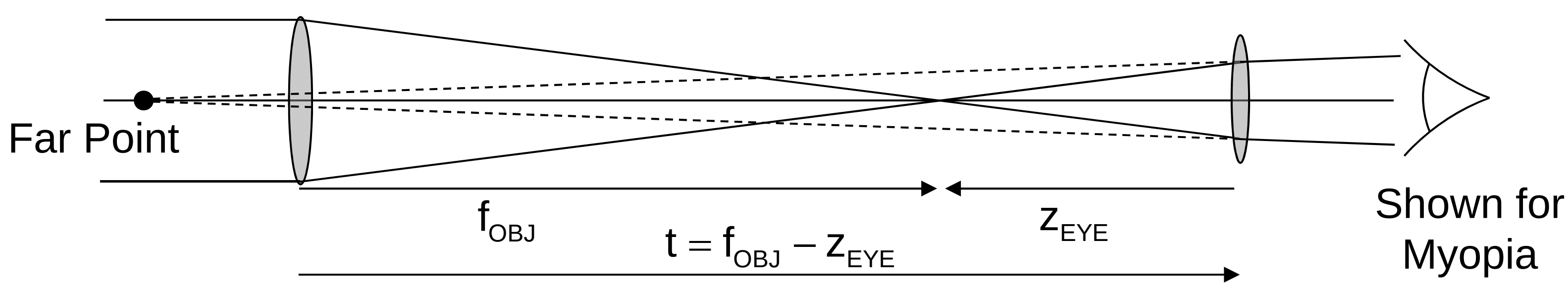
Focusing a Telescope

While telescopes are defined to be afocal, in practice they often deviate from this condition.

When the object is not at infinity, the image must still be projected to infinity for viewing with a relaxed eye. The telescope length is adjusted to place the intermediate image at the front focal point of the eyepiece (or magnifier).



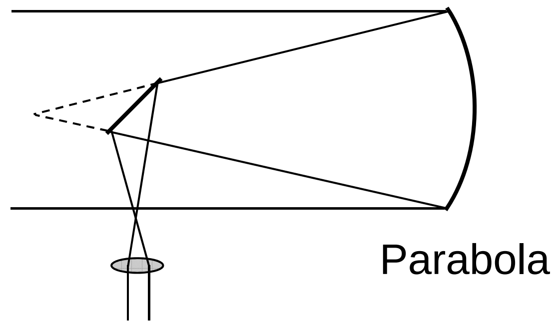
With refractive error, the far point of the relaxed eye is no longer at infinity. The far point is the object distance that is in focus without accommodation. The distance from the intermediate image to the eyepiece can be adjusted to place the image presented to the eye at its far point.



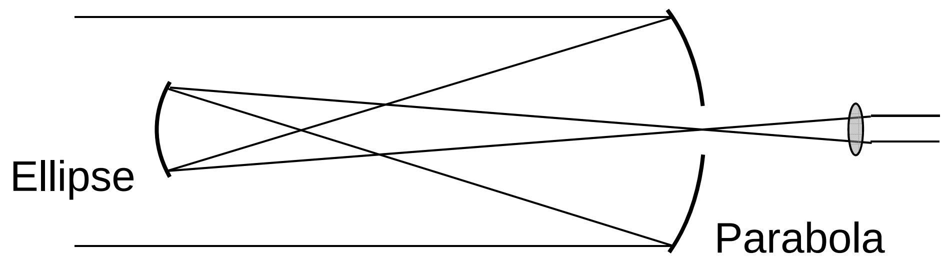
Of course, both corrections can be combined.

Mirror Based Telescopes

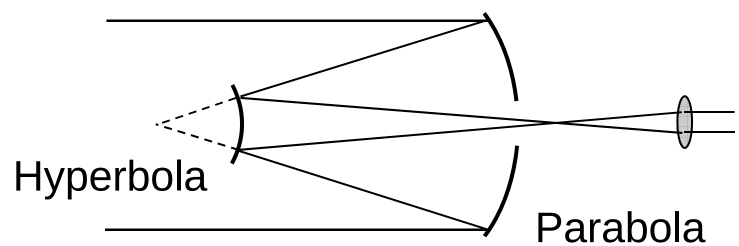
Newtonian telescope: A positive mirror with a fold flat. It is directly analogous to a Keplerian telescope.



Gregorian telescope: The positive primary mirror is followed by a second positive mirror to relay the intermediate image. As with the analogous relayed Keplerian telescope, an erect image is produced.



Cassegrain telescope: The combination of the positive primary with a negative secondary is the mirror equivalent of a telephoto objective.



The choice of specific conics used for these telescopes is based on the aberrations and imaging properties of the conic surfaces.

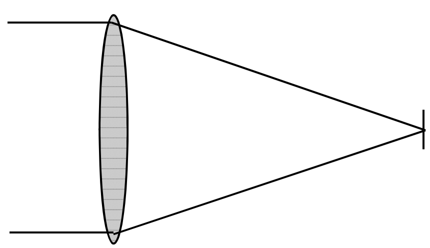
Telescopes and Imaging Detectors

The term telescope has come to mean any system used to view distant objects. In this formal discussion, a telescope specifically refers to an afocal system used with the eye. Large astronomical telescopes are actually objectives or cameras where an image array detector is placed at the system focal point.

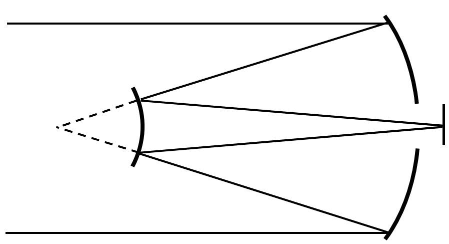
Two examples of imaging detectors are CDD arrays and photographic film. With these detectors, an eye lens is not used and the real image produced by the telescope falls on the detector array.

The detector is placed at an image plane.

Refracting:



Reflecting:



The image can also be relayed to other parts of the instrument.

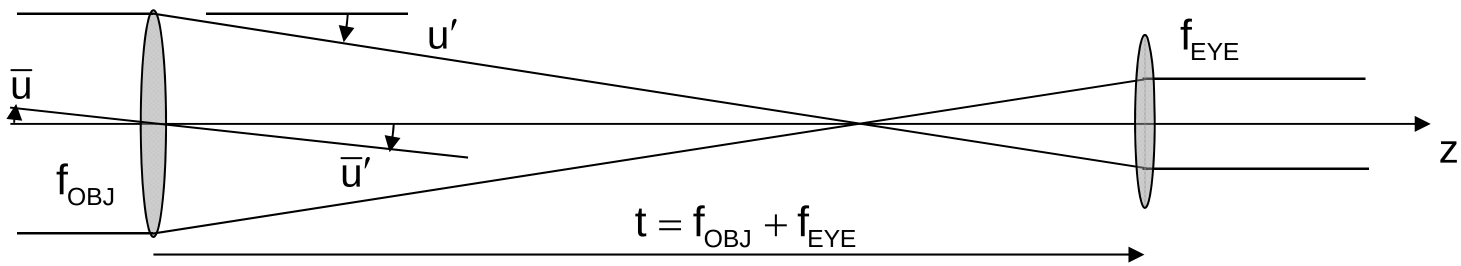
Ignoring diffraction and aberrations, the resolution is determined by the pixel size or the resolution of the film. The pixel size equates to an angular size in object space.

Vignetting Example

Keplerian Telescope: 5X $m = -1/5$

$f_{OBJ} = 250$ $D_{OBJ} = 30$ $a_{OBJ} = 15$

$t = 300$ $f_{EYE} = 50$ $D_{EYE} = 20$ $a_{EYE} = 10$



What are the unvignetted, half-vignetted and fully-vignetted FOVs of this system?
Vignetting will occur at the eye lens, since the stop is at the objective.

Marginal Ray: $u' = -y_{OBJ}\phi_{OBJ} = \dfrac{-15}{250} = -.06$ $y_{OBJ} = 15$

$$y_{EYE} = y_{OBJ} + u't = 15 + (-.06)(300) = -3$$

$$y_{EYE} = m y_{OBJ}$$

Chief Ray: $\bar{u}' = \bar{u}$ $\bar{y}_{OBJ} = 0$

$$\bar{y}_{EYE} = \bar{y}_{OBJ} + \bar{u}'t = 300\bar{u}$$

The chief ray height at the eye lens depends on the FOV.
The marginal ray is independent of FOV.

Vignetting Example – Continued

At the eye lens –

Unvignetted:

$$a_{EYE} \geq |\bar{y}_{EYE}| + |y_{EYE}|$$
$$10 \geq |300\bar{u}| + |-3|$$
$$|\bar{u}| \leq \frac{7}{300} = .02333$$
$$|\bar{U}| \leq 1.34°$$

$$a_{EYE} = 10$$
$$\bar{U} = \tan^{-1} \bar{u}$$

Half Vignetted:

$$a_{EYE} = |\bar{y}_{EYE}| \quad \text{and} \quad a_{EYE} \geq |y_{EYE}|$$
$$10 = |300\bar{u}| \qquad 10 \geq |-3|$$
$$|\bar{u}| = \frac{10}{300} = .03333$$
$$|\bar{U}| = 1.91°$$

Fully Vignetted:

$$a_{EYE} \leq |\bar{y}_{EYE}| - |y_{EYE}| \quad \text{and} \quad a_{EYE} \geq |y_{EYE}|$$
$$10 \leq |300\bar{u}| - |-3| \qquad 10 \geq |-3|$$
$$|\bar{u}| \geq \frac{13}{300} = .04333$$
$$|\bar{U}| \geq 2.48°$$

Vignetting Example – Add a Field Lens

What happens to the FOV if a field lens of diameter 20 is added to the 5X Keplerian telescope?

$$t_{FIELD} = 250$$

At the field lens:

$$y_{FIELD} = y_{OBJ} + u' t_{FIELD}$$

$$y_{FIELD} = 15 + (-.06)(250) = 0$$

It's an image plane!

$$\bar{y}_{FIELD} = \bar{u}' t_{FIELD} = 250\bar{u}$$

Since $y_{FIELD} = 0$, all of the vignetting conditions collapse to one.

The field lens is in an image plane, and it serves as a field stop.

Unvignetted Field:

$$a_{FIELD} \geq |\bar{y}_{FIELD}| + |y_{FIELD}|$$

$$a_{FIELD} = 10$$

$$10 \geq |250\bar{u}|$$

$$\bar{u} \leq .04$$

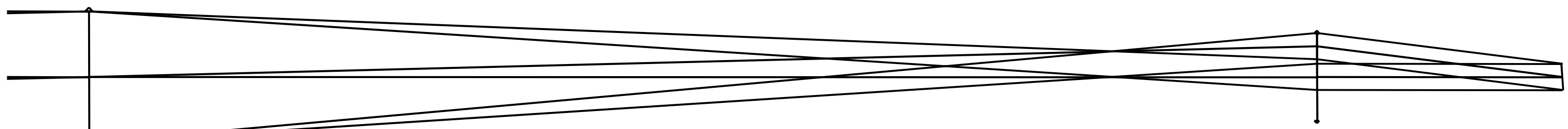
$$|\bar{U}| \leq 2.29°$$

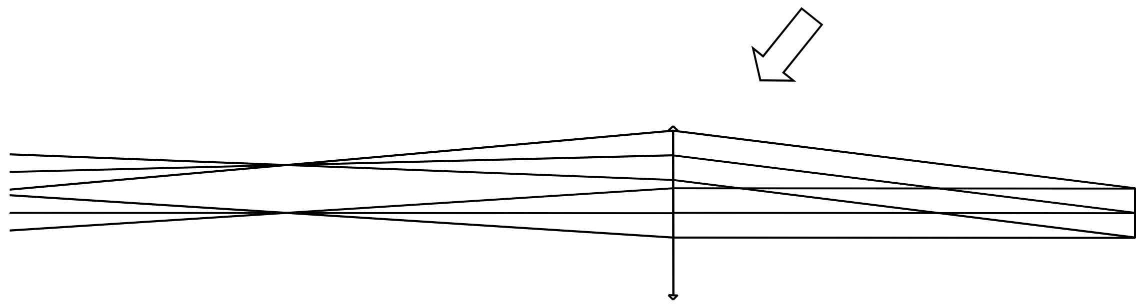
An almost 2X improvement over the base Keplerian telescope results.

Note that the power of the field lens was not required to do this calculation. This power can be used to prevent vignetting at the eye lens and/or to position the exit pupil (eye relief).

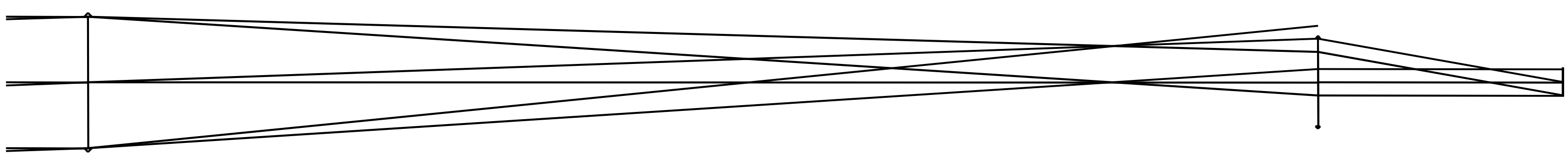
Vignetting Example – 5X Keplerian

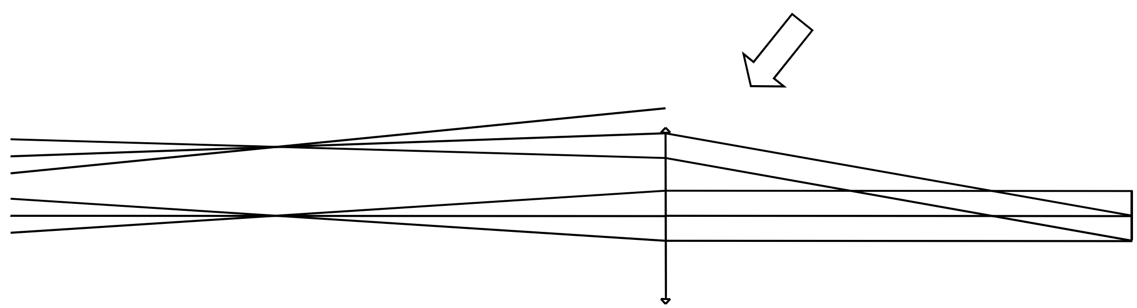
Unvignetted: $\bar{U} = 1.34°$





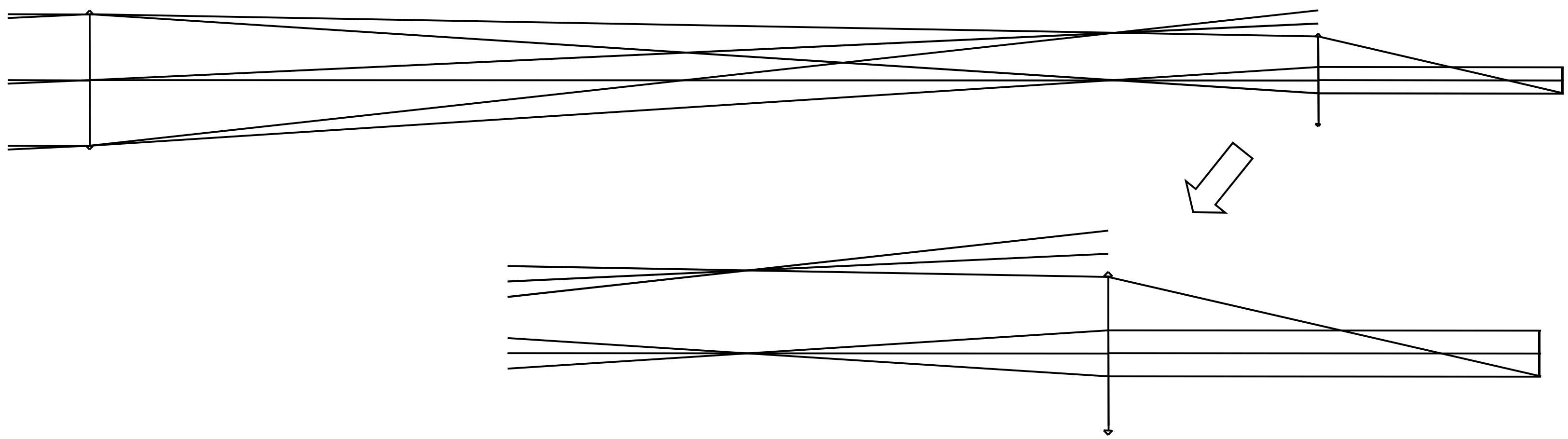
Half Vignetted: $\bar{U} = 1.91°$



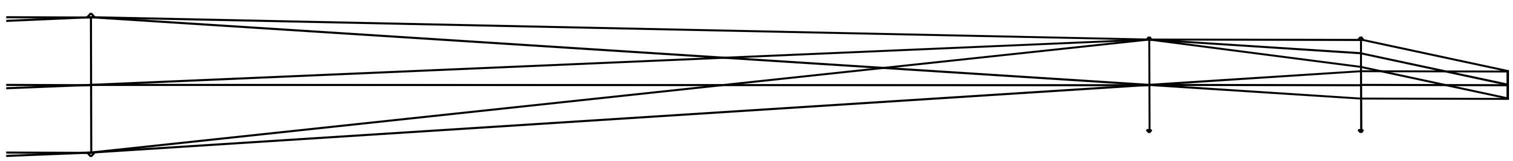


Vignetting Example – 5X Keplerian

Fully Vignetted: $\bar{U} = 2.48°$

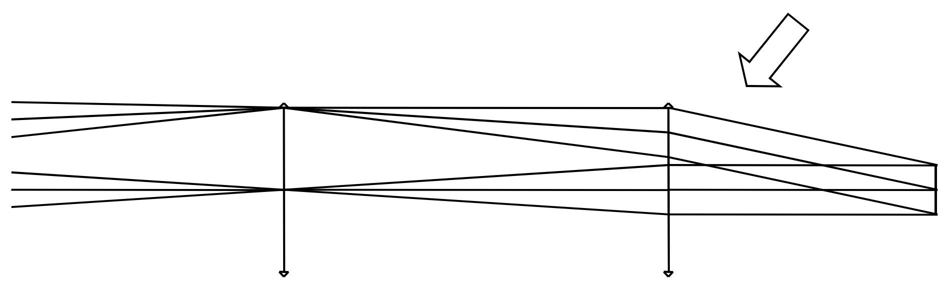


With Field Lens: $\bar{U} = 2.29°$



$f_{FIELD} = 100$

$d' = -25$



The ER is reduced from 60 to 35.

Vignetting and FOV with Galilean Telescopes

In most Galilean telescopes and binoculars, the eye serves as the system stop. The system FOV is limited by vignetting at the objective lens. Note that these systems are not always well defined as there is no prescribed location for the eye relative to the eye lens (no external XP).

Consider this example of a 3X system:

$f_{OBJ} = 150$ mm

$f_{EYE} = -50$ mm

$t = 100$ mm

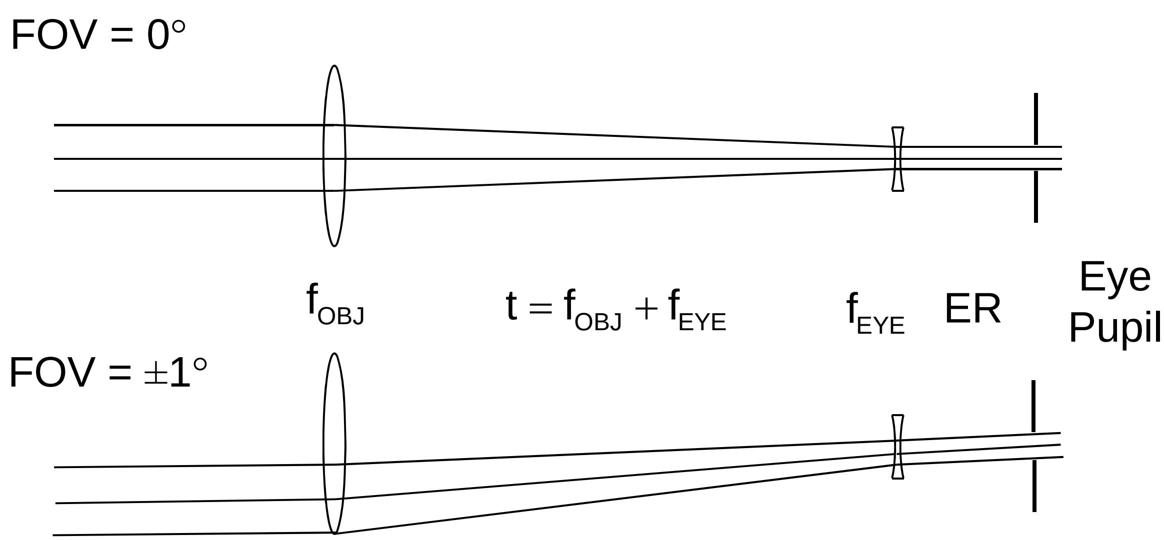
$ER = 25$ mm

Pupil Dia = 4 mm

$D_{OBJ} = 30$ mm

$D_{EYE} = 10$ mm

$MP = 3$



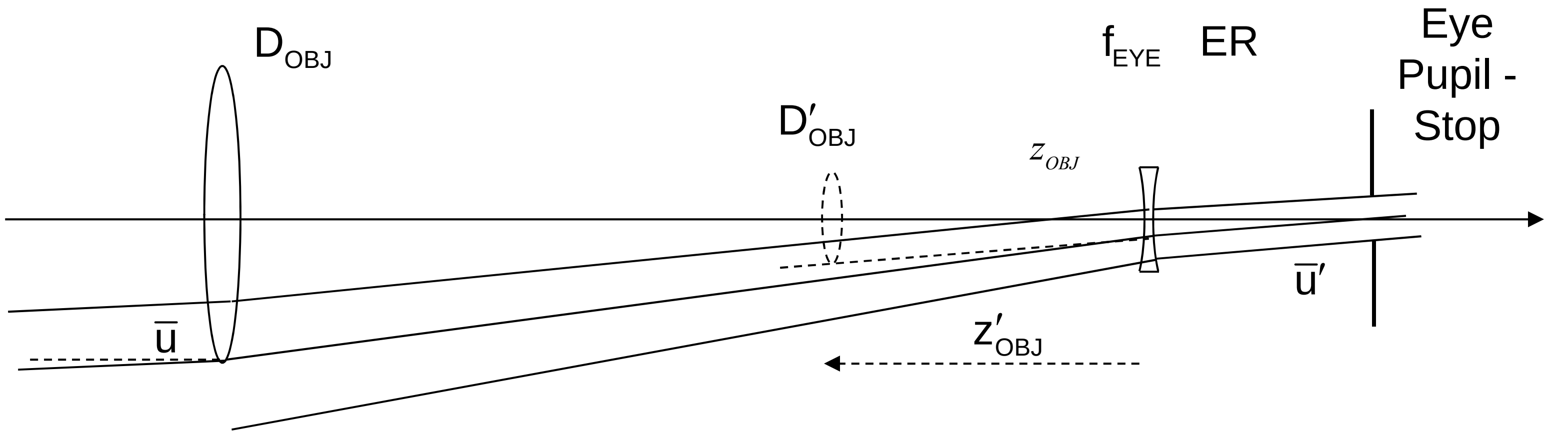
The half vignetted object space FOV is about ±1.6°

Vignetting occurs at the objective lens.

Half-Vignetted FOV of Galilean Telescopes

When looking through a Galilean telescope, it appears that you are looking through a hole well out in front of the telescope. The hole is the image of the objective lens through the negative eye lens. Vignetting at the objective lens usually limits the FOV of a Galilean telescope.

The apparent size of the image of the objective lens (the hole) as viewed from the eye determines the half-vignetted FOV and the chief ray at the eye.



Using the 3X telescope design:

f_{EYE} = -50 mm

t = $-z_{OBJ}$ = 100 mm

D_{OBJ} = 30 mm

$$\frac{1}{z'_{OBJ}} = \frac{1}{z_{OBJ}} + \frac{1}{f_{EYE}} = \frac{1}{-t} + \frac{1}{f_{EYE}}$$

$$z'_{OBJ} = -33.33\ mm$$

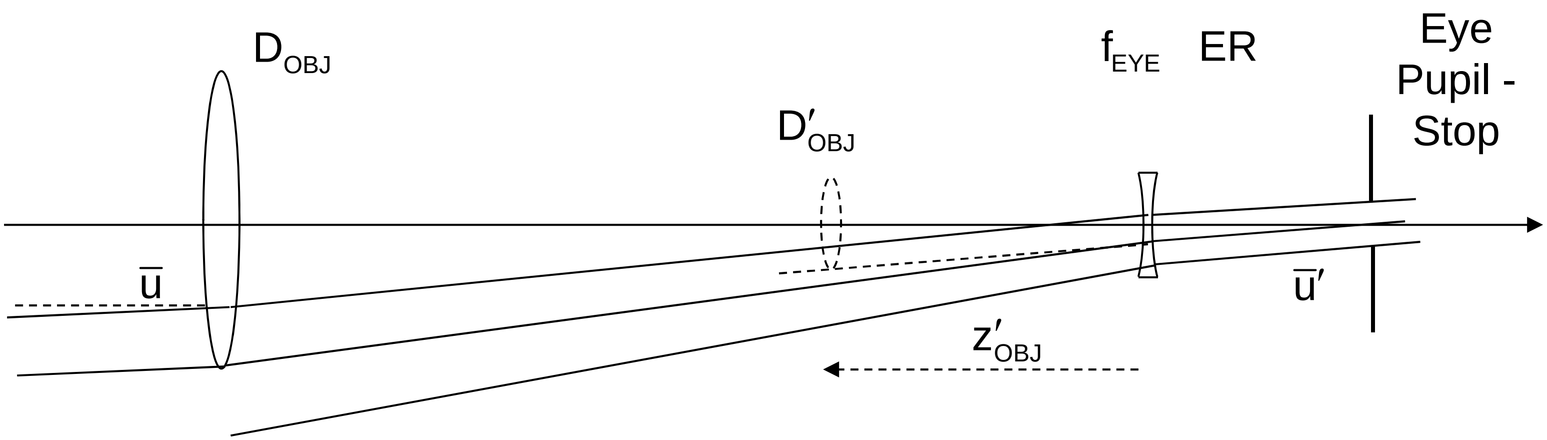
$$m = z'_{OBJ} / z_{OBJ} = 0.3333^*$$

$$D'_{OBJ} = 10\ mm^*$$

*Obvious result: the objective is in object space of the telescope and the objective image is in image space. The diameters must be related by the telescope magnification.

Half-Vignetted FOV of Galilean Telescopes

The apparent size of the image of the objective lens (the hole) as viewed from the eye determines the half-vignetted FOV and the chief ray at the eye. The chief ray in object space also goes through the edge of the objective lens.



$$z'_{OBJ} = -33.33\ mm$$

$$D'_{OBJ} = 10\ mm$$

$$ER = 25mm$$

$$\bar{u}' = \tan(\theta') = \frac{D'_{OBJ}/2}{ER + |z'_{OBJ}|} = \frac{5\ mm}{58.33\ mm}$$

$$\theta' = \pm 4.9^\circ \qquad MP = \frac{\theta'}{\theta}$$

$$\theta = \frac{\theta'}{MP} = \pm 1.6^\circ$$

This is the half-vignetted FOV of the telescope. Note that the FOV depends on the value of the Eye Relief or where the telescope is placed relative to the eye.

Autocollimator

An autocollimator is a widely-used metrology tool for alignment and angle measurement. It is the combination of a collimator and a viewing telescope. The same objective lens is used for both. A point source is placed at the focal point of the objective, and the resulting collimated beam is reflected back into the telescope by the target mirror. The displacement of the returned beam on a measurement reticle measures the mirror tilt.

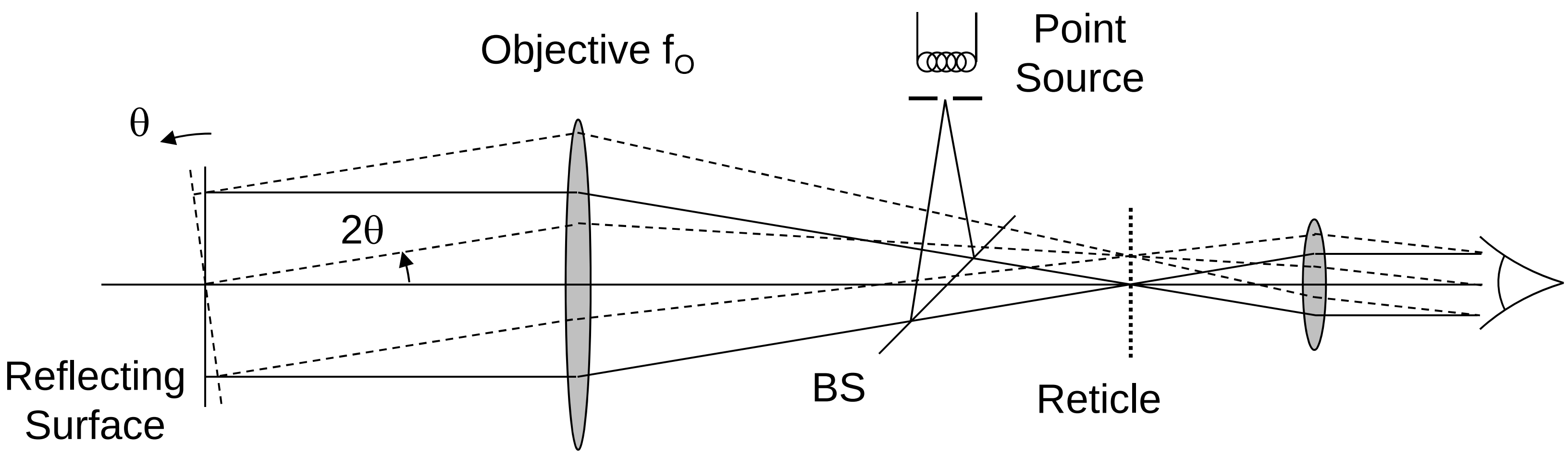


Image displacement in the reticle plane:

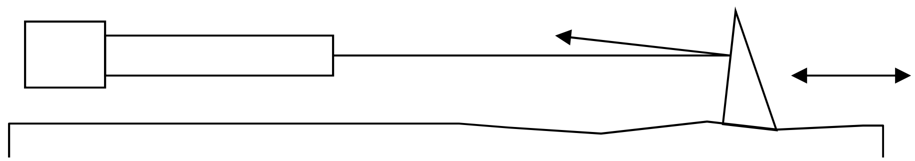
$$D = 2\theta f_O$$

This displacement is independent of the distance to the test surface.

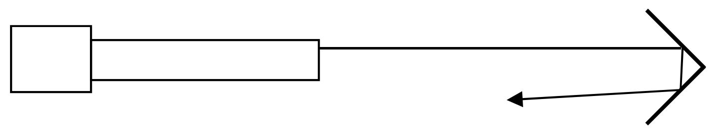
Autocollimator Applications

The autocollimator is used to measure tilt angles and to align parallel surfaces. Other applications include:

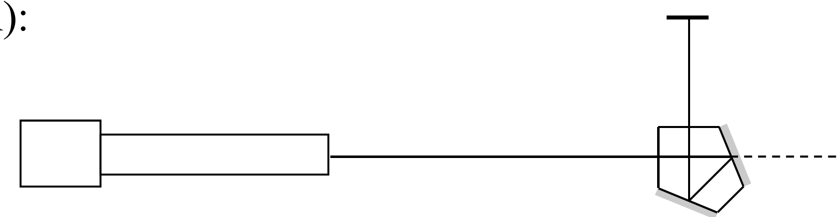
Surface flatness:



Roof test (two returned spots):



Right angles (with a pentaprism):



Resolutions of 0.1 arc sec are quoted for commercial autocollimators.

References: Metrology with Autocollimators, Hume
Geometrical and Instrumental Optics, Ch 4, D. Goodman (D. Malacara, Ed.)