October 18, 2017 Lecture 17 Name SOLUTIONS

Closed book; closed notes. Time limit: 75 minutes.
An equation sheet is attached and can be removed. A spare raytrace sheet is also attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, that method must be used.
Raytraces must be done on the raytrace form. Be sure to indicate the initial conditions for your rays.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Provide your answers in a neat and orderly fashion. No credit if it can't be read/followed.
Use a ruler or straight edge!
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.
Note: On some quantities, only the magnitude of the quantity is provided. The proper sign conventions and reference definitions must be applied.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (10 points) A 10 mm square LCD is used in a projector. A 2 m square screen is located at a distance of 5 m from the projector. Approximately what focal length lens is required for the image to fill the screen?

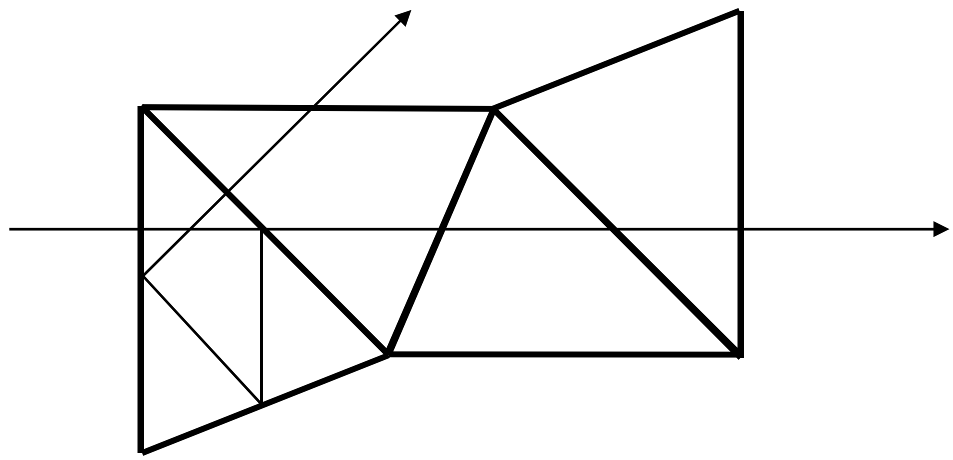
Since this situation requires a distant image, the object or LCD is located at approximately the front focal point of the lens: $z \approx -f$

$$m = \frac{h'}{h} = \frac{-2000\,mm}{10mm} = -200 \qquad m = \frac{z'}{z} \approx \frac{z'}{-f} \qquad f \approx -\frac{z'}{m} = \frac{5m}{200} = \frac{5000mm}{200} = 25\,mm$$

$$m = 200:1 \qquad f \approx \frac{z'}{m} = \frac{5m}{200} = \frac{5000mm}{200} = 25\,mm$$

Focal Length ≈ 25 mm

2) (10 points) Draw the tunnel diagram for this prism and the ray path shown.



3) (15 points) An optical system is to be constructed using two thin lenses in air. The system must have a focal length of 100 mm and a back focal distance of 75 mm. The spacing between the two thin lenses is 50 mm.

Determine the focal lengths of the two thin lenses. Sketch the system noting the positions of P' and F'.

NOTE: Use Gaussian Reduction and Gaussian Imaging for this problem. Cascaded imaging may not be used (you may not image through one lens and then use this image as an object for the other lens).

$$f = 100mm \qquad \phi = \frac{1}{f} = \frac{1}{100mm} = 0.01/mm \qquad BFD = 75mm$$

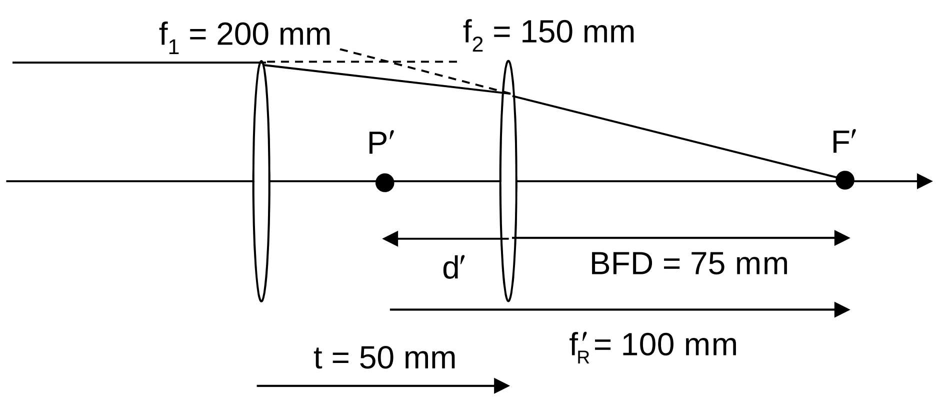
$$d' = BFD - f'_R = BFD - f = 75mm - 100mm = -25mm$$

$$d' = -\frac{\phi_1}{\phi} t = -25mm \qquad t = 50mm$$

$$\phi_1 = 0.005/mm \qquad f_1 = 200mm$$

$$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t = 0.01/mm$$

$$\phi_2 = 0.00667/mm \qquad f_2 = 150mm$$



This configuration of two positive lenses places the Rear Principal Plane P' between the two lenses and is called a Petzval Objective.

f_1 = ____200___ mm f_2 = ___150____ mm

4) (10 points) A thin lens is made of a glass with an index of 1.70. When immersed in water, the rear focal length of the lens is 100 mm. The index of water is 1.33. What is the focal length of this lens in air?

Just as with a single refracting surface, the power of a thin lens depends on the index of the medium. It is two back-to-back refracting surfaces. If the lens happens to be immersed in a medium with the same index of that of the glass, it disappears and the power is zero. In fact, this is a method for measuring the index of refraction of a lens.

In water:

$$f_R' = 100mm \qquad \frac{1}{\phi_{WATER}} = f_{WATER} = \frac{f_R'}{n_{WATER}} = \frac{100mm}{1.33} = 75.2mm$$

$$\phi_{WATER} = 0.01333/mm$$

For a thin lens in water:

$$\phi_{WATER} = \phi_{1W} + \phi_{2W} \qquad \phi_{1W} = \left(n_{LENS} - n_{WATER}\right)C_1 \qquad \phi_{2W} = \left(n_{WATER} - n_{LENS}\right)C_2$$

$$\phi_{WATER} = \left(n_{LENS} - n_{WATER}\right)\left(C_1 - C_2\right) = 0.0133/mm \qquad n_{LENS} = 1.7$$

$$\left(C_1 - C_2\right) = \frac{\phi_{WATER}}{\left(n_{LENS} - n_{WATER}\right)} = \frac{0.0133/mm}{\left(1.7 - 1.33\right)} = 0.0359/mm$$

The difference in curvatures $\left(C_1 - C_2\right)$ does not change with medium.

The same lens in air:

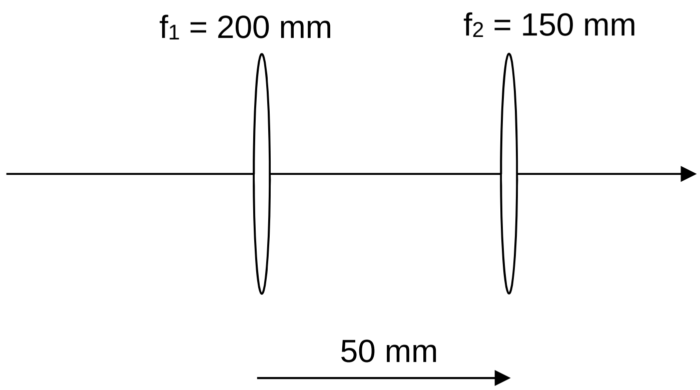
$$\phi_{AIR} = \left(n_{LENS} - n_{AIR}\right)\left(C_1 - C_2\right) = \left(n_{LENS} - 1\right)\left(C_1 - C_2\right) \qquad n_{AIR} = 1$$

$$\phi_{AIR} = \left(0.33\right)\left(0.0359/mm\right) = 0.0252/mm$$

$$f_{AIR} = \frac{1}{\phi_{AIR}} = 39.7mm$$

Focal Length = ____39.7___ mm

5) (15 points) An optical system in air is comprised of two thin lenses:



An object is placed 500 mm to the left of the first lens. The object size is ± 10 mm. Use paraxial raytrace methods to determine the system focal length and the location and size of the image.

Determine:
- System Focal Length
- Back Focal Distance
- Image Location and Size

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the image size must be determined from a separate raytrace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Calculations may NOT be done in the margins of the raytrace sheet. Gaussian imaging methods may not be used for any portion of this problem.

System Focal Length = 100 mm

Back Focal Distance = 75 mm

Image Location = 98.1 mm to the R of the second lens

Image Size = +/- 2.31 mm (Inverted)

Note that this lens system is the same system that was designed in an earlier problem!

Three rays need to be traced:

- Forward infinity ray – parallel to the axis in object space.
- Ray from the axial object location: y = 0 at the object location
- Ray from the edge of the object: h = 10 mm

The distance from the object to the first lens is 500 mm.

	Infinity or Object		f_1		f_2		F' or Image						
Surface	0		1		2		3		4		5		6
f			200		150								
$-\phi$			-.005		-.00667								
t		Inf/500		50		BFD/s'							
Infinity Ray						75.0							
y	1		1		0.75		0						
u		0		-.005		-0.01							
y													
u													
Image Location Ray						98.1							
y	0		50.0		42.5		0						
u		0.1*		-0.15		-0.433							
Image Height Ray													
y	10		10		7.5		-2.31						
u		0*		-0.05		-0.1							
y													
u													
y													
u													

* Arbitrary

Note also that this Image Height Ray is a scaled version of the Infinity Ray extended to the image plane.

Continues…

Method of Solution:

Method of Solution:

From the forward infinity ray – crosses the axis at F':

$$\overline{f_2 F'} = BFD = 75mm \qquad y_0 = 1mm$$

$$u' = -0.01 \qquad \phi = -\frac{u'}{y_0} = 0.01/mm$$

$$f = \frac{1}{\phi} = 100mm$$

Using the rays defining the object location and object height. The axial ray crosses the axis at the image location (s'). The ray from the top of the object determines the image height (h').

The image is 98.1*mm* to the right of the second lens.

The image height is -2.31*mm*. The image is reduced and inverted.

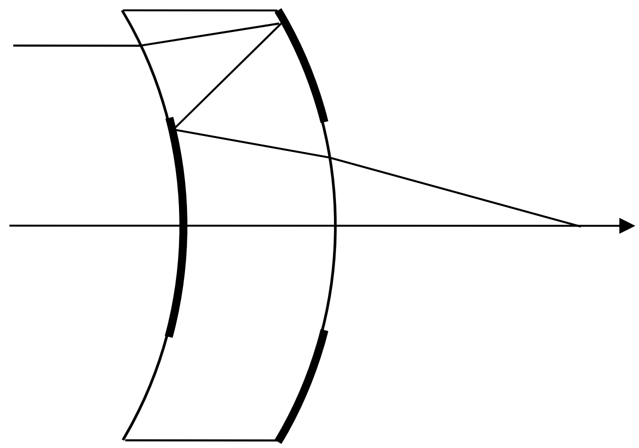
6) (20 points) A catadioptric system uses both reflection and refraction to achieve its focal power. A solid catadioptric system (a solid-cat) can be produced by coating portions of the front and rear surfaces of a thick lens so that there are transmissive and reflective zones on each surface. In this system, both surfaces have the same radius of curvature.

R1 = R2 = -100 mm

t = 30 mm

n = 1.5

The system is in air.



Use a paraxial raytrace to determine the back focal distance and the system focal length.

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from quantities derived from the rays you trace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Calculations may NOT be done in the margins of the raytrace sheet. Gaussian imaging methods may not be used for any portion of this problem.

A sizeable portion of the credit for this problem is properly setting up the raytrace sheet.

There are four surface interactions.

$$C1 = C2 = \frac{1}{R1} = \frac{1}{R2} = -0.01/mm$$

$$t_1 = 30mm \qquad n_1 = 1.5$$

$$t_2 = -30mm \qquad n_2 = -1.5$$

$$t_3 = 30mm \qquad n_3 = 1.5$$

$$t_4 = BFD \qquad n_4 = 1.0$$

Focal Length = ____75.08____ mm Back Focal Distance = ___22.82____ mm

Method of Solution:

A single infinity ray parallel to the axis needs to be traced: u = 0, y = 1.

			Front		Back		Front		Back		F'		
Surface	0		1		2		3		4		5		6
C			-0.01		-.0.01		-0.01		-0.01				
t		Inf		30		-30		30		BFD			
n		1.0		1.5		-1.5		1.5		1.0			
$-\phi$			0.005		-.030		.030		-0.005				
t/n		Inf		20		20		20		22.82			
y	1		1		1.10		0.54		0.304		0		
nu		0		0.005		-0.028		-0.0118		-.01332			
u										-.01332			
y													
nu													
u													

From this ray:

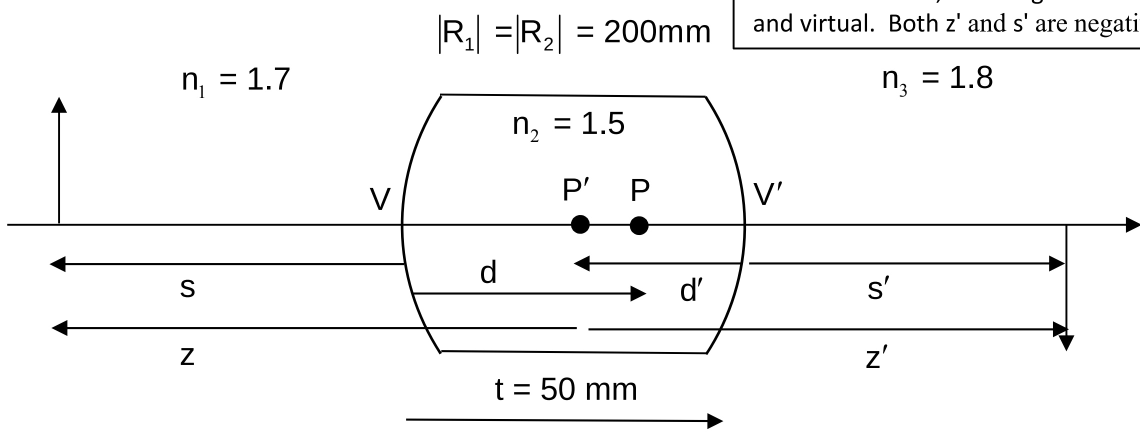
$$BFD = 22.82mm \qquad y_0 = 1mm$$

$$u' = -0.01332 \qquad \phi = -\frac{y_0}{u'} = 0.01332/mm$$

$$f = \frac{1}{\phi} = 75.08mm$$

7) (20 points) An equi-convex thick lens of index 1.5 separates media of indices 1.7 and 1.8. Both radii are 200 mm and the lens is 50 mm thick.

Note: This is a generalized drawing. In the final solution, the image is erect and virtual. Both z' and s' are negative.



A 10 mm high object is located 100 mm to the left of the front vertex of the lens. Determine image size and the location of the image relative to the rear vertex of the lens.

NOTE: Use Gaussian Reduction and Gaussian Imaging for this problem. Explain your process. Cascaded imaging may not be used for this problem. No credit will be given for any answers obtained by raytrace methods.

$$R_1 = 200mm \qquad C_1 = 0.005/mm \qquad R_2 = -200mm \qquad C_2 = -0.005/mm$$

$$\phi_1 = (n_2 - n_1)C_1 = (1.5 - 1.7)C_1 \qquad \phi_2 = (n_3 - n_2)C_2 = (1.8 - 1.5)C_2$$

$$\phi_1 = -0.001/mm \qquad \phi_2 = -0.0015/mm$$

Since the lens index is lower than the surrounding media, both lens surfaces have negative power. The lens will also have negative power:

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2\tau \qquad \tau = \frac{t}{n_2} = \frac{50mm}{1.5} = 33.33mm$$

$$\phi = -0.00255/mm$$

$$f = \frac{1}{\phi} = -392.2mm$$

Continues…

Now, the Principal Plane locations:

$$\delta' = \frac{d'}{n_3} = -\frac{\phi_1}{\phi}\tau = -13.07mm \qquad \delta = \frac{d}{n_1} = \frac{\phi_2}{\phi}\tau = 19.61mm$$

$$d' = -23.52mm \qquad n_3 = 1.8 \qquad d' = 33.33mm \qquad n_1 = 1.7$$

The object position relative to the Front Principal Plane P:

$$s = -100mm$$

$$z = s - d = -133.33mm$$

Image!

$$\frac{n_3}{z'} = \frac{n_1}{z} + \phi$$

$$\frac{z'}{n_3} = -65.36mm$$

$$z' = -117.6mm$$

Convert to vertex distance:

$$s' = z' + d' = -117.6mm - 23.52mm$$

$$s' = -141.1mm$$

Virtual image to the left of the rear vertex.

Image magnification and size:

$$m = \frac{z'/n_3}{z/n_1} = \frac{-117.6mm/1.8}{-133.33mm/1.7}$$

$$m = 0.833$$

$$h' = mh \qquad h = 10mm$$

$$h' = 8.33mm$$

Erect image.

Image: Located __141.1___ mm to the _L_ of the rear vertex Size = __8.33__ mm