

October 19, 2016 Lecture 17

Name SOLUTIONS

Closed book; closed notes. Time limit: 75 minutes.

An equation sheet is attached and can be removed. A spare raytrace sheet is also attached.

Use the back sides if required.

Assume thin lenses in air if not specified.

If a method of solution is specified in the problem, that method must be used.

Raytraces must be done on the raytrace form. Be sure to indicate the initial conditions for your rays.

You must show your work and/or method of solution in order to receive credit or partial credit for your answer.

Provide your answers in a neat and orderly fashion. No credit if it can't be read/followed.

Use a ruler or straight edge!

Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.

Note: On some quantities, only the magnitude of the quantity is provided. The proper sign conventions and reference definitions must be applied.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (10 points) A 2 m tall zebra is to be imaged onto a 1 cm detector. The zebra is about 30 m away, and the image of the zebra completely fills the detector. Approximately what focal length lens is required?

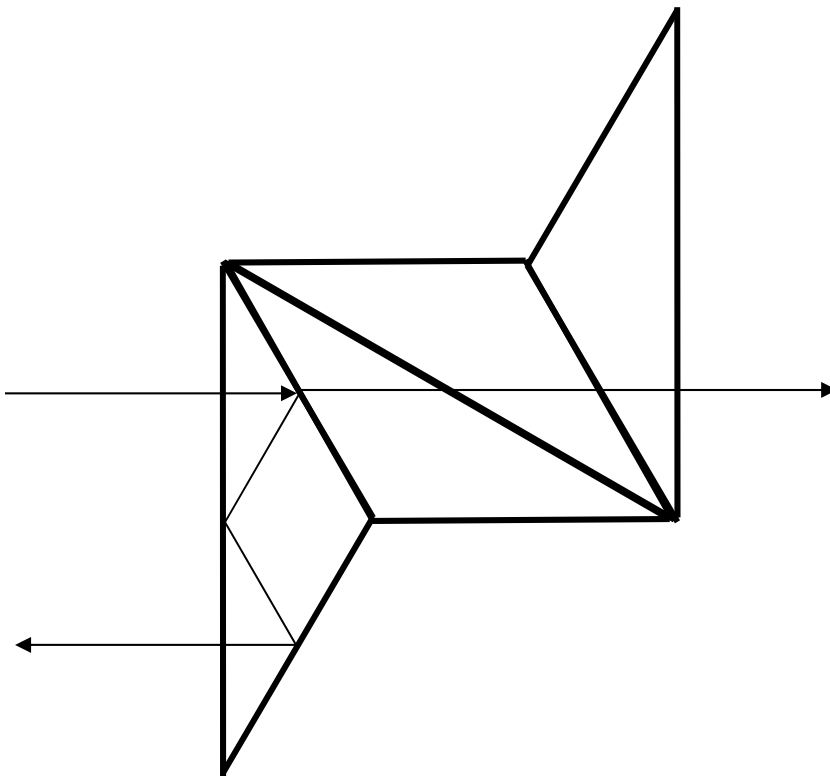
$$m = \frac{h'}{h} = \frac{-10 \text{ mm}}{2,000 \text{ mm}} = -0.005 \quad m = \frac{z'}{z} \approx \frac{f}{z} \quad f \approx mz = (-0.005)(-30,000 \text{ mm}) = 150 \text{ mm}$$

or

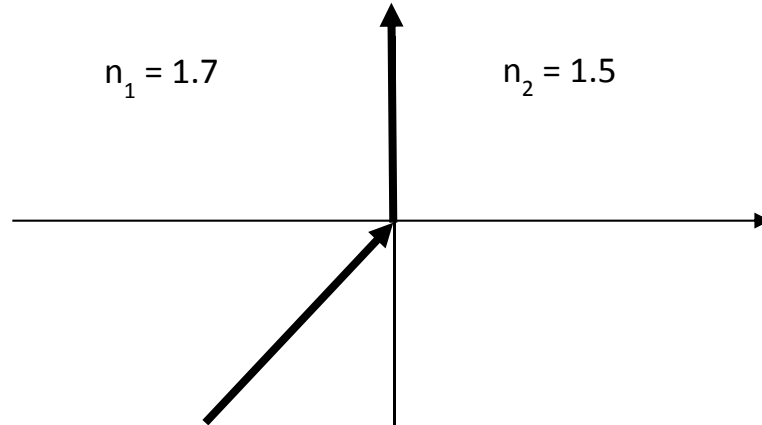
$$m = 200 : 1 \quad f \approx \frac{z}{200} = \frac{30 \text{ m}}{200} = \frac{30,000 \text{ mm}}{200} = 150 \text{ mm}$$

Focal Length \approx 150 mm

2) (10 points) Draw the tunnel diagram for this prism and the ray path shown.



3) (10 points) A flat refractive surface separates indices of refraction of 1.7 and 1.5. What is the critical angle for total internal reflection associated with this surface?



$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

For the critical angle $\theta_2 = 90 \text{ deg}$

$$n_1 \sin \theta_c = n_2$$

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.5}{1.7} = 0.8824$$

$$\theta_c = 61.9 \text{ deg}$$

Critical Angle = 61.9 deg

4) (25 points) An imaging system is comprised of two thin lenses in air. The focal length of the first lens is 40 mm, and the focal length of the second lens is 60 mm. The lenses are separated by 25 mm. The system is to be used in conjunction with a cube of glass with an index of refraction of 1.6. The object is 100 mm to the left of the first thin lens. What is the size of the largest glass cube that will fit between the second lens of this system and the image plane? Consider that the image is formed on the rear surface of the cube, and the cube is in contact with the second thin lens.

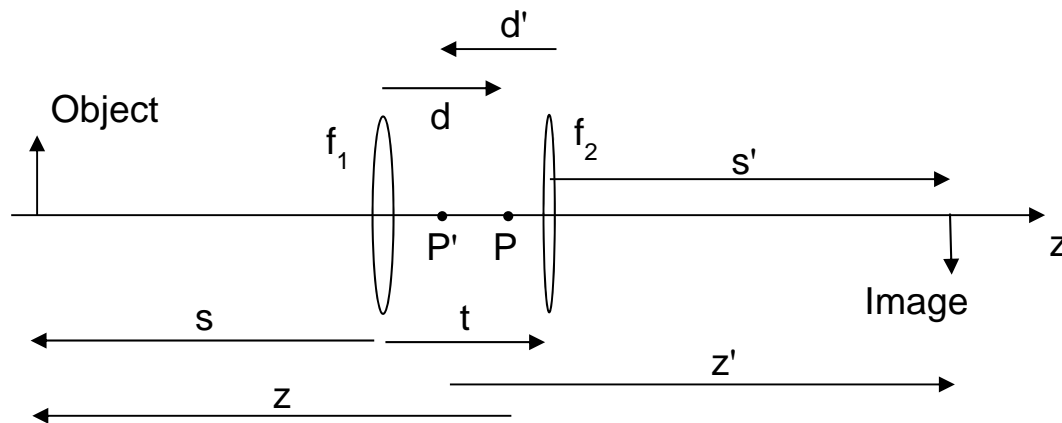
Glass Cube: Index of refraction = 1.6

System: A 40 mm focal length thin lens and a 60 mm focal length thin lens separated by 25 mm

Object: The object is 100 mm to the left of the first thin lens.

NOTE: Use Gaussian Reduction and Gaussian Imaging for this problem. Cascaded imaging may not be used (you may not image through one lens and then use this image as an object for the other lens).

First consider the optical system without the glass blocks:



$$s = -100\text{mm} \quad t = 25\text{mm}$$

$$f_1 = 40\text{mm} \quad f_2 = 60\text{mm} \quad \phi_1 = 0.025 / \text{mm} \quad \phi_2 = 0.01667 / \text{mm}$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2t \quad \phi = 0.03126 / \text{mm}$$

$$f = 32.0\text{mm}$$

Continues...

$$d' = -\frac{\phi_1}{\phi} t = -20.0mm$$

$$d = \frac{\phi_2}{\phi} t = 13.33mm$$

$$z = s - d = -113.33mm$$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$z' = 44.59mm$$

z' is the distance from the rear principal plane P' to the image plane.

s' is the distance from the second lens to the image plane.

$$s' = z' + d' = 24.59mm$$

Glass Cube Size – the reduced thickness of the cube must equal the image distance s' for the largest possible cube.

$L =$ Cube Dimension

$$n = 1.6$$

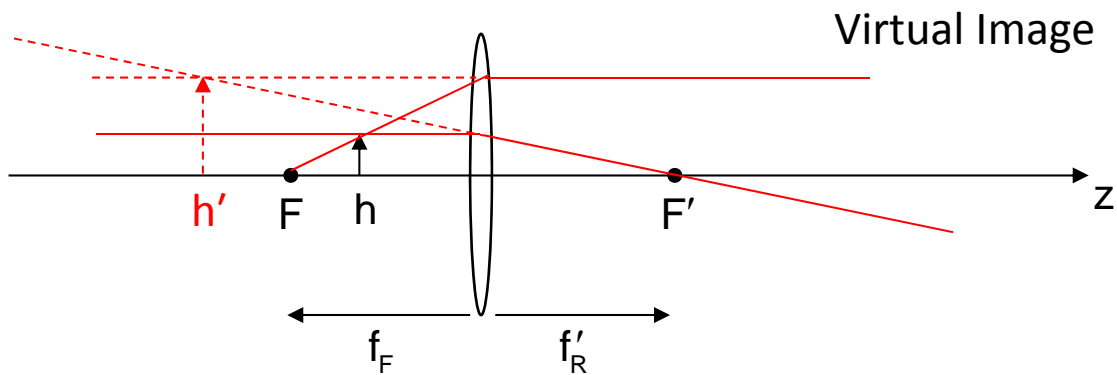
$$\frac{L}{n} = s'$$

$$L = 39.34mm$$

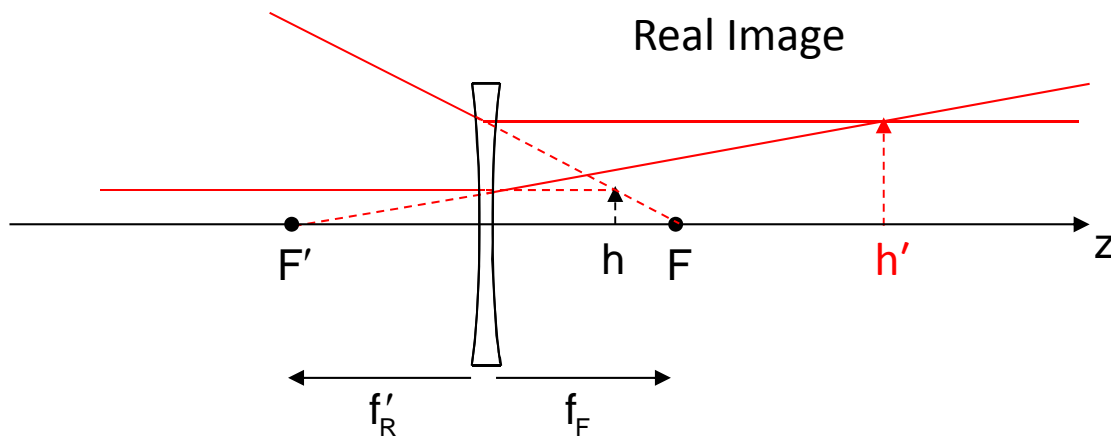
Maximum Glass Cube Dimension = 39.34 mm

5) (10 points) Two imaging configurations are given, each showing an object. Determine the image location and size by using a ray construction. Both real and virtual objects are shown. Indicate if the image is real or virtual. Rays must be drawn using a straight edge.

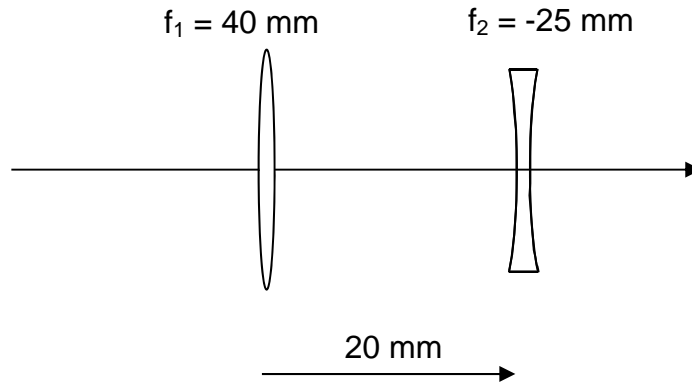
Positive Thin Lens – Real Object



Negative Thin Lens – Virtual Object



6) (20 points) An optical system in air is comprised of two thin lenses:



An object is placed 300 mm to the left of the first lens. The object size is ± 10 mm. Use paraxial raytrace methods to determine the system focal length and the location and size of the image.

Determine:

- System Focal Length
- Back Focal Distance
- Front Focal Distance
- Image Location and Size

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the image size must be determined from a raytrace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Gaussian imaging methods may not be used for any portion of this problem.

System Focal Length = 200 mm

Back Focal Distance = 100 mm

Front Focal Distance = -360 mm

Image Location = 566.7 mm to the Left of the second lens (*Virtual*)

Image Size = +/- 33.33 mm (*Erect and magnified*)

A total of four rays must be traced:

- Forward infinity ray – parallel to the axis in object space
- Reverse infinity ray – parallel to the axis in image space
- Ray from the axial object location: $y = 0$ at object location
- Ray from the top of the object: $h = 10$ mm

The distance from the object to the first lens is 300 mm.

	F or Object	f_1	f_2	F' or Image					
Surface	0	1	2	3	4	5	6		
f		40	-25						
$-\phi$		-.025	.040						
t			20						
				100					
π_i	y	1	1	0.5	0				
	u	0	-.025	-.005					
	y	0	1.80	1	1				
	u	.005	-.040	0					
	y	0	300						
	u	0.1*	-.65	.03					
	y	10	10	5	33.33				
	u	0*	-.25	-.05					
	y								
	u								
	y								
	u								

* arbitrary

Method of Solution:

From the forward infinity ray – crosses the axis at F':

$$\overline{f_2 F'} = BFD = 100mm$$

$$u' = -0.005 \quad \phi = -\frac{y_0}{u'} = 0.005 / mm$$

$$f = \frac{1}{\phi} = 200mm$$

From the reverse infinity ray – crosses the axis at F:

$$\overline{F f_1} = -FFD = 360mm \quad FFD = -360mm$$

$$u = 0.005 \quad \phi = \frac{y'}{u} = 0.005 / mm$$

$$f = \frac{1}{\phi} = 200mm$$

Using the rays defining the object location and object height. The axial ray crosses the axis at the image location (s'). The ray from the top of the object determines the image height (h').

The image is 566.7mm to the left of the second lens.

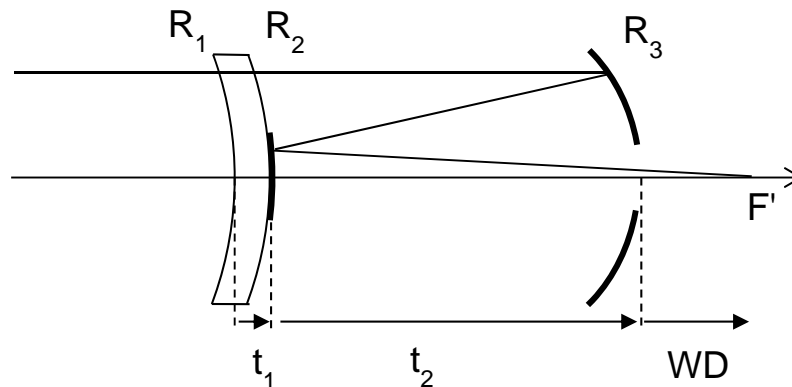
The image height is 33.33mm. The image is enlarged and erect.

The image is virtual and the system is operating as a magnifier.

7) (15 points) In the very first homework problem of the semester, we found by using Fermat's Principle that the proper mirror shape to image a distant point source (star) was a parabola. Unfortunately, parabolas are more difficult to polish than spheres.

An all-spherical alternative design is the Maksutov-Cassegrain. A thin meniscus lens is used with a spherical primary mirror to correct the aberrations of the telescope. You do not need to know any of these details for this problem, however!

To simplify fabrication, the secondary mirror is an aluminized patch on the rear surface of this meniscus corrector lens. The secondary mirror therefore has the same radius of curvature as the second surface of the lens. Assume the coating for the secondary mirror has zero thickness.



Specifications:

$R_1 = 140.0$ mm	
$t_1 = 10.0$ mm	$n_1 = 1.5$
$R_2 = 150.0$ mm	
$t_2 = 200.0$ mm	
$R_3 = 500.0$ mm	

The surface curvatures are diagrammatically correct in the figure. Apply the proper sign conventions to all quantities.

Determine the System Focal Length and the Working Distance (WD). The working distance is measured from the vertex plane of the primary mirror (R_3).

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from quantities derived from the rays you trace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Gaussian imaging methods may not be used for any portion of this problem.

A sizeable portion of the credit for this problem is properly setting up the raytrace sheet.

All of the surfaces have negative radii of curvature.

$$C_1 = -1/140.0mm = -0.007143 / mm$$

$$C_2 = -1/150.0mm = -0.006667 / mm$$

$$C_3 = -1/480.0mm = -0.002083 / mm$$

Only a single ray needs to be traced: $y_0 = 1mm$ $u_0 = 0$

F'

Surface	0	1	2	3	4	5	6
C		-0.007143	-0.006667	-0.002083	-0.006667	-	
t		10	200	-200	200	200	∞
n		1.0	1.5	1.0	-1.0	1.0	1.0

-φ		.003572	-.003333	-.004080	.013333	-	
t/n		6.6667	200	200	200	200	95.7

y	1	1.02381	1.05574	.2431	.07867	0	
nu	0	.003572	.000596	-.004083	-.000823	-.000823	
u							

y							
nu							
u							

y							
nu							
u							

Method of Solution:

A “dummy” surface can be placed 200mm to the right of the secondary mirror at the physical location of the primary mirror vertex. This allows the Working Distance to be computed directly.

From the raytrace:

$$u' = \omega' = -0.0008221$$

$$\phi = -\frac{y_1}{\omega'} = 0.0008221 / mm$$

$$f = \frac{1}{\phi} = 1216mm$$

The Working Distance is the distance from the vertex of the primary mirror (the dummy surface) to the rear focal point F' .

$$WD = 95.7mm$$

In the dummy surface is not used, the raytrace sheet gives the distance from the secondary mirror (R_2) to the rear focal point F' :

$$\overline{R_2 F'} = 295.7mm$$

$$WD = \overline{R_2 F'} - t_2 = 295.7mm - 200mm$$

$$WD = 95.7mm$$

Focal Length = 1216 mm

Working Distance = 95.7 m