Name_____**SOLUTIONS**______

Closed book; closed notes. Time limit: 120 minutes.
An equation sheet is attached and can be removed. A spare raytrace sheet is also attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
As usual, only the magnitude of a magnification or magnifying power may be given.
If a method of solution is specified in the problem, that method must be used.
Raytraces must be done on the raytrace form. Be sure to indicate the initial conditions for your rays.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Provide your answers in a neat and orderly fashion.
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.
Note: On some quantities, only the magnitude of the quantity is provided. The proper sign convention must be applied.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (5 points) A 100 mm focal length thin lens (in air) in made out of glass N-LaSF46A. The glass code for this glass is 904313. What is the longitudinal chromatic aberration of this lens?

From the glass code: $n = 1.904 \qquad \nu = 31.3$

$$\frac{\delta f}{f} = \frac{1}{\nu} \qquad \delta f = \frac{f}{\nu} \qquad f = 100mm$$

$$\delta f = \frac{100mm}{31.3} = 3.19mm$$

Longitudinal Chromatic Aberration = δf = __3.19__mm

2) (10 points) You are riding in the passenger seat of a car traveling at 50 km/hour. You look out the passenger window and notice that objects close to the road appear to be "moving" faster than more distant objects. In fact, the mountains in the distance appear to be stationary.

As an experiment, you take a picture out the open passenger window. The optical axis of the camera is perpendicular to the direction of motion of the car. A slow f/# is used, so depth of field is not an issue, and any recorded image blur will be due only to the motion of the car. You use a shutter speed of 0.01 sec.

The focal length of the camera lens is 50 mm, and the pixel size on the detector is 10 μm.

At what distance from the car will the motion blur equal the pixel size? Use and note reasonable assumptions.

From the reference frame of the car, the world is moving backwards at a speed of 50 km/hour or

$$50 km/hour \frac{1000m}{km}\frac{1hour}{60\,min}\frac{1min}{60\,sec} = 13.88 m/sec$$

During an exposure time of 0.01 sec, the world will "move" by

$$Blur_{WORLD} = 13.88 m/sec\left(0.01 sec\right) = 0.1388 m = 133.8 mm$$

This is the object blur and it is independent of object distance. On the detector, this object blur is is reduced by the magnification of the camera lens. The image distance is approximately the focal length f. The blur retuirement is met when the image blur equals the pixel size of 10 μm (0.010 mm).

$$|m| = \frac{|z'|}{|z|} \approx \frac{f}{|z|} \qquad f = 50mm \qquad Blur_{IMAGE} = |m|\, Blur_{OBJECT} = \frac{f}{|z|} Blur_{OBJECT}$$

$$|m| = \frac{Blur_{IMAGE}}{Blur_{OBJECT}} = \frac{0.010mm}{133.8mm} = 0.0000747$$

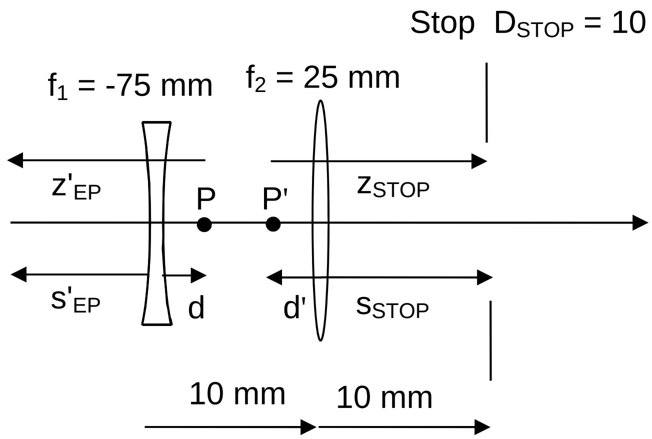
$$|m| \approx \frac{f}{|z|} = \frac{50mm}{|z|} = 0.0000747$$

$$|z| = 694m$$

Objects at a distance of greater than 694 m will not show motion blur. Of course the motion blur decreases with distance, so this is not an abrupt transition.

Distance = ____694_____ m

3) (15 points) A 10 mm diameter stop is located to the right of an optical system comprised of two thin lenses in air as shown:



Determine the entrance pupil location and diameter. The entrance pupil is to be located relative to the first lens.

NOTE: Use Gaussian Reduction and Gaussian Imaging for this problem. Cascaded imaging may not be used (you may not image through one lens and then use this image as an object for the other lens).

Reduce the front group of elements:

$$\phi_1 = 1/f_1 = -0.01333/mm \qquad \phi_2 = 1/f_2 = 0.04/mm$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t \qquad t = 10mm$$

$$\phi = 0.0320/mm \qquad f = 1/\phi = 31.25mm$$

$$d = \frac{\phi_2}{\phi}t = \frac{0.040/mm}{0.0320/mm}10mm \qquad d' = -\frac{\phi_1}{\phi}t = \frac{0.01333/mm}{0.0320/mm}10mm$$

$$d = 12.50mm \qquad d' = 4.167mm$$

Note that both Principal Planes are to the right of the lens system as would be expected for a reverse telephoto lens.

Continues…

The EP is found by imaging the real stop through the front group. The object distance is measured from the rear principal plane of the front group.

The light is traveling from Right to Left for this imaging so $n = n' = -1$

$$z_{STOP} = s_{STOP} - d' = 10mm - 4.167mm = 5.833mm$$

$$\frac{-1}{z'_{EP}} = \frac{-1}{z_{STOP}} + \phi = \frac{-1}{5.833mm} + 0.0320 / mm$$

$$z'_{EP} = 7.172mm \qquad \text{from P of the system.}$$

EP Size:

$$m_{EP} = \frac{z'_{EP} / n'}{z_{STOP} / n} = \frac{7.172mm}{5.833mm} = 1.230 \qquad D_{STOP} = 10mm$$

$$D_{EP} = m_{EP} D_{STOP} = 12.30mm$$

Determine the EP location relative to the first lens:

$$s'_{EP} = z'_{EP} + d = 7.172mm + 12.50mm$$

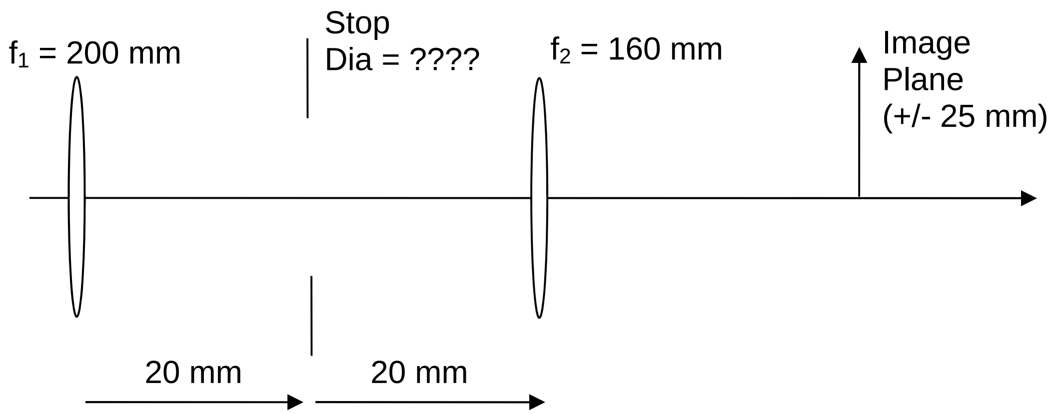
$$s'_{EP} = 19.67mm$$

The EP is 19.67 mm to the Right of the first lens element.

EP: D_{EP} = ___12.30_ mm; Located __19.67__ mm to the __Right__ of the first lens.

4) (25 points) The following diagram shows the design of an objective that is comprised of two thin lenses in air. The system stop is located between the two lenses.

The system operates at f/4.
The object is at infinity.
The maximum image size is +/- 25 mm.



Determine the following:
- Entrance pupil and exit pupil locations and sizes.
- System focal length and back focal distance.
- Stop diameter.
- Angular field of view (in object space).
- Required diameters for the two lenses for the system to be unvignetted over the specified maximum image size.

NOTE: This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem. The field of view must be determined from the chief ray.

Be sure to clearly label your rays on the raytrace form. Your answers must be entered below. Be sure to provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.).

Entrance Pupil: __22.22__ mm to the ___R___ of the first lens. D_{EP} = ___25.0__ mm

Exit Pupil: __22.86__ mm to the ___L___ of the second lens. D_{XP} = __25.72__ mm

System Focal Length = ___100___ mm Back Focal Distance = ___80____mm

Stop Diameter = ____22.5__ mm FOV = +/- __14.0__ deg in object space

Lens 1 Diameter = __36.1___ mm Lens 2 Diameter = ___31.1___ mm

													F′
	Obj		EP		L1		Stop		L2		XP		Image
Surface	0		1		2		3		4		5		6
f			-		200		—		160		—		
$-\phi$					-.005		-		-.00625		-		
t				(-22.222)		20		20		(-22.857)		102.86	
Potential Chief Ray													
$\tilde{\bar{y}}$			0		-2.0		0		2.0		0		
$\tilde{\bar{u}}$				.09		.1 *		.1 *		.0875			
Potential Marginal Ray (102.86)													
$\tilde{y}$	1 *		1		1		0.9		0.8		1.0286		0
$\tilde{u}$		0		0		-.005		-.005		-.01		-.01	
Potential Chief Ray - Extended													
$\tilde{\bar{y}}$			0		-2.0		0		2.0		0		9.00
$\tilde{\bar{u}}$				.09		.1		.1		.0875		.0875	
Marginal Ray - Scale Factor = 12.5													
y	12.5		12.5		12.5		11.25		10.0		12.86		0
u		0		0		-.0625		-.0625		-.125		-.125	
Chief Ray - Scale Factor = 2.778													
$\bar{y}$			0		-5.56		0		5.56		0		25.0
$\bar{u}$				.250		.2778		.2778		.2431		.2431	
y													
u													
y													
u													

* arbitrary

Continues…

Provide Method of Solution:

EP/XP Location:

Trace a potential chief ray that starts at the center of the stop. The pupils are located where this ray crossed the axis in object/image space.

$$L_1 \rightarrow EP = 22.22mm \quad \text{(Right of L1)}$$
$$L_2 \rightarrow XP = -22.86mm \quad \text{(Left of L2)}$$

Focal Length:

Trace a potential marginal ray parallel to the axis in object space ($\tilde{y}_1 = 1$). The rear focal point is located where this ray crossed the axis.

$$XP \rightarrow F' = 102.86mm$$

$$BFD = \left(L_2 \rightarrow XP\right) + \left(XP \rightarrow F'\right) = -22.86mm + 102.86mm = 80.0mm$$

$$BFD = 80.0mm$$

$$\phi = -\frac{\tilde{u}'}{\tilde{y}_1} \qquad \tilde{u}' = -0.010 \qquad \tilde{y}_1 = 1$$

$$\phi = 0.010 / mm$$

$$f = \frac{1}{\phi} = 100mm$$

Extend the potential chief ray to the image plane F'

Entrance Pupil:

$$f/\# = f/4 = f/D_{EP}$$

$$D_{EP} = \frac{100mm}{4} = 25.0mm$$

Continues…

Provide Method of Solution:

Pupil/Stop Sizes: $r_{EP} = D_{EP} / 2 = 12.5mm$

Scale the marginal ray to the proper r_{EP}:

$$\text{Scale Factor } = \frac{12.5mm}{1.0mm} = 12.5$$

$$r_{STOP} = 11.25mm \qquad r_{XP} = 12.86mm$$

$$D_{STOP} = 22.5mm \qquad D_{XP} = 25.72mm$$

FOV: Scale the potential chief ray to the desired image height of 25.0 mm (from the current or potential chief ray value of 9.0mm)

$$\text{Scale Factor } = \frac{25.0mm}{9.0mm} = 2.778$$

Object Space Chief Ray:

$$\bar{u}_0 = 0.250$$

$$HFOV = \tan^{-1}\left(0.250\right) = 14.0°$$

$$FOV = 28.0° \text{ or } \pm 14.0°$$

Vignetting:

L1: $y_1 = 12.5mm \qquad \bar{y}_1 = -5.56mm$

$$a_1 = |y_1| + |\bar{y}_1| = 18.06mm$$

$$D_1 = 36.1mm$$

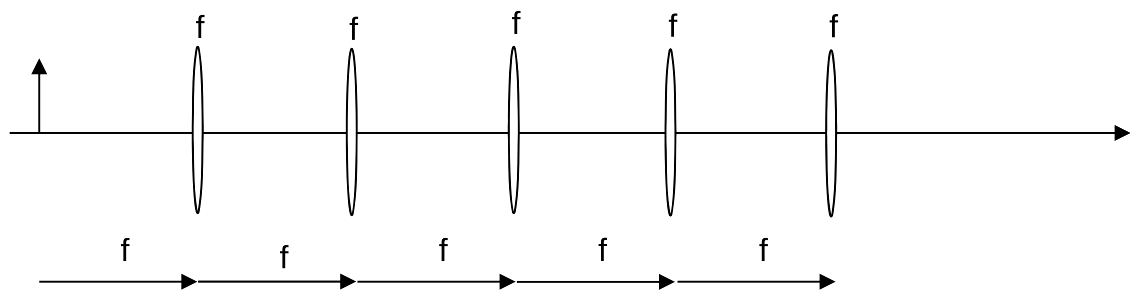
L2: $y_2 = 10.0mm \qquad \bar{y}_2 = 5.56mm$

$$a_2 = |y_2| + |\bar{y}_2| = 15.56mm$$

$$D_2 = 31.1mm$$

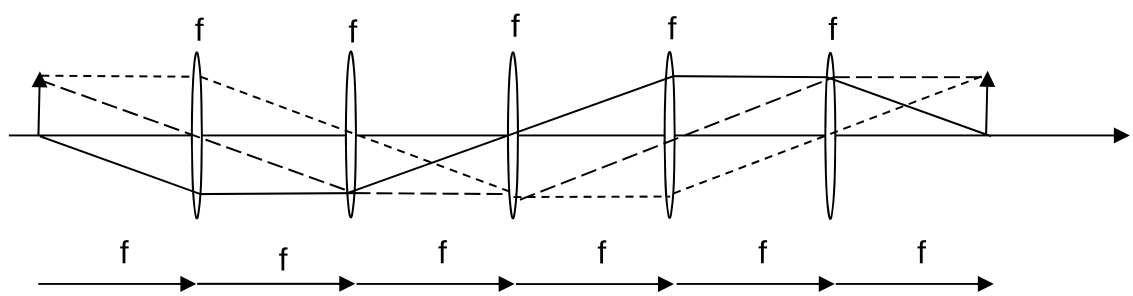
5) (10 points) Consider the following optical system comprised of five identical thin lenses of focal length f that are each separated by this same distance f.

An object is located at the front focal point of the first lens element. Determine the image location and size by sketching rays. Please use a straightedge. No calculations are required or permitted.



Sketch rays starting at the axial object location and from the top of the object. Make use of the properties of focal points. There are two possible rays from the top of the object.

Two of the three rays shown must be used.



The image is at the rear focal point of the final lens element.

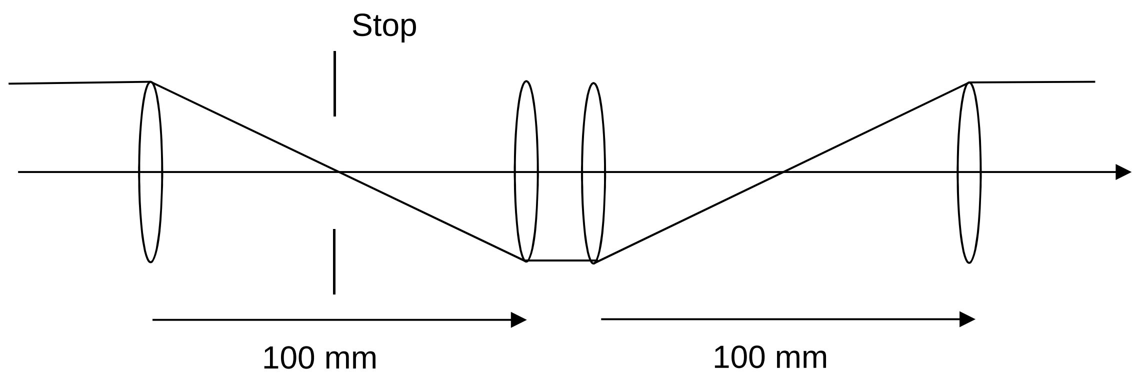
The system operates at a true 1:1 magnification. The image is at the rear focal point of the final lens. The rear focal point of the system is located at the final lens element.

6) (10 points) Using only 50 mm focal length thin lenses, provide the layout of a double-telecentric system with a lateral magnification of +1.0. You must use four of these thin lenses in your design. Provide a sketch of the system clearly indicating the spacings of the lenses and the location of the system stop.

Note: The system magnification must be POSITIVE.
The lens diameters are not required.

A double-telecentric system must be afocal. Two of the 50 mm focal length lenses separated by 100 mm will result in an afocal system with a magnification of -1.0.

Combining or cascading two of these afocal systems will result in an afocal system with a magnification of +1.0. It will also utilize the required four lens elements.



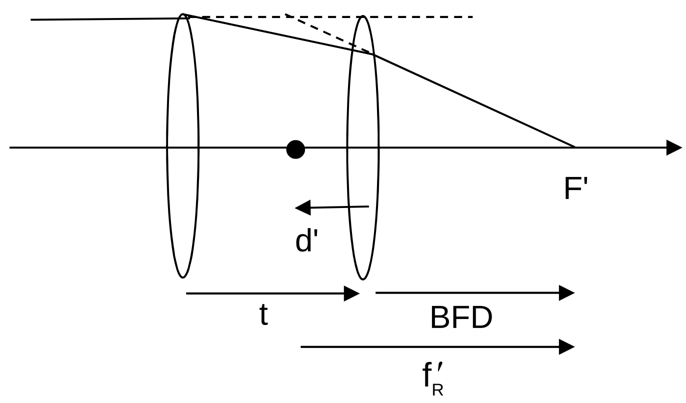
In order to make the system telecentric, the stop must be placed at eaither the rear focal point of the first lens or the front focal point of the fourth lens.

The separation between the second and third lenses is arbitrary. It does not affect the system magnification, stop location or telecentricity. This separation would however affect the conjugate locations associated with an object that would be imaged by this system.

This final system can also be thought of as a single afocal system (first and fourth lenses) with the addition of a 1:1 relay ($m_R = -1.0$). The net magnification of the system is then positive.

Other solutions are possible.

7) (10 points) Design a thin-lens Petzval objective with the following specifications:
Separation of the two elements = 50 mm
Focal length = 100 mm
Back focal distance = 75 mm



$$BFD = 75mm = d' + f'_R = d' + f \qquad f = 100mm$$

$$d' = -25mm$$

$$d' = -\frac{\phi_1}{\phi}t \qquad \phi = \frac{1}{f} = 0.01/mm \qquad t = 50mm$$

$$\phi_1 = 0.005/mm$$

$$f_1 = 200mm$$

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t = 0.01/mm$$

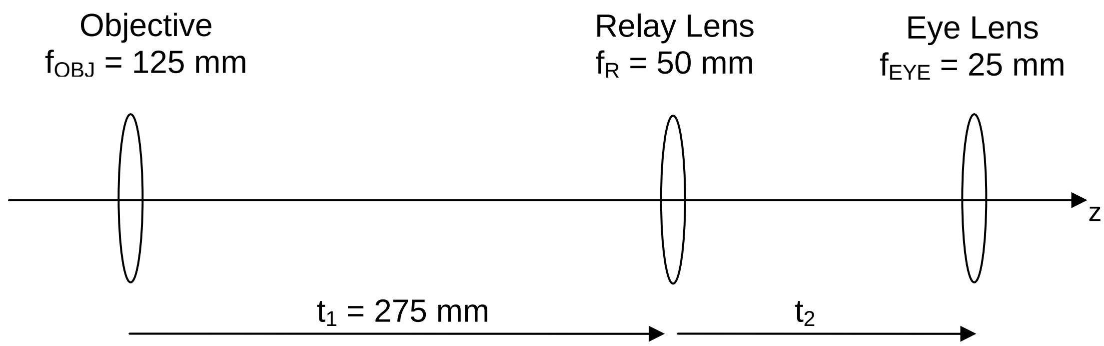
$$\phi_2 = 0.00667/mm$$

$$f_2 = 150mm$$

The rear Principal Plane of the system is located between the two lens elements.

f1 = ___200_ mm f2 = __150__ mm

8) (15 points) A relayed Keplarian telescope is constructed with three thin lenses in air as shown. The objective lens serves as the system stop. The stop diameter is 25 mm.

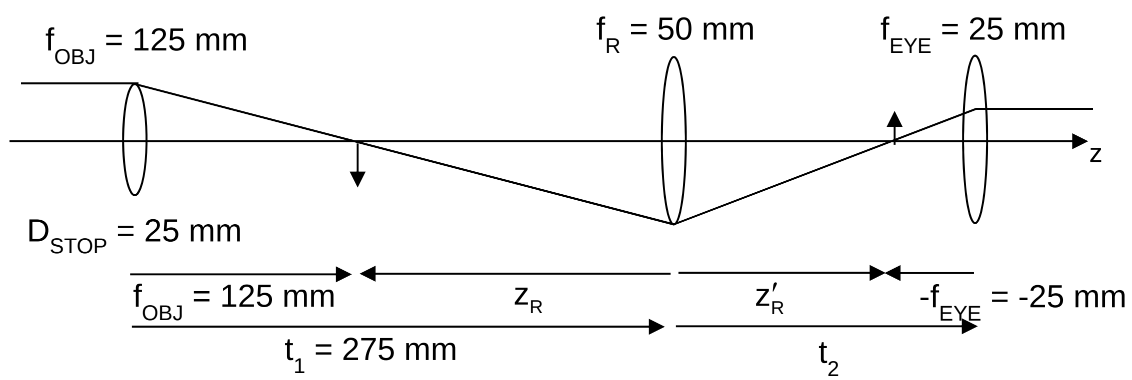


Determine: The Magnifying Power of the telescope.
The Magnification of the relay m_R.
The separation between the Relay Lens and the Eye Lens.
The Eye Relief ER (or the location of the Exit Pupil) of the telescope.
The diameter of the Exit Pupil.

Provide a clear explanantion of your method of solution.

Note: The solution of this problem does not require the use of raytrace. However, a raytrace sheet is provided should you choose to use raytrace.

For the telescope to be afocal, the first intermediate image is at the rear focal point of the objective lens and the relayed intermediate image is at the front focal point of the eye lens.



Base Keplerian Telescope:

$$MP_K = -\frac{f_{OBJ}}{f_{EYE}} = -\frac{125mm}{25mm} = -5.0$$

Continues…

Relay Lens:

$$t_1 = 275mm = f_{OBJ} - z_R = 125mm - z_R \qquad z_R = -150mm$$

$$\frac{1}{z'_R} = \frac{1}{z_R} + \frac{1}{f_R} \qquad f_R = 50mm$$

$$z'_R = 75mm$$

$$m_R = \frac{z'_R}{z_R} = \frac{75mm}{-150mm} = -0.5$$

Lens Separation: $t_2 = z'_R + f_{EYE} = 75mm + 25mm = 100mm$

System MP: $MP = m_R MP_K = (-0.5)(-5.0) = +2.5$

XP Size:

$$MP = 2.5 = \frac{D_{EP}}{D_{XP}} = \frac{D_{STOP}}{D_{XP}} \qquad D_{STOP} = 25mm$$

$$D_{XP} = 10.0mm$$

ER or EX Location: Image the stop through the relay lens to find the intermediate pupil

$$z_{STOP} = -t_1 = -275mm \qquad \frac{1}{z'_{PUPIL}} = \frac{1}{z_{STOP}} + \frac{1}{f_R}$$

$$z'_{PUPIL} = 61.11mm$$

Now image this pupil through the eye lens to locate the XP and find the ER. The intermediate pupil is to the left of the eye lens.

$$z_{PUPIL} = t_2 - z'_{PUPIL} = -(100mm - 61.11mm) = -38.89mm$$

$$\frac{1}{z_{XP}} = \frac{1}{z_{PUPIL}} + \frac{1}{f_{EYE}} = \frac{1}{-38.89mm} + \frac{1}{25mm}$$

$$z_{XP} = ER = 70mm$$

MP = ___2.5X___ m_R = ___-0.50____ t_2 = ___100__mm

ER = __70___ mm D_{XP} = ___10.0__ mm

For raytrace analysis, trace a marginal ray and a chief ray. The chief ray angle is arbitrary and the initial marginal ray height is half the stop diameter ($y_{OBJ} = 12.5mm$).

For the relay magnification, surfaces at both intermediate images must be used.

		Objective	Image 1	Relay	Image 2	Eye	XP
Surface	0	1	2	3	4	5	6
f		125	–	50	–	25	
$-\phi$		-.008		-.02		-.04	
t		125	150	(75)	25	(70)	

Marginal Ray

	0	1	2	3	4	5	6
y	12.5	12.5	0	-15.0	0	5.0	5.0
u	0	-0.1	-0.1	0.2	0.2	0	

Chief Ray

	0	1	2	3	4	5	6
$\bar{y}$		0	-1.25	-2.75	.625	1.75	0
$\bar{u}$	-.01*	-.01	-.01	.045	.045	-.025	

*arbitrary

y							
u							

System magnification: $m = \frac{y_{EYE}}{y_{OBJ}} = \frac{5.0}{12.5} = 0.4$ $MP = \frac{1}{m} = 2.5$

XP Size: $D_{XP} = 2y_{XP} = 10mm$

ER: The XP is located where the chief ray crosses the axis. $ER = 70mm$

Relay Magnification: $m_R = \frac{Image\ Size\ 2}{Image\ Size\ 1} = \frac{\bar{y}_2}{\bar{y}_2} = \frac{0.625mm}{-1.25mm} = -0.5$

Lens Separation: $t_2 = 75mm + 25mm = 100mm$