Elementary Optical Systems

Section 12

Objectives

Many optical systems can be understood when treated as combinations of thin lenses. Mirror equivalents exist for many. The goal of this approach is to examine the paraxial properties (image size and location; entrance and exit pupils; etc.) of a variety of systems.

The types of systems examined include:

Objectives
Collimators
Magnifiers
Field lenses
Telescopes
Eyepieces
Microscopes
Telecentric systems
Relays
Illumination systems
Scanners
Objectives – Simple and Petzval

Objectives are lens element combinations used to image (usually) distant objects. To classify the objective, separated groups of lens elements are modeled as thin lenses. The simple objective is represented by a positive thin lens.

\[ BFD = f \]

The Petzval objective consists of two separated positive groups of elements. The system rear principal plane is located between the groups.

\[ BFD < f \]

Objectives – Telephoto and Reverse Telephoto

The telephoto objective produces a system focal length longer than the overall system length \((t + BFD)\). It consists of a positive group followed by a negative group.

\[ BFD \ll f \]

The reverse telephoto objective or retrofocus objective consists of a negative group followed by a positive group. This configuration is used to produce a system with a BFD larger than the system focal length. While this configuration is used for many wide angle objectives, the term reverse telephoto specifically refers to the configuration, not the FOV.
Collimator

A collimator is a reversed objective. It creates a collimated beam from a source at the system front focal point, and the image of the source is projected to infinity. The degree of collimation is determined by the source size.

\[ f_p = -f \]

Depth of Focus and Depth of Field

There is often some allowable image blur that defines the performance requirement of an optical system. This maximum acceptable blur may result from the detector resolution or just the overall system or display resolution requirement. This blur requirement results in a first-order geometrical tolerance for the longitudinal position of the object or the image plane. No diffraction or aberrations are included.

The depth of focus DOF describes the amount the detector can be shifted from the nominal image position for a given position before the resulting blur exceeds the blur diameter criterion \( B' \).

\[ DOF = \pm b' \]

\[ b' = \frac{B' L_{o'}}{D_{xp}} = \frac{B' z'}{D_{xp}} \]

\[ DOF = \pm b' = \pm B' f / \#P \]

\[ DOF = \pm \frac{B'}{2NA} \]
**Depth of Focus and F/#**

The depth of focus is directly proportional to the f/# of the lens:

\[ \text{DOF} = \pm B' \frac{f}{\#} \]

As a result, as a lens of a given focal length is stopped down (its f/# is increased), an increased depth of focus results.

**Depth of Field**

When a camera is focused at a particular object distance \( L_0 \), there is some range of object positions \( L_{\text{FAR}} \) to \( L_{\text{NEAR}} \), the depth of field, that will appear in focus for a given detector or image plane position. The image plane blur criterion \( B' \) is met for these object positions.

Consider the nominal object location:

\[ L_0 < 0 \]

\( L_0 \) is the object plane where the camera is focused, and \( L_0' \) is the corresponding image plane where an in-focus image is produced. The detector is located at this position.

These results assume a thin lens with the stop at the lens.
Depth of Field

The same image plane/detector location is maintained. When an object is at a distance greater than $L_{OL}$, the resulting image will move closer to the lens. A blur will form and be seen on the detector. At $L_{FAR}$, this blur equals the blur criteria $B'$. 

The similar scenario exists for an object at a distance less than $L_{OL}$. At $L_{NEAR}$, this blur equals the blur criteria $B'$. 

All object positions between $L_{FAR}$ to $L_{NEAR}$ will meet the blur criteria and appear to be in focus.

$L_{FAR} = \frac{L_{OL} f D}{f D + L_{OL} B'}$  

$L_{NEAR} = \frac{L_{OL} f D}{f D - L_{OL} B'}$

$L_{OL}$ is the object plane where the camera is focused – this nominal object plane is conjugate to the detector. All objects positioned between $L_{FAR}$ and $L_{NEAR}$ will produce images on the detector that have geometrical blurs less than the blur criterion $B'$. 

This linear blur condition is called the photographic depth of focus as it constrains the blur on a print or film to be smaller than a certain diameter. Historically, this was probably related to the grain size in the film. 

These results assume a thin lens with the stop at the lens.
Depth of Field – Derivation

Consider the image side:

\[ \frac{L_o + L'_{o}}{D} = \frac{L'_{o}}{B'} \]

\[ \frac{L'_o + L''_{o}}{D} = \frac{L''_{o}}{B''} \]

\[ L'_o \left( \frac{1}{D} + \frac{1}{B'} \right) = \frac{L'_{o}}{D} \]

\[ L'_o \left( \frac{1}{D} + \frac{1}{B''} \right) = \frac{L''_{o}}{D} \]

\[ B' << D \quad \frac{1}{B''} >> \frac{1}{D} \]

\[ L'_o = \frac{B'L'_{o}}{D} \quad L'_o = \frac{B'L'_o}{D} \]

\[ L' = -L'_o = L''_{o} = \text{DOF} = \frac{B'L'_{o}}{D} \]

Depth of Field – Derivation – Continued

The nominal object position \( L_o \) is conjugate to \( L'_{o} \):

\[ \frac{1}{z} = \frac{1}{L'_{o}} + \frac{1}{L} + \frac{1}{f} \]

\[ L'_o = \frac{fL_o}{f + L_o} \]

The image distances corresponding to the image limits are:

\[ z' = L'_{o} + L' = L''_{o} + \frac{B'L'_{o}}{D} \]

Solve for the corresponding object positions \( L_{\text{FAR}} \) and \( L_{\text{NEAR}} \):

\[ \frac{1}{z'} = \frac{1}{L'_{o}} + \frac{1}{L} + \frac{1}{f} \]

\[ L = \frac{L'_o f(D + B')}{fD - L'_o (D + B')} \]

\[ \frac{1}{L} = \frac{f - (L'_{o} + L')}{(L'_{o} + L')f} \]

\[ L = \frac{(L'_o + B'L'_o / D)f}{f - (L'_o + B'L'_o / D)} \]

\[ L = \frac{L'_o f(D + B')}{fD - L'_o (D + B')} \]

\[ L = \frac{L'_o f(D + B')}{fD + L'_o B'} \]
**Depth of Field – Derivation – Continued**

\[ L = \frac{L_0 f (D + B')}{fD + L_0 B'} \]

\[ L_{\text{FAR}} = \frac{L_0 f (D - B')}{fD + L_0 B'} \]

\[ z' = L_0' - L' \]

\[ L_{\text{NEAR}} = \frac{L_0 f (D + B')}{fD - L_0 B'} \]

\[ z' = L_0' + L' \]

**Hyperfocal Distance**

An important condition occurs when the far point of the depth of field \( L_{\text{FAR}} \) extends to infinity. The optical system is focused at the hyperfocal distance \( L_{\text{hf}} \) and all objects from \( L_{\text{NEAR}} \) to infinity meet the image plane blur criterion and are in focus.

\[ L_{\text{FAR}} = \infty \frac{L_0 f (D - B')}{fD + L_0 B'} = \frac{L_0 f (D - B')}{fD} = L_0 = L_{\text{hf}} \]

\[ fD + L_0 B' = 0 \]

\[ L_{\text{hf}} = \frac{fD}{B'} = -\frac{f^2}{(f / \#)B'} \]

Hyperfocal Focus Position \( f / \# = \frac{f}{D} \)

Where is \( L_{\text{NEAR}} \) when the system is focused at \( L_{\text{hf}} \)?

\[ L_{\text{NEAR}} = \frac{L_0 f (D + B')}{fD - L_0 B'} \]

\[ L_0 = L_{\text{hf}} = \frac{fD}{B'} \]

\[ L_{\text{NEAR}} = -\frac{fD^2 / B'}{fD + fD} \]

\[ L_{\text{NEAR}} = \frac{fD}{2B'} = \frac{L_0}{2} \]

The near focus object limit is approximately half the hyperfocal object distance.
Hyperfocal Distance and Depth of Focus

The detector is placed at the conjugate to the hyperfocal distance:

\[ L_{\text{HEAR}} = \frac{1 - B'/D}{f} \]

Of course, objects at infinity will focus at the rear focal point of the lens, but produce an acceptable blur on the detector.

The separation between the sensor and the rear focal point is given by the Depth of Focus.

Objects at the Near Point will focus a Depth of Focus behind the sensor, and will also produce an acceptable blur on the detector.

\[ \text{DOF} = \frac{B'}{\#} \]

\[ \text{DOF} \approx L'_{\text{HEAR}} \]

Hyperfocal Distance

If the camera were focused at infinity, the depth of field actually extends beyond infinity. Focusing at the hyperfocal distance maximizes the use of the available depth of field that includes infinity.
Hyperfocal Distance and Depth of Focus and Field

There are many assumptions in these Depth of Focus/Field calculations. The two most important are that there is no diffraction (Airy disc) and that there are no aberrations. Once again, a thin lens with the stop at the lens is assumed.

However, these results are very important as the limitations to system performance are often these first-order geometrical considerations:

- Depth of Focus - Film plane flatness
- Depth of Field - Focus precision
- Number of autofocus zones
- Artistic considerations
- Hyperfocal Distance - Why fixed-focus cameras work

Number of autofocus zones:
- The most distant zone will be from infinity to half the hyperfocal distance.
- The second zone extends from half the hyperfocal distance to its near point.
- The next zone starts at this near point, etc.
- There can be overlap between the zones.
- The object position only needs to be determined within a zone.
- The zones get shorter as the object distance get closer.

Example – The Fixed-Focus Camera

System specification: 35 mm film (24 x 36 mm)
  4R Print (4 x 6 inch or 100 x 150 mm)
  Maximum blur on the print is 0.006" (0.15 mm)
  Near focus is 4 ft (1200 mm)
  Focal length = 38 mm (sets angular FOV with film)

Print Magnification = 4X

\[ B' = \frac{.006''}{4} = \frac{.15 mm}{4} = .038 mm \]

\[ L_n = 2L_{NEAR} = -8 ft = -2438 mm \]

\[ L_n = -\frac{fD}{B'} \quad f = 38 mm \]

\[ D = 2.44 mm \]

\[ f / \# = f / D = f / 15.5 \]

The exposure is set by the shutter speed and the film speed (ISO).

Dividing the format size with the blur on the film provides about 632 x 947 effective “pixels.” This is approximately SVGA resolution.
Example – The Fixed-Focus Digital Phone Camera

System specification: Sensor 1/3.2” Format (4.54 x 3.42 mm)
Number of Pixels = 3264 x 2488 (8MP)
Pixel Size = 1.4 μm
Near focus is 4 ft (1200 mm)
Focal length = 4.8 mm (35 mm equivalent = 38 mm)

Set the image blur equal to twice the pixel size.

\[ B' = 2.8 \mu m = 0.0028 \text{ mm} \]
\[ L_H = 2L_{NEAR} = -8\text{ ft} = -2438 \text{ mm} \]

\[ L_H = \frac{fD}{B'} \quad f = 4.8 \text{ mm} \]
\[ D = 1.4 \text{ mm} \]
\[ f/\# = f/D = f/2.9 \]

Most camera phones seem to operate at f/2.5 to f/2.2.

Depth of Field and F/\#

As a lens of a given focal length is stopped down, the depth of field increases with the increased depth of focus.

As the f/\# of a lens increases (the lens is stopped down), the hyperfocal distance also moves closer to the lens:

\[ L_H = -\frac{fD}{B'} = -\frac{f^2}{B'f/\#} \quad L_{NEAR} = \frac{L_H}{2} = -\frac{f^2}{2B'f/\#} \]
Depth of Field and F/#

f/32

f/5.6

Depth of Field and F/#

f/22 - small aperture
Deep Depth of Field

f/2.8 - large aperture
Shallow Depth of Field

wikipedia
Scheimpflug Condition

First-order optical systems image points to points, lines to lines, and planes to planes. This condition holds even if the line or plane is not perpendicular to the optical axis. The Scheimpflug condition states that a tilted plane images to another tilted plane, and for a thin lens, the line of intersection lies in the plane of the lens.

Proof: Consider any ray in the plane of the object. The conjugate to this ray must lie in the plane of the image. The intersection point of this object ray and this image ray is the refraction point in the plane of the lens. The object plane and the image plane must also intersect at this same point. By extension, the intersection line between the object plane and the image plane also lies in the plane of the lens.

This condition easily extends to a thick lens or system: the line of intersection is coincident in the front and rear principal planes of the system.

Even though the image is in focus, it will exhibit keystone distortion as the lateral magnification varies along the tilted object.
Scheimpflug Condition – For Architectural Photography

If the object plane is vertical, the optical system (camera) and detector are tilted to achieve the Scheimpflug Condition:

In the image, the top of the building will have a smaller magnification than the bottom of the building.

A square or rectangular window on the building will show keystone distortion and be imaged as a trapezoid.

Zoom Lenses

A zoom lens is a variable focal length objective with a fixed image plane. The simplest example consists of two lens elements or groups (powers $\phi_1$ and $\phi_2$) where both the system focal length $f$ and BFD vary with element spacing $t$.

$$\phi = \frac{1}{f} = \phi_1 + \phi_2 - \phi_1 \phi_2 f$$

$$BFD = f + d' = f - \frac{\phi_1 t}{\phi}$$

This type of lens is also called a varifocal lens. In order to maintain a fixed image plane with a variable focal length:
- Vary the element spacing to change the system focal length.
- Move the pair of elements relative to the image plane to maintain focus.
Telephoto Zoom

The element positions and spacing for a telephoto zoom are plotted as a function of system focal length:

As the element separation approaches the sum of the element focal lengths \((f_1 + f_2)\), the system focal length approaches infinity \((f \rightarrow \infty)\).

The short end of the zoom range of this configuration is limited by the BFD as the rear element runs into the image plane when the element separation approaches \(f_1\).

Reverse Telephoto Lens

The element positions and spacing for a reverse telephoto zoom are plotted as a function of system focal length:

As the element separation approaches the sum of the element focal lengths \((f_1 + f_2)\), the system focal length approaches infinity \((f \rightarrow \infty)\).

This configuration does not have the BFD issue of the telephoto zoom. Of the two configurations, the reverse telephoto zoom is the more commonly used.
Mechanically Compensated Zoom Lenses

A mechanical cam provides the complicated lens motions required for these mechanically compensated zoom lenses. Zoom lenses often use multiple groups of moving elements.

A common three group configuration uses a fixed front element and moving second and third groups. The fixed element provides the bulk of the system power, and the two moving elements vary the system power and maintain the image plane.

Optically Compensated Zoom Lenses

These lenses consist of a number of spaced elements, and they are often alternated positive/negative. The focal length of the system is changed by linking alternate elements and moving these elements relative to the other elements.

This lens does not need a complicated cam, but is generally significantly longer than a mechanically compensated system. Early zooms used this configuration as the mechanical cams could not be made to the required precision or were prone to wear. Numerically controlled machining techniques now allow cams to be easily fabricated, and an optically compensated zoom is almost never used today.