Section 12

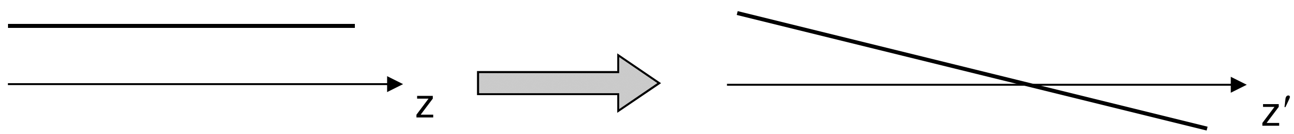
Afocal Systems

Gaussian Optics Theorems

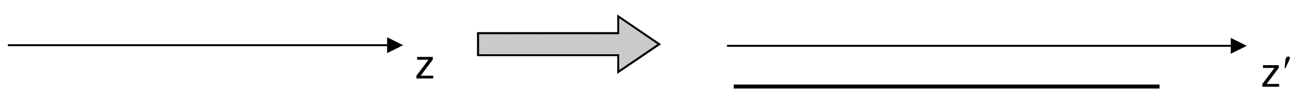
In the initial discussion of Gaussian optics, one of the theorems defined the two different types of optical systems that were possible for a rotationally symmetric system:

- Lines parallel to the axis in one space map to conjugate lines in the other space that either intersect the axis at a common point (focal system), or are also parallel to the axis (afocal system).

Focal System:



Afocal System:



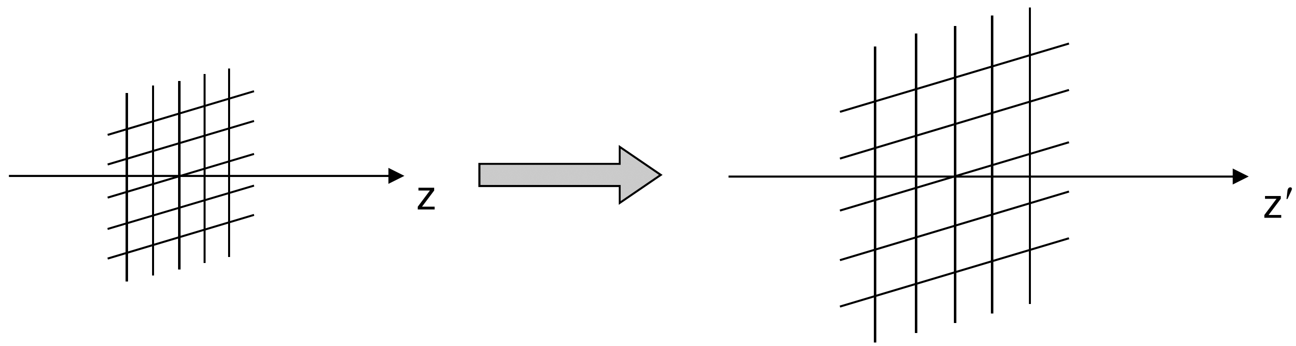
In a Gaussian or collinear mapping, lines must map to lines and are conjugate elements.

Gaussian Optics Theorems - Continued

As a reminder, the other two theorems of Gaussian optics apply to both focal and afocal systems:

- Planes perpendicular to the axis in one space are mapped to planes perpendicular to the axis in the other space.

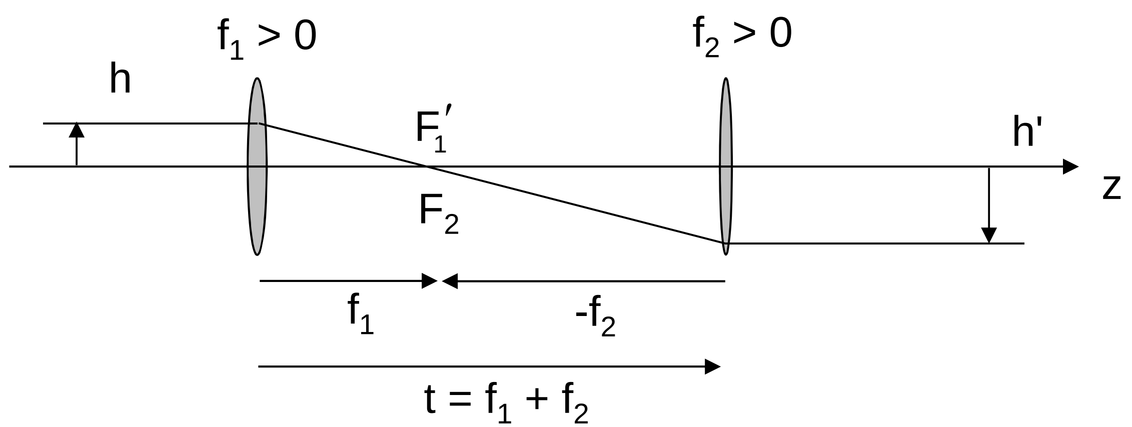
- The transverse magnification is constant in conjugate planes perpendicular to the axis.



Thin Lens Afocal Systems

Combinations of lenses can produce afocal systems. A ray parallel to the axis produces an image ray that is also parallel to the axis. Common afocal systems are telescopes, binoculars and beam expanders.

Example: Two thin lenses separated by the sum of their focal lengths $t = f_1 + f_2$



The rear focal point of the first lens F_1' is coincident with the front focal point of the second lens F_2.

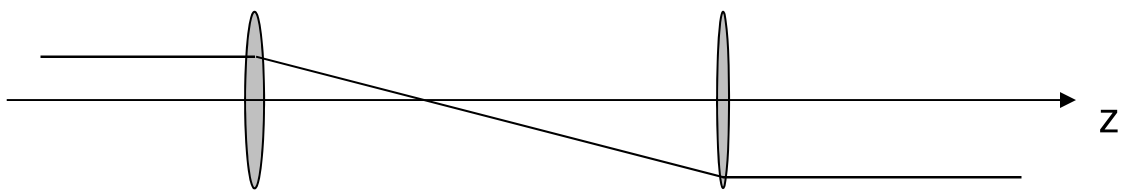
Since the object and image rays are both parallel to the axis, the magnification of the system is constant:

$$\frac{h}{-f_1} = \frac{h'}{f_2} \qquad m = \frac{h'}{h} = \frac{-f_2}{f_1}$$

This configuration forms the basis for refracting telescopes (Keplerian).
The system magnification is negative.

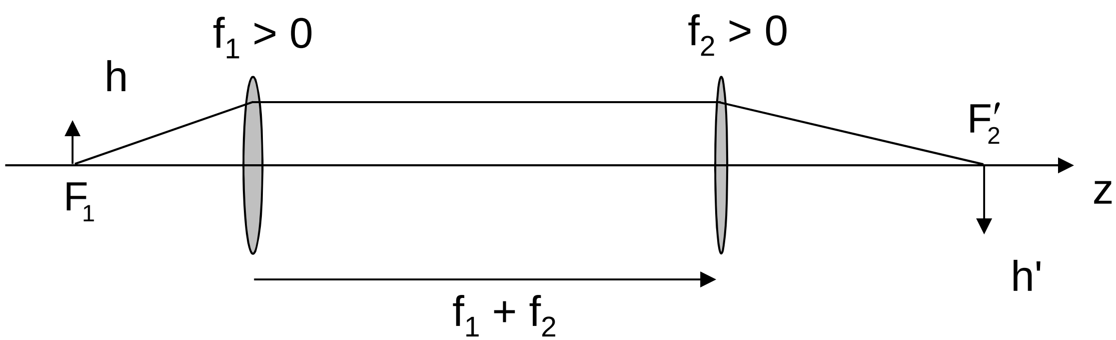
Imaging With Afocal Systems

Since the output ray of an afocal system is parallel to the axis, a possible description places the focal points at infinity. The focal length of an afocal system is then infinite. However, it is much more accurate to say that an afocal system does not have a focal length or focal points.



While it may seem impossible at first glance, afocal systems can be used to form images.

Consider an object at the front focal point of the first lens. It is imaged to the rear focal point of the second lens with the magnification of the afocal system.



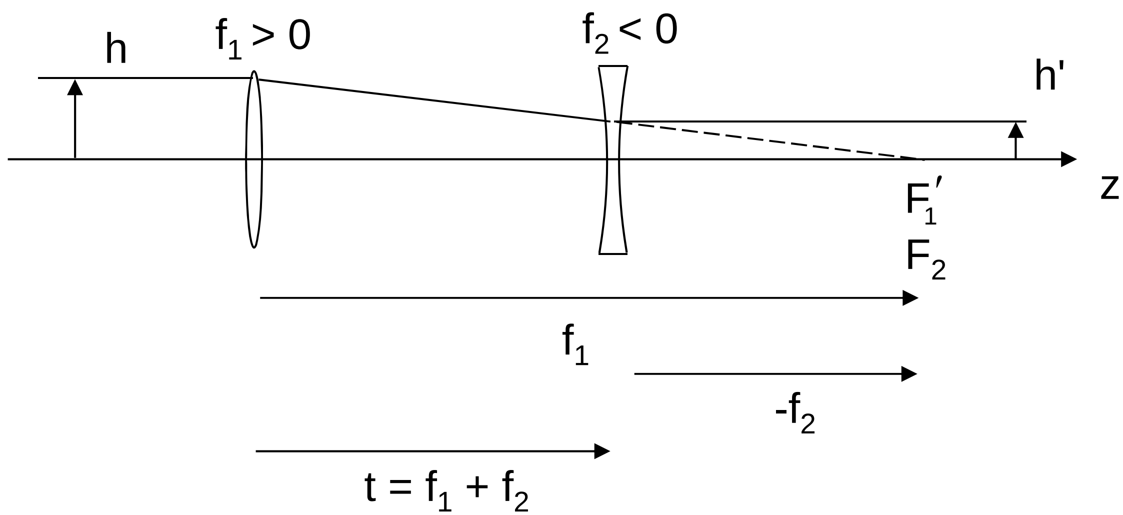
$$m = \frac{h'}{h} = -\frac{f_2}{f_1}$$

The imaging equations for focal systems do not apply to afocal systems as there are no system focal points. However, afocal systems can form images, and this will be discussed later.

Thin Lens Afocal Systems - Galilean

A positive and a negative lens can also be used for an afocal system (Galilean telescope).

The same relationships and requirements hold.



$$t = f_1 + f_2$$

$$\frac{h}{-f_1} = \frac{h'}{f_2}$$

$$m = \frac{h'}{h} = \frac{-f_2}{f_1}$$

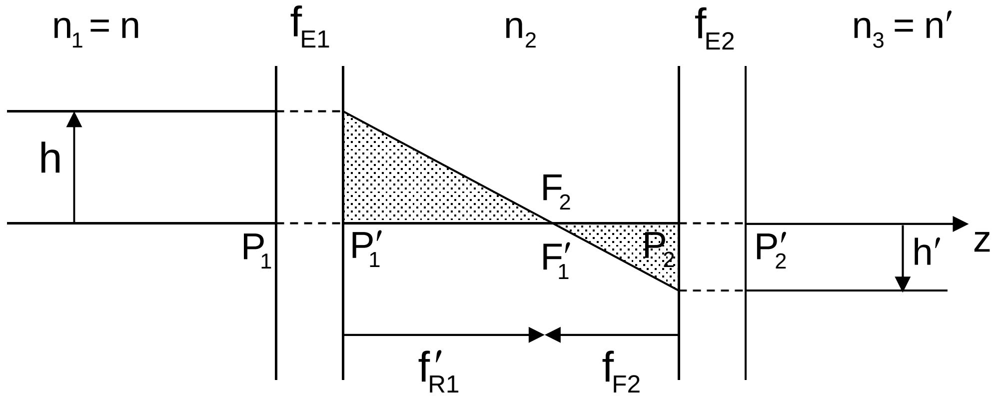
A positive system magnification is obtained in this configuration.

The system magnification is constant.

The lenses are still separated by the sum of the element focal lengths.

Generalized Afocal Systems

An afocal system is formed by the combination of two focal systems. The rear focal point of the first system is coincident with the front focal point of the second system. Rays parallel to the axis in object space are conjugate to rays parallel to the axis in image space.



Transverse Magnification
(Use similar triangles)

$$\frac{h}{-f'_{R1}} = \frac{h'}{-f_{F2}}$$

$$f'_{R1} = n_2 f_{E1}$$
$$f_{F2} = -n_2 f_{E2}$$

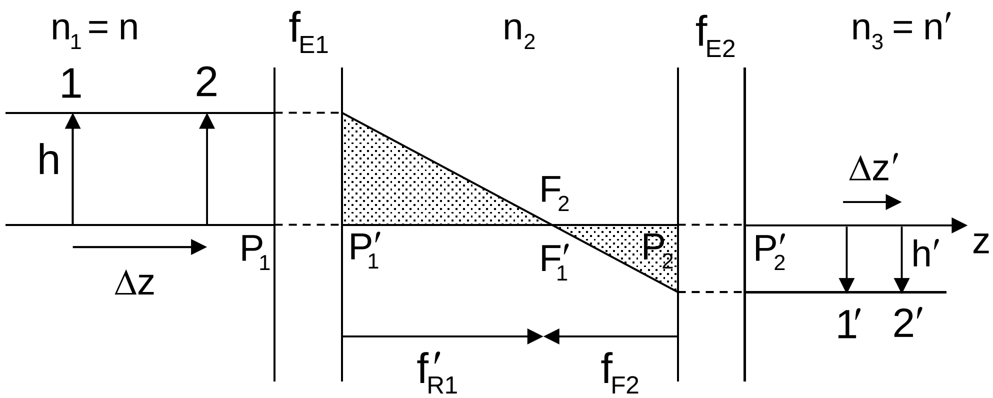
$$m \equiv \frac{h'}{h} = \frac{f_{F2}}{f'_{R1}} = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1} = \text{constant}$$

The transverse or lateral magnification of an afocal system is constant.

$$m = -\frac{f_2}{f_1}$$

Afocal Systems – Longitudinal Magnification

Consider two separated objects producing two separated images:



Longitudinal Magnification (Derivation follows)

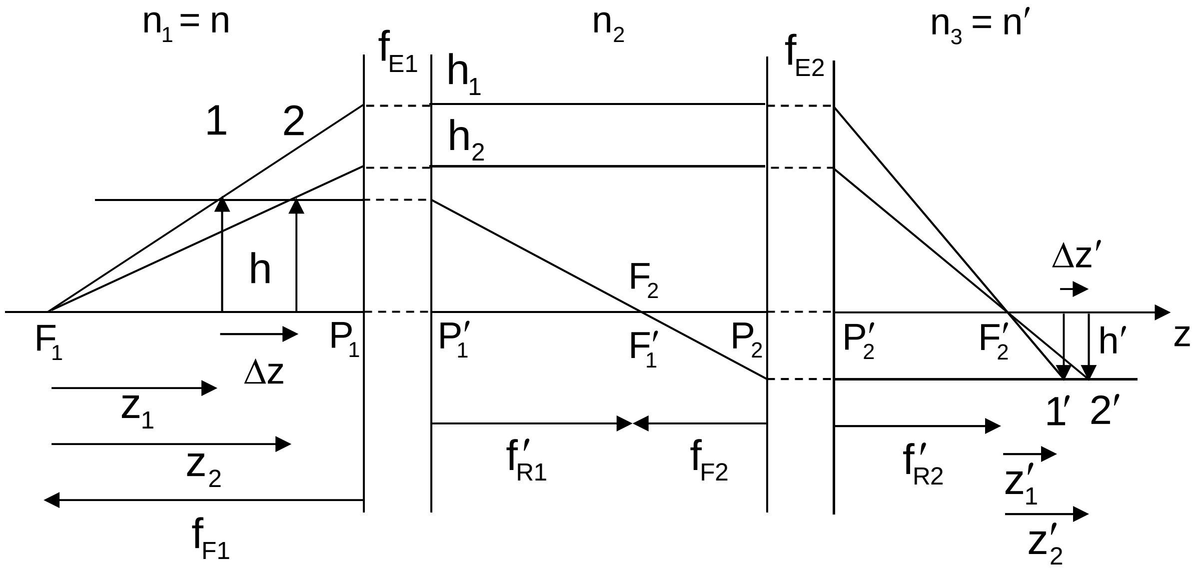
$$\bar{m} = \frac{\Delta z'}{\Delta z} = \left(\frac{n'}{n}\right) m^2 = \text{constant}$$

$$m \equiv \frac{h'}{h} = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1} = \text{constant}$$

The transverse and longitudinal magnifications are constant. Equispaced planes map into equispaced planes. The relative axial spacing changes by the longitudinal magnification.

Because the magnification is constant, the cardinal points are not defined for an afocal system, and the Gaussian and Newtonian equations cannot be used to determine conjugate planes.

Afocal Systems – Longitudinal Magnification – Derivation



$$z_1 = -\frac{h}{h_1} f_{F1}$$

$$z_1' = -\frac{h'}{h_1} f_{R2}'$$

$$z_2 = -\frac{h}{h_2} f_{F1}$$

$$z_2' = -\frac{h'}{h_2} f_{R2}'$$

$$\Delta z = z_2 - z_1 = h\, f_{F1} \left(\frac{1}{h_1} - \frac{1}{h_2} \right)$$

$$\Delta z' = z_2' - z_1' = h'\, f_{R2}' \left(\frac{1}{h_1} - \frac{1}{h_2} \right)$$

$$f_{F1} = -n f_{E1} \qquad f_{R2}' = n' f_{E2}$$

Afocal Systems – Longitudinal Magnification – Derivation Page 2

$$\Delta z = z_2 - z_1 = -hnf_{E1}\left(\frac{1}{h_1} - \frac{1}{h_2}\right) \qquad \Delta z' = z_2' - z_1' = n'h' f_{E2}\left(\frac{1}{h_1} - \frac{1}{h_2}\right)$$

$$\frac{\Delta z'}{\Delta z} = -\frac{n'}{n}\frac{h'}{h}\frac{f_{E2}}{f_{E1}} = \left(\frac{n'}{n}\right)m^2 = \text{constant}$$

$$m \equiv \frac{h'}{h} = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1} = \text{constant}$$

$$\bar{m} = \frac{\Delta z'}{\Delta z} = \left(\frac{n'}{n}\right)m^2$$

$$\frac{\Delta z' / n'}{\Delta z / n} = m^2$$

The transverse or lateral magnification of an afocal system is constant.

The longitudinal or axial magnification of an afocal system is proportional to the square of the transverse magnification, and it is also constant.

Longitudinal Magnification – Afocal Systems

$$\bar{m} = \frac{\Delta z'}{\Delta z} = \left(\frac{n'}{n}\right) m^2 = \text{constant}$$

$$m = -\frac{f_{E2}}{f_{E1}} = -\frac{f_2}{f_1}$$

Equispaced planes map into equispaced planes.

Assume $\left(\frac{n'}{n}\right) = 1$

$|m| < 1$ $\quad$ $\Delta z' < \Delta z$

$|m| > 1$ $\quad$ $\Delta z' > \Delta z$

Angular Magnification – Afocal Systems

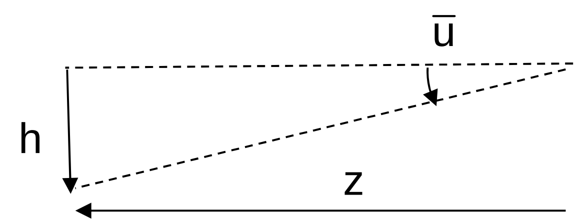
$$\bar{m} = \frac{\Delta z'}{\Delta z} = m^2 = \text{constant} \qquad\qquad m = -\frac{f_2}{f_1}$$

The angular size of an object is the ratio of the height of an object to its distance. The angular magnification is the change of this angular size and is equal to ratio of the lateral magnification to the longitudinal magnification.

$$\text{Angular Magnification} = \frac{m}{\bar{m}} = \left(\frac{n}{n'}\right)\frac{1}{m}$$

$$\text{Angular Magnification} = \frac{m}{\bar{m}} = \frac{1}{m} \qquad \text{when } \left(\frac{n}{n'}\right) = 1$$

The angular size is the object or image height or size divided by its distance from the observer.



$$\bar{u} = \frac{h}{z} \qquad \bar{u}' = \frac{h'}{z'} = \frac{mh}{\bar{m}z} = \frac{mh}{m^2 z} = \frac{h}{mz} \qquad \frac{\bar{u}'}{\bar{u}} = \frac{1}{m} \qquad \left(\frac{n}{n'}\right) = 1$$

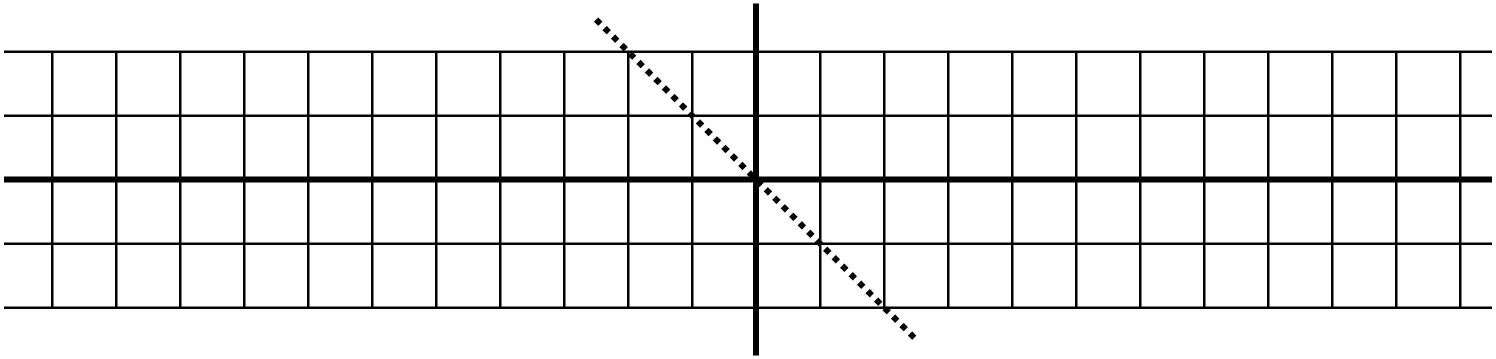
Longitudinal Mapping – Afocal System

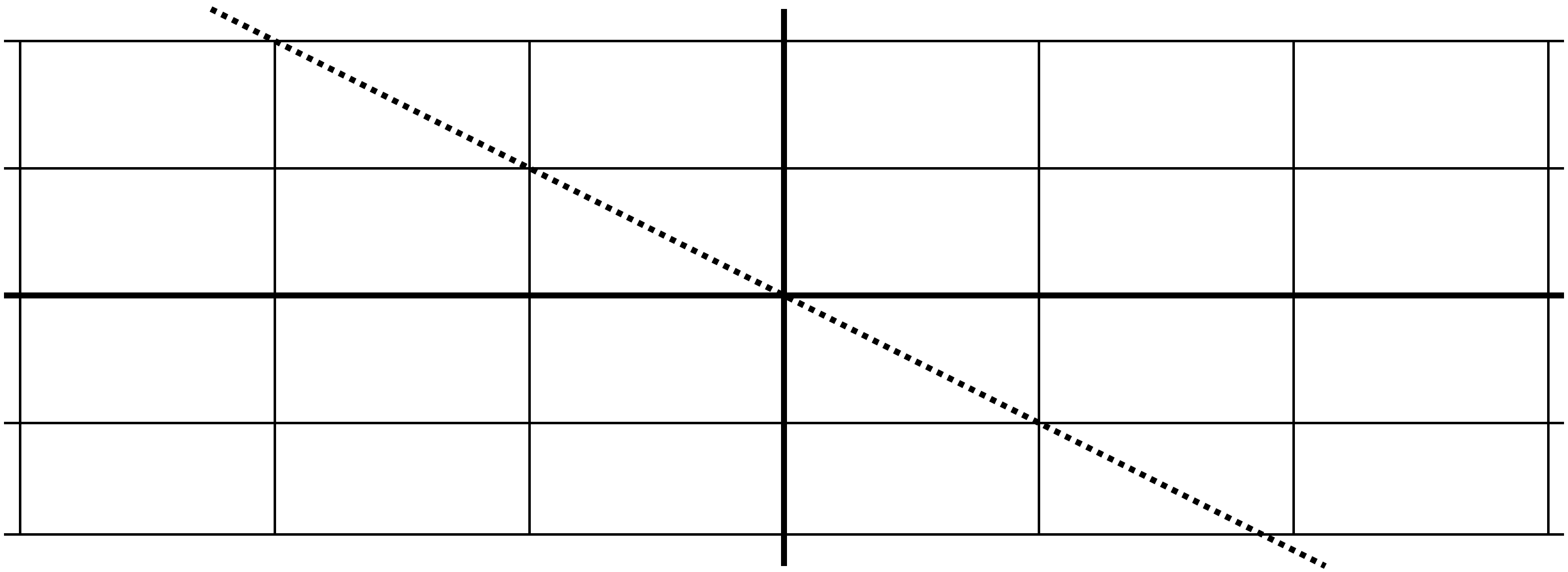
$m = 2$

$\bar{m} = 4$

Angular Mag $= 1/2$

$n'/n = 1$





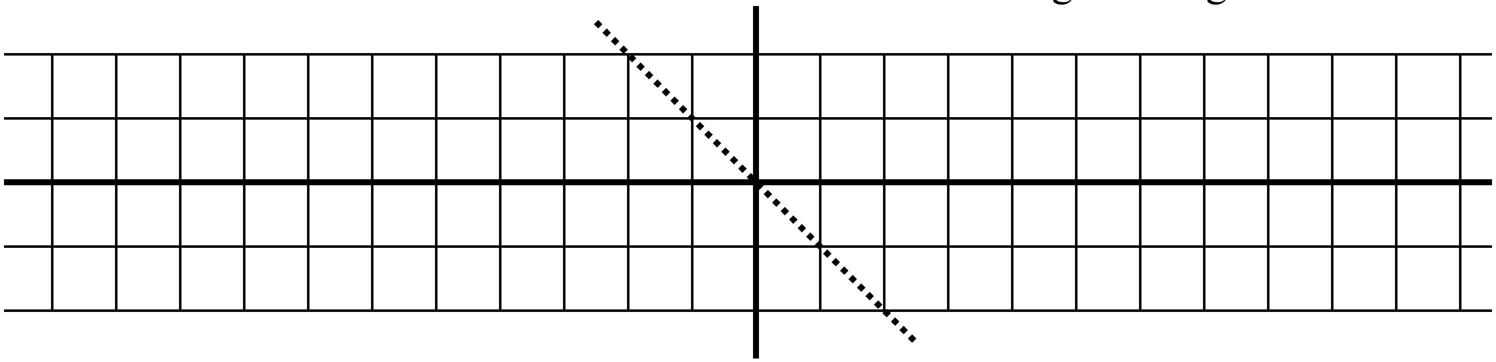
Longitudinal Mapping – Afocal System

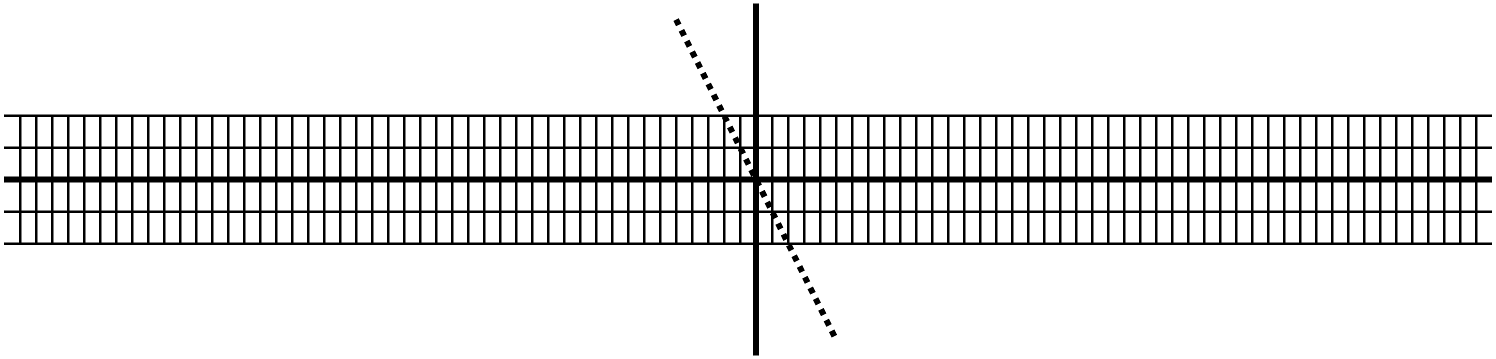
$m = 1/2$

$\bar{m} = 1/4$

Angular Mag = 2

$n'/n = 1$





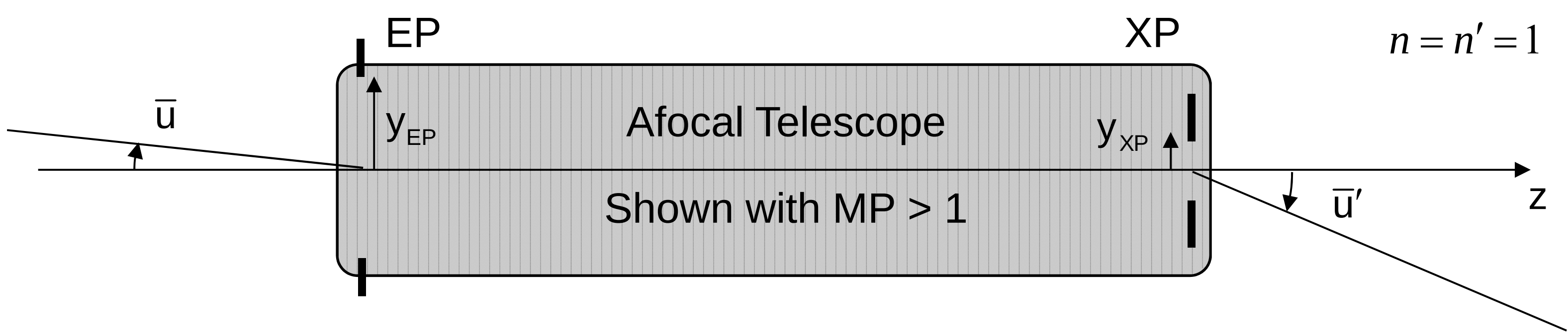
Telescopes and the Lagrange Invariant

The angular magnification of a telescope is also known as the Magnifying Power:

$$MP = \text{Angular Magnification} = \frac{\theta'}{\theta}$$

where the angle subtended by the object is θ, and the image size as seen through the telescope subtends an angle θ'. The angles θ and θ' are often considered to be the paraxial chief ray angles as measured in the Entrance and Exit Pupils.

$$MP = \frac{\theta'}{\theta} \approx \frac{\bar{u}'}{\bar{u}}$$



In the pupils: $Ж = \bar{u} y_{EP} = \bar{u}' y_{XP}$ $\quad MP = \frac{\bar{u}'}{\bar{u}} = \frac{y_{EP}}{y_{XP}}$ $\quad D_{EP} = 2y_{EP}$ $\quad D_{XP} = 2y_{XP}$

The pupils are conjugate and their sizes are related by the afocal lateral magnification m:

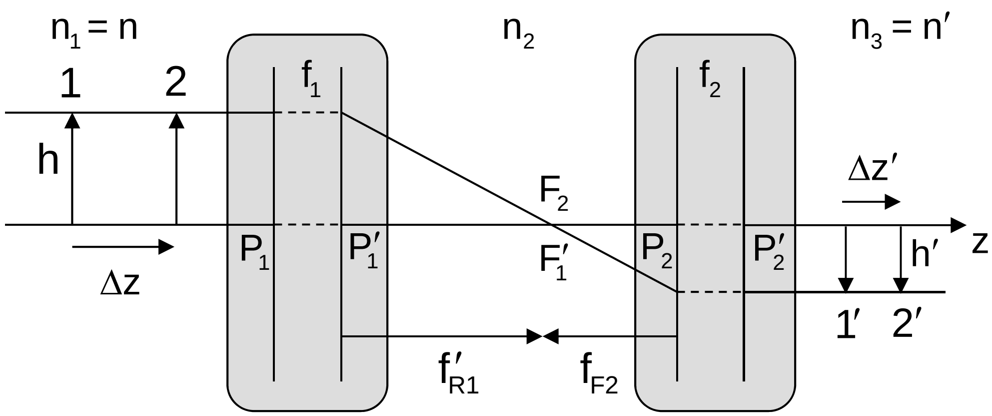
$$mD_{EP} = D_{XP} \qquad MP = \frac{\bar{u}'}{\bar{u}} = \frac{D_{EP}}{D_{XP}} = \frac{1}{m}$$

For a telescope that magnifies:

$$|m| < 1 \qquad D_{EP} > D_{XP}$$

Afocal Systems - Summary

An afocal system is formed by the combination of two focal systems. The rear focal point of the first system is coincident with the front focal point of the second system.



Lateral Magnification $m = -\frac{f_2}{f_1}$

Longitudinal Magnification $\bar{m} = \left(\frac{n'}{n}\right) m^2$ $\frac{\Delta z'/n'}{\Delta z/n} = m^2$ In air: $\bar{m} = m^2$

Angular Magnification $\frac{m}{\bar{m}} = \left(\frac{n}{n'}\right)\frac{1}{m}$ In air: $\frac{m}{\bar{m}} = \frac{1}{m}$

The lateral, longitudinal and angular magnifications are all constant. Equispaced planes map into equispaced planes.