1) (10 points) A 10 mm square LCD is used in a projector. A 2 m square screen is located at a distance of 5 m from the projector. Approximately what focal length lens is required for the image to fill the screen?

Since this situation requires a distant image, the object or LCD is located at approximately the front focal point of the lens: \( z \approx -f \)

\[
m = \frac{h'}{h} = \frac{-2000 \text{mm}}{10 \text{mm}} = -200 \quad m = \frac{z'}{z} \approx \frac{-z'}{-f} \quad f \approx -\frac{z'}{m} = \frac{5 \text{m}}{200} = \frac{5000 \text{mm}}{200} = 25 \text{mm}
\]

\[
m = 200 : 1 \quad f \approx \frac{z'}{m} = \frac{5 \text{m}}{200} = \frac{5000 \text{mm}}{200} = 25 \text{mm}
\]

Focal Length \( \approx 25 \) mm
2) (10 points) Draw the tunnel diagram for this prism and the ray path shown.
3) (15 points) An optical system is to be constructed using two thin lenses in air. The system must have a focal length of 100 mm and a back focal distance of 75 mm. The spacing between the two thin lenses is 50 mm.

Determine the focal lengths of the two thin lenses. Sketch the system noting the positions of P' and F'.

NOTE: Use Gaussian Reduction and Gaussian Imaging for this problem. Cascaded imaging may not be used (you may not image through one lens and then use this image as an object for the other lens).

\[ f = 100 \text{mm} \quad \phi = \frac{1}{f} = \frac{1}{100\text{mm}} = 0.01/\text{mm} \quad \text{BFD} = 75\text{mm} \]

\[ d' = \text{BFD} - f'_1 = \text{BFD} - f = 75\text{mm} - 100\text{mm} = -25\text{mm} \]

\[ d' = -\frac{\phi_1}{\phi} t = -25\text{mm} \quad t = 50\text{mm} \]

\[ \phi_1 = 0.005/\text{mm} \quad f_1 = 200\text{mm} \]

\[ \phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t = 0.01/\text{mm} \]

\[ \phi_2 = 0.00667/\text{mm} \quad f_2 = 150\text{mm} \]

This configuration of two positive lenses places the Rear Principal Plane P' between the two lenses and is called a Petzval Objective.

\[ f_1 = \underline{200}\text{ mm} \quad f_2 = \underline{150}\text{ mm} \]
4) (10 points) A thin lens is made of a glass with an index of 1.70. When immersed in water, the rear focal length of the lens is 100 mm. The index of water is 1.33. What is the focal length of this lens in air?

Just as with a single refracting surface, the power of a thin lens depends on the index of the medium. It is two back-to-back refracting surfaces. If the lens happens to be immersed in a medium with the same index of that of the glass, it disappears and the power is zero. In fact, this is a method for measuring the index of refraction of a lens.

In water:

$$f'_R = 100\text{mm} \quad \frac{1}{\phi_{\text{WATER}}} = f_{\text{WATER}} = \frac{f'_R}{n_{\text{WATER}}} = \frac{100\text{mm}}{1.33} = 75.2\text{mm}$$

$$\phi_{\text{WATER}} = 0.01333/\text{mm}$$

For a thin lens in water:

$$\phi_{\text{WATER}} = \phi_{W} + \phi_{2W} \quad \phi_{W} = (n_{\text{LENS}} - n_{\text{WATER}})C_1 \quad \phi_{2W} = (n_{\text{WATER}} - n_{\text{LENS}})C_2$$

$$\phi_{\text{WATER}} = (n_{\text{LENS}} - n_{\text{WATER}})(C_1 - C_2) = 0.0133/\text{mm} \quad n_{\text{LENS}} = 1.7$$

$$(C_1 - C_2) = \frac{\phi_{\text{WATER}}}{(n_{\text{LENS}} - n_{\text{WATER}})} = \frac{0.0133/\text{mm}}{(1.7 - 1.33)} = 0.0359/\text{mm}$$

The difference in curvatures $(C_1 - C_2)$ does not change with medium.

The same lens in air:

$$\phi_{\text{AIR}} = (n_{\text{LENS}} - n_{\text{AIR}})(C_1 - C_2) = (n_{\text{LENS}} - 1)(C_1 - C_2) \quad n_{\text{AIR}} = 1$$

$$\phi_{\text{AIR}} = (0.33)(0.0359/\text{mm}) = 0.0252/\text{mm}$$

$$f_{\text{AIR}} = \frac{1}{\phi_{\text{AIR}}} = 39.7\text{mm}$$

Focal Length = 39.7 mm
5) (15 points) An optical system in air is comprised of two thin lenses:

\[ f_1 = 200 \text{ mm} \quad f_2 = 150 \text{ mm} \]

An object is placed 500 mm to the left of the first lens. The object size is ± 10 mm. Use paraxial raytrace methods to determine the system focal length and the location and size of the image.

Determine:
- System Focal Length
- Back Focal Distance
- Image Location and Size

**NOTE:** This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the image size must be determined from a separate raytrace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Calculations may NOT be done in the margins of the raytrace sheet. Gaussian imaging methods may not be used for any portion of this problem.

System Focal Length = __100___ mm

Back Focal Distance = _____75____ mm

Image Location = ____98.1____ mm to the ___R__ of the second lens

Image Size = +/- ____2.31____ mm (Inverted)

Note that this lens system is the same system that was designed in an earlier problem!
Three rays need to be traced:
- Forward infinity ray – parallel to the axis in object space.
- Ray from the axial object location: \( y = 0 \) at the object location
- Ray from the edge of the object: \( h = 10 \) mm

The distance from the object to the first lens is 500 mm.

Note also that this Image Height Ray is a scaled version of the Infinity Ray extended to the image plane.

* Arbitrary
Method of Solution:

From the forward infinity ray – crosses the axis at F':

\[
\frac{f_2 F'}{BFD} = 75 \text{mm} \quad \quad y_0 = 1 \text{mm}
\]

\[
u' = -0.01 \quad \quad \phi = \frac{-y_0}{u'} = 0.01/\text{mm}
\]

\[
f = \frac{1}{\phi} = 100 \text{mm}
\]

Using the rays defining the object location and object height. The axial ray crosses the axis at the image location (s'). The ray from the top of the object determines the image height (h').

The image is 98.1 mm to the right of the second lens.

The image height is -2.31 mm. The image is reduced and inverted.
6) (20 points) A catadioptric system uses both reflection and refraction to achieve its focal power. A solid catadioptric system (a solid-cat) can be produced by coating portions of the front and rear surfaces of a thick lens so that there are transmissive and reflective zones on each surface. In this system, both surfaces have the same radius of curvature.

\[ R_1 = R_2 = -100 \text{ mm} \]

\[ t = 30 \text{ mm} \]

\[ n = 1.5 \]

The system is in air.

Use a paraxial raytrace to determine the back focal distance and the system focal length.

**NOTE:** This problem is to be worked using raytrace methods only. All answers must be determined directly from quantities derived from the rays you trace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Calculations may NOT be done in the margins of the raytrace sheet. Gaussian imaging methods may not be used for any portion of this problem.

A sizeable portion of the credit for this problem is properly setting up the raytrace sheet.

There are four surface interactions.

\[ C_1 = C_2 = \frac{1}{R_1} = \frac{1}{R_2} = -0.01/\text{mm} \]

\[ t_1 = 30\text{mm} \quad n_1 = 1.5 \]

\[ t_2 = -30\text{mm} \quad n_2 = -1.5 \]

\[ t_3 = 30\text{mm} \quad n_3 = 1.5 \]

\[ t_4 = \text{BFD} \quad n_4 = 1.0 \]

Focal Length = 75.08 mm  
Back Focal Distance = 22.82 mm
Method of Solution:

A single infinity ray parallel to the axis needs to be traced: \( u = 0, \ y = 1 \).

\[
\begin{array}{cccccc}
\text{Surface} & \text{Front} & \text{Back} & \text{Front} & \text{Back} & \text{F'} \\
0 & 1 & 2 & 3 & 4 & 5 & 6 \\
C & -0.01 & -0.01 & -0.01 & -0.01 \\
t & \text{Inf} & 30 & -30 & 30 & \text{BFD} \\
n & 1.0 & 1.5 & -1.5 & 1.5 & 1.0 \\
-\phi & 0.005 & -0.030 & 0.030 & -0.005 \\
t/n & \text{Inf} & 20 & 20 & 20 & 22.82 \\
y & 1 & 1 & 1.10 & 0.54 & 0.304 & 0 \\
u & 0 & 0.005 & -0.028 & -0.0118 & -0.01332 \\
u' & \text{Inf} & 0.10 & 0.54 & 0.304 & 0.10 \\
\end{array}
\]

From this ray:

\[
\begin{align*}
BFD &= 22.82 \text{mm} & y_0 &= 1 \text{mm} \\
u' &= -0.01332 & \phi &= -\frac{y_0}{u'} = 0.01332 / \text{mm} \\
f &= \frac{1}{\phi} = 75.08 \text{mm}
\end{align*}
\]
7) (20 points) An equi-convex thick lens of index 1.5 separates media of indices 1.7 and 1.8. Both radii are 200 mm and the lens is 50 mm thick. 

Note: This is a generalized drawing. In the final solution, the image is erect and virtual. Both $z'$ and $s'$ are negative.

A 10 mm high object is located 100 mm to the left of the front vertex of the lens. Determine image size and the location of the image relative to the rear vertex of the lens.

**NOTE:** Use Gaussian Reduction and Gaussian Imaging for this problem. Explain your process. Cascaded imaging may not be used for this problem. No credit will be given for any answers obtained by raytrace methods.

\[
R_1 = 200 \text{mm} \quad C_1 = 0.005 / \text{mm} \quad R_2 = -200 \text{mm} \quad C_2 = -0.005 / \text{mm}
\]

\[
\phi_1 = (n_2 - n_1) C_1 = (1.5 - 1.7) C_1 \quad \phi_2 = (n_3 - n_2) C_2 = (1.8 - 1.5) C_2
\]

\[
\phi_1 = -0.001 / \text{mm} \quad \phi_2 = -0.0015 / \text{mm}
\]

Since the lens index is lower than the surrounding media, both lens surfaces have negative power. The lens will also have negative power:

\[
\phi = \phi_1 + \phi_2 - \frac{d_1 d_2}{2n_2} \tau \quad \tau = \frac{t}{n_2} = \frac{50 \text{mm}}{1.5} = 33.33 \text{mm}
\]

\[
\phi = -0.00255 / \text{mm}
\]

\[
f = \frac{1}{\phi} = -392.2 \text{mm}
\]

Continues...
Now, the Principal Plane locations:

\[ \delta' = \frac{d'}{n_3} = -\frac{\phi_1}{\phi} \tau = -13.07\text{mm} \quad \delta = \frac{d}{n_1} = \frac{\phi_1}{\phi} \tau = 19.61\text{mm} \]

\[ d' = -23.52\text{mm} \quad n_3 = 1.8 \quad d = 33.33\text{mm} \quad n_1 = 1.7 \]

The object position relative to the Front Principal Plane \( P \):

\[ s = -100\text{mm} \]

\[ z = s - d = -133.33\text{mm} \]

Image!

\[ \frac{n_3}{z'} = \frac{n_1}{z} + \phi \]

\[ \frac{z'}{n_3} = -65.36\text{mm} \]

\[ z' = -117.6\text{mm} \]

Convert to vertex distance:

\[ s' = z' + d' = -117.6\text{mm} - 23.52\text{mm} \]

\[ s' = -141.1\text{mm} \]

Virtual image to the left of the rear vertex.

Image magnification and size:

\[ m = \frac{z'}{n_3} = \frac{-117.6\text{mm}}{1.8} \]

\[ \frac{z}{n_1} = \frac{-133.33\text{mm}}{1.7} \]

\[ m = 0.833 \]

\[ h' = mh \quad h = 10\text{mm} \]

\[ h' = 8.33\text{mm} \]

Erect image.

Image: Located \( 141.1 \) mm to the \_L\_ of the rear vertex  
Size = \( 8.33 \) mm