Name_______SOLUTIONS________

Closed book; closed notes. Time limit: 120 minutes.
An equation sheet is attached and can be removed. Spare raytrace sheets are attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, that method must be used.
Raytraces must be done on the raytrace form. Be sure to indicate the initial conditions for your rays.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Provide your answers in a neat and orderly fashion. No credit if it can't be read/followed.
Use a ruler or straight edge!
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.
Note: On some quantities, only the magnitude of the quantity is provided. The proper sign conventions and reference definitions must be applied.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (10 points) Design a Galilean telescope constructed out of two thin lenses in air. The telescope must have a magnifying power of 3X and a length of 75 mm.

The Galilean telescope consists of a positive lens followed by a negative lens.
The MP is positive.

$$MP = -\frac{f_{OBJ}}{f_{EYE}} = 3 \qquad f_{OBJ} = -3 f_{EYE}$$

$$t = 75mm = f_{OBJ} + f_{EYE} = -3 f_{EYE} + f_{EYE} = -2 f_{EYE}$$

$$f_{EYE} = -37.5mm \qquad f_{OBJ} = 112.5mm$$

f_{OBJ} = ___112.5__ mm f_{EYE} = ___-37.5___ mm

2) (15 points) Design a spectacle lens for a patient with corneal astigmatism. The patient requires a lens that has powers of -4 D and -6 D in orthogonal meridians on the lens. The lens has an index of 1.5, and the shape of this thin lens in air is a meniscus. The concave spherical surface of the lens has a radius of curvature of 80 mm. The convex surface of the lens is toroidal producing the required power variation.

a) Determine the two principal radii of curvature for the convex surface of the lens.

b) What is the surface sag difference of the concave surface along the two principal meridians? Determine this sag at a radius of 20 mm from the surface vertex.

a) Required Radii:

$$\phi_1 = -4D = -4/m = -0.004/mm \qquad \phi_2 = -6D = -6/m = -0.006/mm$$

For a thin lens, assuming the concave surface is the second surface of the lens:

$$R_{CC} = 80mm \qquad n = 1.5$$

$$\phi_1 = -0.004/mm = (n-1)\left(\frac{1}{R_{CV1}} - \frac{1}{R_{CC}}\right) \qquad \phi_2 = -0.006/mm = (n-1)\left(\frac{1}{R_{CV2}} - \frac{1}{R_{CC}}\right)$$

$$R_{CV1} = 222.2mm \qquad R_{CV2} = 2000mm$$

Alternate Solution:

$$\phi = \phi_{CV} + \phi_{CC} \qquad \phi_{CC} = \frac{1-n}{R_{CC}} = -0.00625/mm$$

$$\phi_1 = -0.004/mm = \phi_{CV1} + \phi_{CC} \qquad \phi_2 = -0.006/mm = \phi_{CV2} + \phi_{CC}$$

$$\phi_{CV1} = 0.00225/mm = \frac{n-1}{R_{CX1}} \qquad \phi_{CV2} = 0.00025/mm = \frac{n-1}{R_{CX2}}$$

$$R_{CV1} = 222.2mm \qquad R_{CV2} = 2000mm$$

b) Sag Difference at 20 mm: $y = 20mm$

$$Sag_1 = \frac{y^2}{2R_{CV1}} \qquad Sag_2 = \frac{y^2}{2R_{CV2}}$$

$$Sag_1 = 0.900mm \qquad Sag_2 = 0.100mm \qquad \Delta Sag = 0.800mm$$

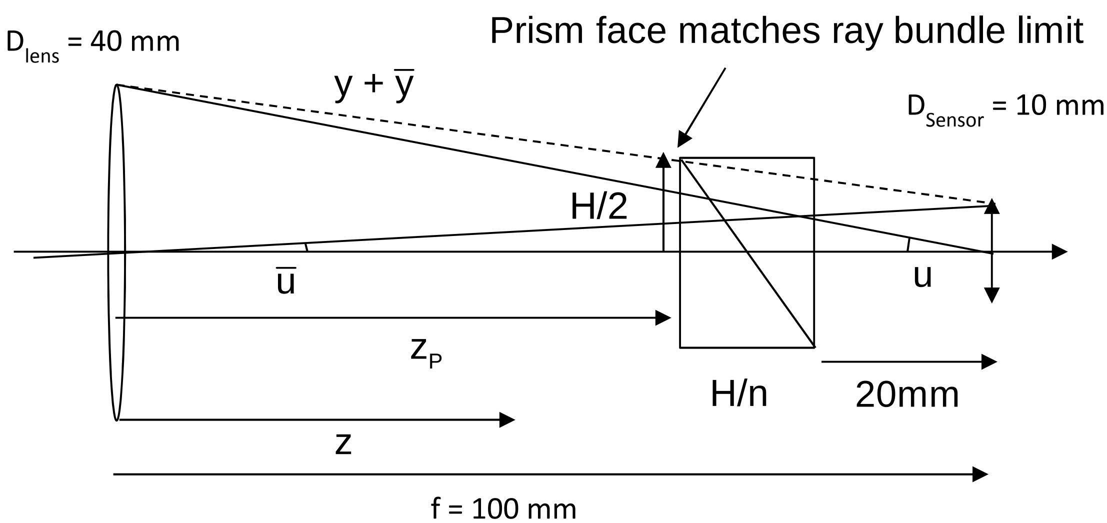
R1 = __222.2__ mm R2 = __2000_ mm ΔSag = ___0.800__ mm

3) (15 points) As shown below, a 100 mm focal length thin lens is used to image an object at infinity. The lens has a diameter of 40 mm. The sensor used with the lens has a width of 10 mm (± 5 mm). The system stop is at the lens.

A right angle prism is to be inserted between the lens and the sensor. The exit face of the prism must be spaced 20 mm from sensor. The prism has an index of refraction of 1.5.

What is the smallest prism that can be used in the system with no vignetting? In other words, the system is unvignetted over the full width of the sensor.

Let H be the width of the face of the prism. You may consider this to be a one-dimensional problem and consider vignetting only in the plane of the paper.



Determine the ray bundle limit for no vignetting (dashed line):

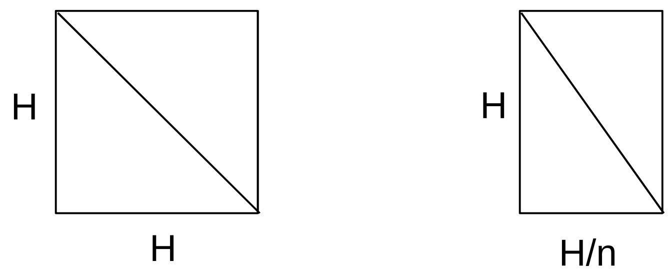
$$\bar{u} = \frac{D_{Sensor1}/2}{f} = \frac{5mm}{100mm} = 0.05 \qquad u = -\frac{D_{LENS}/2}{f} = \frac{20mm}{100mm} = -0.2$$

$$\bar{y}(z) = \bar{u}z \qquad y(z) = D_{LENS}/2 - uz = 20mm - uz$$

$$Ray\ Bundle\ Limit = |y| + |\bar{y}| = 20mm - uz + \bar{u}z = 20mm - 0.15z$$

Continues…

The Tunnel Diagram and the Reduced Tunnel Diagram for the Right Angle Prism:



Place the reduced tunnel diagram on the ray drawing and scale so that the size of the front prism face matches the ray bundle limit. The prism must be spaced 20 mm from the detector. The location of the front face of the prism:

$$z_P = 100mm - 20mm - H/n = 80mm - H/n \qquad n = 1.5$$

The vignetting condition will be met when the ray bundle limit equals H/2 at z_P:

$$Ray\ Bundle\ Limit = |y| + |\bar{y}| = 20mm - 0.15z$$

$$H/2 = 20mm - 0.15(80mm - H/1.5)$$

Solve for the prism size H:

$$H/2 = 20mm - 0.15(80mm - H/1.5)$$

$$0.5H = 20mm - 12mm + 0.1H$$

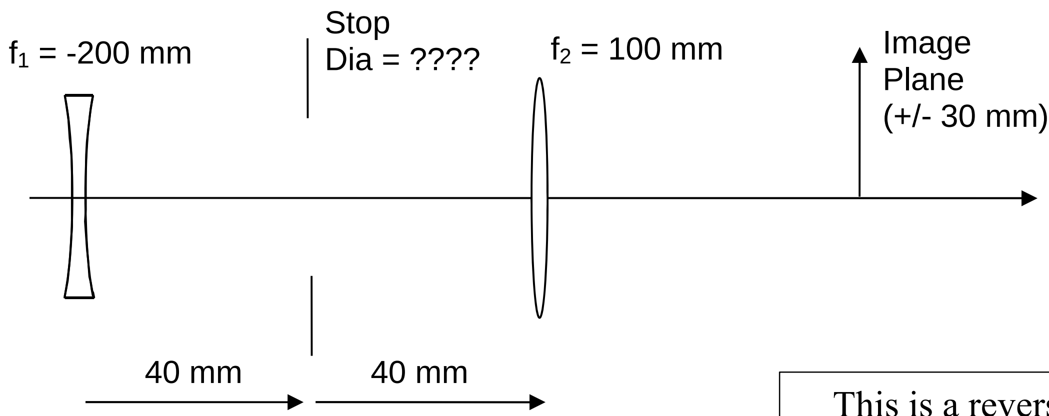
$$H = 20mm$$

A similar method of solution was used in the design of the Porro-Prism Binoculars in the Homework.

Minimum Prism Size H = __20_ mm

4) (25 points) The following diagram shows the design of an objective that is comprised of two thin lenses in air. The system stop is located between the two lenses.

The system operates at f/4.
The object is at infinity.
The maximum image size is +/- 30 mm.



This is a reverse telephoto objective
BFD > f

Determine the following:
- Entrance pupil and exit pupil locations and sizes.
- System focal length and back focal distance.
- Stop diameter.
- Angular field of view (in object space).
- Required diameters for the two lenses for the system to be unvignetted over the specified maximum image size.

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the FOV must be determined from a raytrace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Calculations may NOT be done in the margins of the raytrace sheet. Gaussian imaging methods may not be used for any portion of this problem.

Entrance Pupil: __33.33__ mm to the ___R___ of the first lens. D_{EP} = __27.78__ mm

Exit Pupil: __66.67__ mm to the ___L___ of the second lens. D_{XP} = __55.56__ mm

System Focal Length = __111.11__ mm Back Focal Distance = ___155.56___mm

Stop Diameter = ___33.33__ mm FOV = +/- __15.1__ deg in object space

Lens 1 Diameter = ___45.78__ mm Lens 2 Diameter = __56.9__ mm

	Object	EP	L1	Stop	L2	XP	F′ Image
Surface	0	1	2	3	4	5	6
f			-200		100		
$-\phi$			0.005		-0.01		

	0–1	1–2	2–3	3–4	4–5	5–6
t		-33.333	40	40	-66.67	222.22

Potential Chief Ray

Surface	0	1	2	3	4	5	6
$\tilde{\bar{y}}$		0	-4.0	0	4.0	0	

	0–1	1–2	2–3	3–4	4–5	5–6
$\tilde{\bar{u}}$		0.12	0.1*	0.1*	0.06	

Potential Marginal Ray

222.22

Surface	0	1	2	3	4	5	6
$\tilde{y}$	1	1	1	1.2	1.4	2.00	0

	0–1	1–2	2–3	3–4	4–5	5–6
$\tilde{u}$	0	0	0.005	0.005	-0.009	-0.009

Potential Chief Ray - Extended

Surface	0	1	2	3	4	5	6
$\tilde{\bar{y}}$		0	-4.0	0	4.0	0	13.33

	0–1	1–2	2–3	3–4	4–5	5–6
$\tilde{\bar{u}}$		1.12	0.1	0.1	0.06	0.06

Marginal Ray – Scale Factor = 13.89

Surface	0	1	2	3	4	5	6
y	13.89	13.89	13.89	16.67	19.45	27.78	0

	0–1	1–2	2–3	3–4	4–5	5–6
u	0	0	0.0695	0.0695	-0.125	-0.125

Chief Ray – Scale Factor = 2.25

Surface	0	1	2	3	4	5	6
$\bar{y}$		0	-9.0	0	9.0	0	30

	0–1	1–2	2–3	3–4	4–5	5–6
$\bar{u}$		0.270	0.225	0.225	0.135	0.135

Surface	0	1	2	3	4	5	6
y							

	0–1	1–2	2–3	3–4	4–5	5–6
u						

Surface	0	1	2	3	4	5	6
y							

	0–1	1–2	2–3	3–4	4–5	5–6
u						

* Arbitrary

Continues…

Provide Method of Solution:

EP/XP Location:

Trace a potential chief ray that starts at the center of the stop. The pupils are located where this ray crossed the axis in object/image space.

$$L_1 \rightarrow EP = 33.33mm \quad \text{(Right of L1)}$$
$$L_2 \rightarrow XP = -66.67mm \quad \text{(Left of L2)}$$

Focal Length:

Trace a potential marginal ray parallel to the axis in object space ($\tilde{y}_1 = 1$). The rear focal point is located where this ray crossed the axis.

$$XP \rightarrow F' = 222.22mm$$

$$BFD = (L_2 \rightarrow XP) + (XP \rightarrow F') = -66.67mm + 222.22mm = 155.56mm$$

$$BFD = 155.56mm$$

$$\phi = -\frac{\tilde{u}'}{\tilde{y}_1} \qquad \tilde{u}' = -0.009 \qquad \tilde{y}_1 = 1$$

$$\phi = 0.009 / mm$$

$$f = \frac{1}{\phi} = 111.11mm$$

Extend the potential chief ray to the image plane F'

Entrance Pupil:

$$f/\# = f/4 = f/D_{EP}$$

$$D_{EP} = \frac{111.11mm}{4} = 27.4mm$$

Continues…

Provide Method of Solution:

Pupil/Stop Sizes: $r_{EP} = D_{EP}/2 = 13.89mm$

Scale the marginal ray to the proper r_{EP}:

$$\text{Scale Factor} = \frac{13.89mm}{1.0mm} = 13.89$$

$$r_{STOP} = 16.67mm \qquad r_{XP} = 27.78mm$$

$$D_{STOP} = 33.33mm \qquad D_{XP} = 55.56mm$$

FOV: Scale the potential chief ray to the desired image height of 30.0 mm (from the current or potential chief ray value of 13.33mm)

$$\text{Scale Factor} = \frac{30.0mm}{13.33mm} = 2.25$$

Object Space Chief Ray:

$$\bar{u}_0 = 0.270$$

$$HFOV = \tan^{-1}(0.270) = 15.1°$$

$$FOV = 30.2° \text{ or } \pm 15.1°$$

Vignetting:

L1: $y_1 = 13.89mm \qquad \bar{y}_1 = -9.0mm$

$$a_1 = |y_1| + |\bar{y}_1| = 22.89mm$$

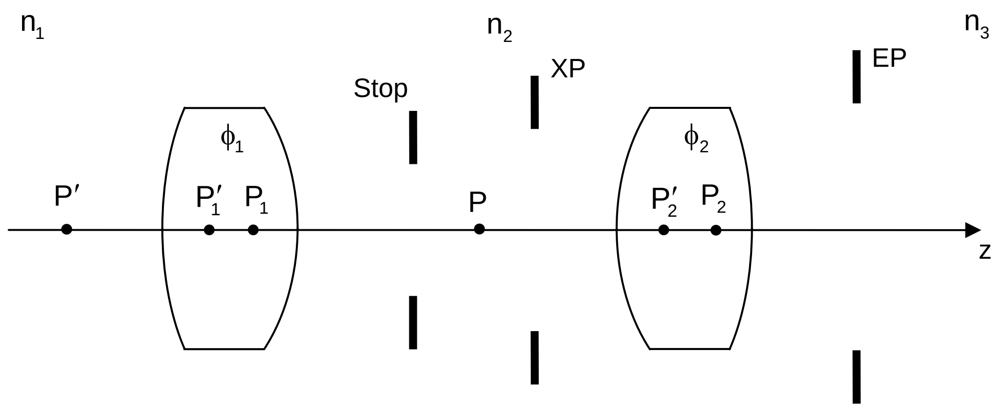
$$D_1 = 45.78mm$$

L2: $y_2 = 19.45mm \qquad \bar{y}_2 = 9.0mm$

$$a_2 = |y_2| + |\bar{y}_2| = 28.45mm$$

$$D_2 = 56.9mm$$

5) (10 points) An optical system is comprised of two elements separating indices of refraction n_1, n_2 and n_3. Subscript 1 designates element 1, subscript 2 designates element 2, and quantities without subscripts (EP, XP, P and P′) are associated with the total system.



Circle the index of refraction (and therefore the corresponding optical space) associated with each of the following:

EP:	**n_1**	n_2	n_3	XP:	n_1	n_2	**n_3**
P_1:	**n_1**	n_2	n_3	P_1':	n_1	**n_2**	n_3
P_2:	n_1	**n_2**	n_3	P_2':	n_1	n_2	**n_3**
P :	**n_1**	n_2	n_3	P′:	n_1	n_2	**n_3**

For any two element system, the details of the system do not matter. The above diagram is unnecessary!

P and EP are always in object space (n_1).

P′ and XP are always in image space (n_3).

P_i is always in the object space of the element.

P_i' is always in the image space of the element.

6) (15 points) A doubly telecentric system is constructed out of two thin lenses in air. The spacing between the lenses is 250 mm, and the magnitude of the magnification |m| is 1/4.

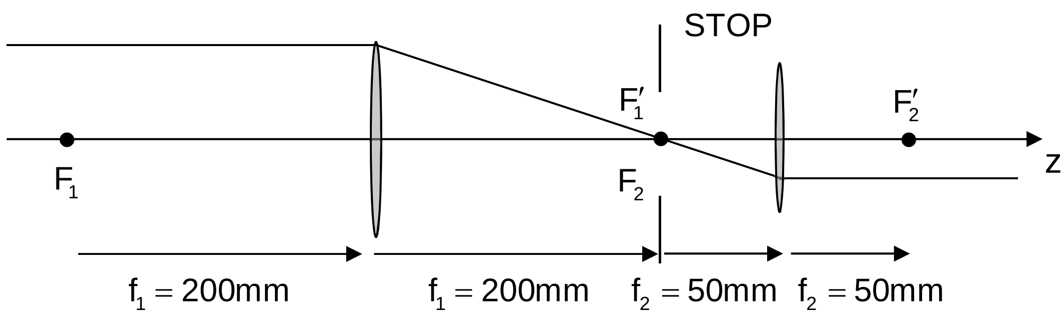
a) Design and sketch the layout of the system. Provide the required focal lengths.

A doubly telecentric system must be afocal (two positive lenses) with the stop at the common focal point. The system magnification must be negative.

$$m = -\frac{1}{4} = -\frac{f_2}{f_1}$$

$$t = f_1 + f_2 = 250mm$$

$$f_1 = 4f_2 = 200mm \qquad f_2 = 50mm$$



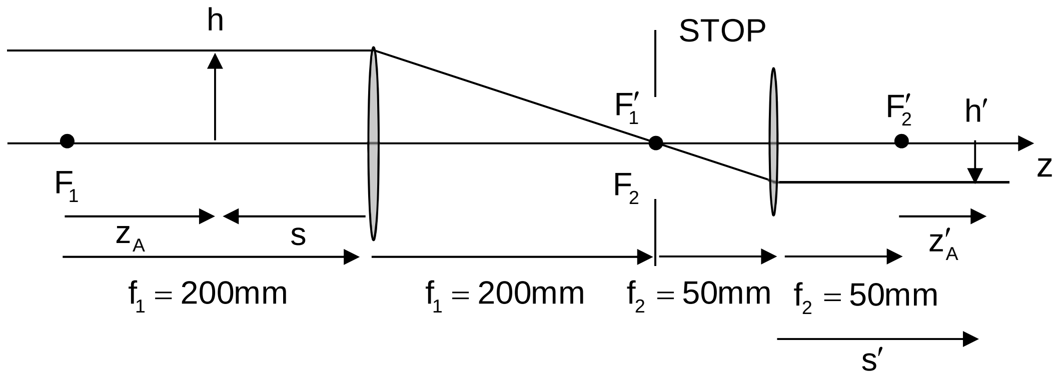
f_1 = ______200___mm f_2 = ______50____ mm

Continue to Part b…

b) A 12 mm high object is located 100 mm to the left of the first lens of this system. Determine the location and size of the image.

Cascaded imaging may not be used (you may not image through one lens and then use this image as an object for the other lens).

Raytrace methods may not be used for this problem.



For imaging, use the longitudinal magnification with the focal points as the reference points.

$$m = -\frac{1}{4} \qquad h = 12mm$$

$$\overline{m} = m^2 = \frac{1}{16} \qquad h' = mh = -3.0mm$$

The object is 100 mm to the left of the first lens:

$$s = -100mm \qquad z_A = 100mm$$

$$z'_A = \overline{m} z_A = 6.25mm$$

$$s' = f_2 + z'_A = 50mm + 6.25mm = 56.25mm$$

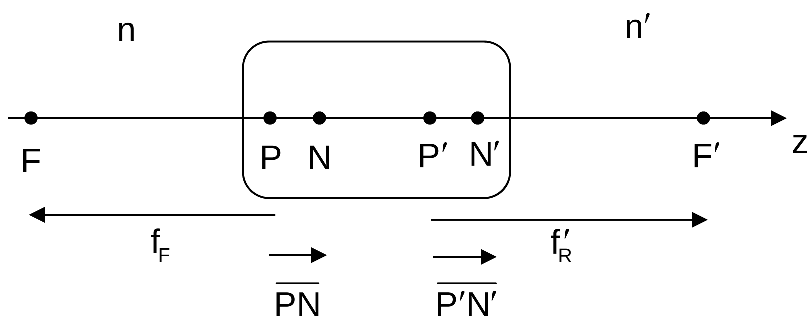
The image is to the right of the second lens.

The image is __56.25_ mm to the __R__ of L2. The image height is __-3.0___ mm.

7) (10 points) Prove the unlikely result that for a general optical system, the distance from the rear nodal point of the system to the rear focal point of the system $\left(\overline{N'F'}\right)$ is equal to minus the front focal length of the system $\left(-f_F\right)$.

Prove $\overline{N'F'} = -f_F$ for a general optical system.

A general system:



$$\overline{PN} = \overline{P'N'} = f_F + f'_R$$

$$f'_R = \overline{P'F'} = \overline{P'N'} + \overline{N'F'}$$

$$f'_R = f_F + f'_R + \overline{N'F'}$$

$$\overline{N'F'} = -f_F$$

In a similar fashion: $\overline{NF} = -f'_R$