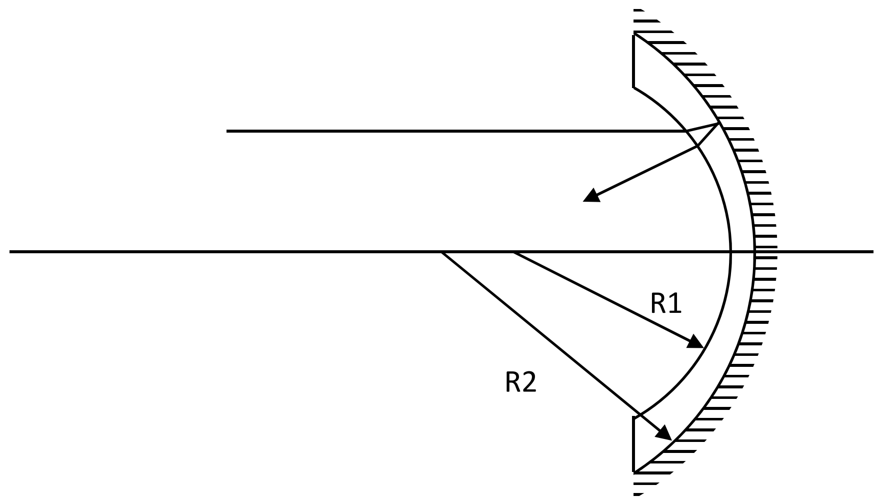
Mangin Mirror

A Mangin mirror consists of a second surface spherical reflector, with a different first surface curvature. (See figure):



Specifications: R1 = 100.0 R2 = 150.0 t = 10.0 n = 1.5

a) Do a paraxial ray trace of this Mangin mirror to determine the effective focal length, rear principal plane location, and back focal distance.

As usual, be sure to apply the reference definitions and sign conventions. Note that the refracting surface is used twice.

b) Where is the front principal plane, and where is the front focal point?

Solution:

Be careful with the sign conventions. As measured relative to the surface vertices, both radii are negative.

The air glass interface is used twice.

Ray from infinity:

Surface	0		1		2		3		4		5		6
r			-100		-150		-100						
n		1.0		1.5		-1.5		-1.0					
t		∞		10		-10		?					
$-\phi$			0.005		-0.02		0.005						
t/n		∞		6.66		6.66		84.45					
y	1		1		1.0333		0.929		0				
nu		0		0.005		-0.01567		-0.0110					
u													

Surface	0		1		2		3		4		5		6
y													
nu													
u													

The system rear vertex is the glass/air interface (used the second time).

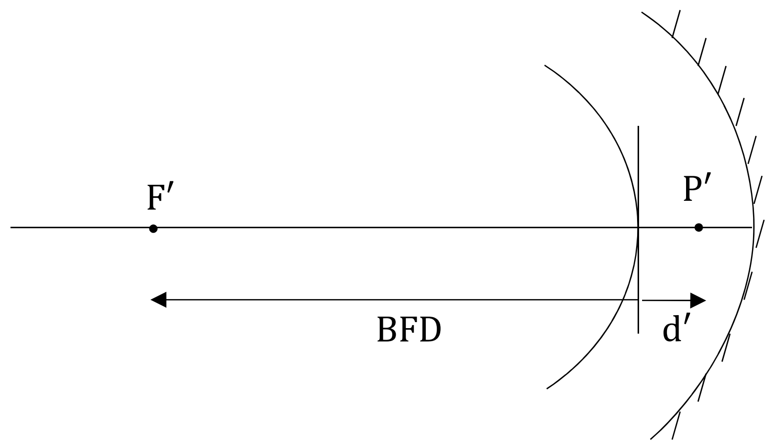
$$\omega' = -0.0110 \qquad \frac{V'F'}{n'} = \frac{BFD}{n'} = 84.45$$

$$\phi = -\frac{\omega'}{y_1} = 0.0110 \qquad f_E = \frac{1}{\phi} = 90.91$$

$$n' = -1$$

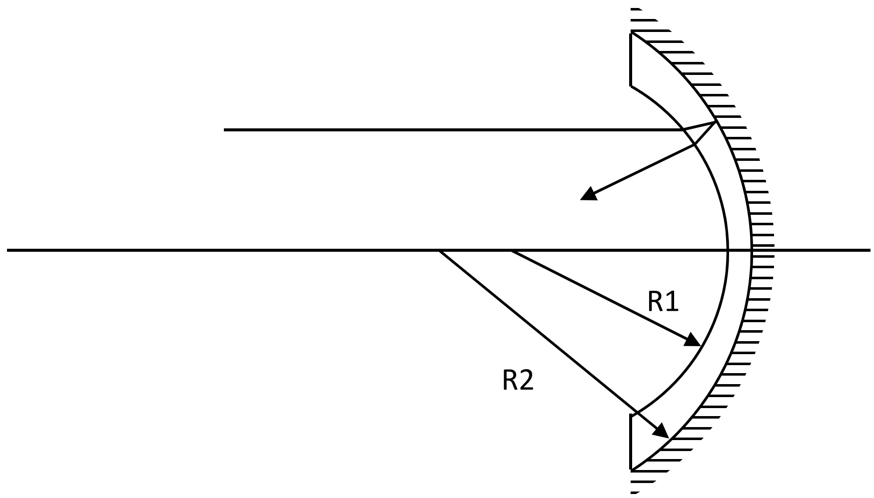
$$BFD = -84.45 \qquad f_R' = \frac{n'}{\phi} = n' f_E = -90.91$$

$$d' = BFD - f' = -84.45 + 90.91 = 6.46$$



b) Since an image space ray parallel to the axis is the same as an object space ray parallel to the axis, P and F are at the same locations as P′ and F′.

Bonus Solution - Gaussian Reduction



Specifications: R1 = 100.0 R2 = 150.0 t = 10.0 n = 1.5

$n_1 = 1.0$	$n_2 = 1.5$	$n_3 = -1.5$	$n_4 = -1$

$R_1 = -100\,\text{mm}$ $\phi_1 = \frac{n_2 - n_1}{R_1}$ $\phi_1 = \frac{1.5 - 1.0}{-100}$ $\phi_1 = -0.005\text{mm}^{-1}$	$R_2 = -150\,\text{mm}$ $\phi_2 = \frac{n_3 - n_2}{R_2}$ $\phi_2 = \frac{-1.5 - 1.5}{-150}$ $\phi_2 = 0.02\,\text{mm}^{-1}$	$R_3 = R_1 = -100\,\text{mm}$ $\phi_3 = \frac{n_4 - n_3}{R_3}$ $\phi_3 = \frac{-1 - (-1.5)}{-100}$ $\phi_3 = -0.005\,\text{mm}^{-1}$

$t'_1 = 10\,\text{mm}$	$t'_2 = -10\,\text{mm}$
$\tau'_1 = \frac{t'_1}{n_2} = \frac{10}{1.5} = 6.66\,\text{mm}$	$\tau'_2 = \frac{t'_2}{n_3} = \frac{-10}{-1.5} = 6.66\,\text{mm}$

First, reduce the first two surfaces

$$\phi_{12} = \phi_1 + \phi_2 - \phi_1\phi_2\tau_1'$$

$$\phi_{12} = -0.005 + 0.02 - ((-0.005)(0.02)10)$$

$$\phi_{12} = 0.0156\text{mm}^{-1}$$

$$d'_{12} = n_3\delta'_{12} = n_3\left(-\frac{\phi_1}{\phi_{12}}\right)\tau_1'$$

$$d'_{12} = (-1.5)\left(-\frac{-0.005}{0.0156}\right)6.66$$

$$d'_{12} = -3.1884\,\text{mm}$$

$$t'_{12} = t'_2 - d'_{12} = -10 - (-3.1884)$$

$$t'_{12} = -6.8115\,\text{mm}$$

$$\tau'_{12} = \frac{t'_{12}}{n_3} = \frac{-6.8115}{-1.5}$$

$$\tau'_{12} = 4.5410\text{mm}$$

Add the third surface:

$$\phi = \phi_{12} + \phi_3 - \phi_{12}\phi_3\tau_{12}'$$

$$\phi = 0.0156 + (-0.005) - ((0.0156)(-0.005)(4.5410))$$

$$\phi = 0.011\text{mm}^{-1}$$

$$\boxed{f_E = \frac{1}{\phi} = 90.91\text{ mm}}$$

$$d'_{123} = n_4\delta'_{123} = n_4\left(-\frac{\phi_{12}}{\phi}\right)\tau_{12}'$$

$$d'_{123} = n_4 \delta'_{123} = -1\left(-\frac{0.0156}{0.011}\right)4.5410$$

$$\boxed{d'_{123} = 6.46\text{mm}}$$

This distance is measured from the third surface, which is the concave glass/air interface (the second time it is used).