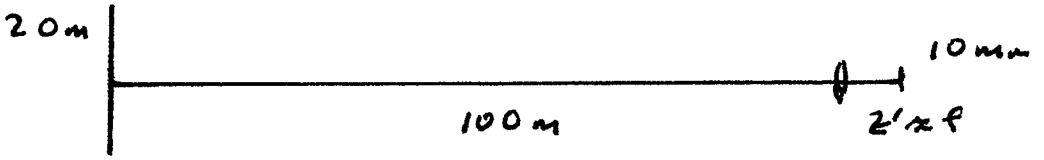
OPTI-502 | **Midterm Exam** Page 1/15 | John E. Greivenkamp Fall, 2015

October 27, 2015   Lecture 19   Name Solutions

Closed book; closed notes. Time limit: 75 minutes.
An equation sheet is attached and can be removed. A spare raytrace sheet is also attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, that method must be used.
Raytraces must be done on the raytrace form. Be sure to indicate the initial conditions for your rays.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Provide your answers in a neat and orderly fashion.
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.
Note: On some quantities, only the magnitude of the quantity is provided. The proper sign conventions and reference definitions must be applied.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (10 points) A 20 m object is to be imaged onto a 1 cm detector. The image must fill the detector size. The object is located at a distance of 100 m. Determine the approximate focal length of the required imaging lens.



$$m = \frac{h'}{h} = \frac{10\,mm}{-20\,m} = -0.0005 \qquad (2000:1)$$

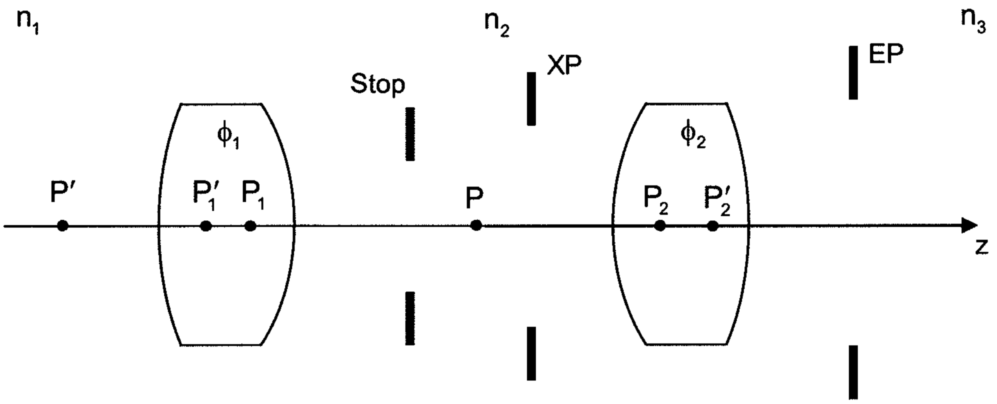
$$z \approx -100\,m \qquad z' \approx f$$

$$m = \frac{z'}{z} \approx \frac{f}{z} = \frac{f}{-100\,m} = -.0005 \qquad \underline{or} \quad f = \frac{100\,m}{2000}$$

$$\underline{f = 50\,mm} \qquad f = 0.05\,m = 50\,mm$$

Focal Length ≈ 50 mm

2) (10 points) An optical system is comprised of two elements separating indices of refraction n_1, n_2 and n_3. Subscript 1 designates element 1, subscript 2 designates element 2, and quantities without subscripts are associated with the total system.



Circle the index of refraction (and therefore the corresponding optical space) associated with each of the following:

EP: [✓ n_1] n_2 n_3

XP: n_1 n_2 [✓ n_3]

P_1: [✓ n_1] n_2 n_3

P_1': n_1 [✓ n_2] n_3

P_2: n_1 [✓ n_2] n_3

P_2': n_1 n_2 [✓ n_3]

P: [✓ n_1] n_2 n_3

P′: n_1 n_2 [✓ n_3]

The actual system does not matter!

P and EP are always in object space (n_1).

P' and XP are always in image space (n_3).

P_i is always in the object space of the element.

P_i' is always in the image space of the element.

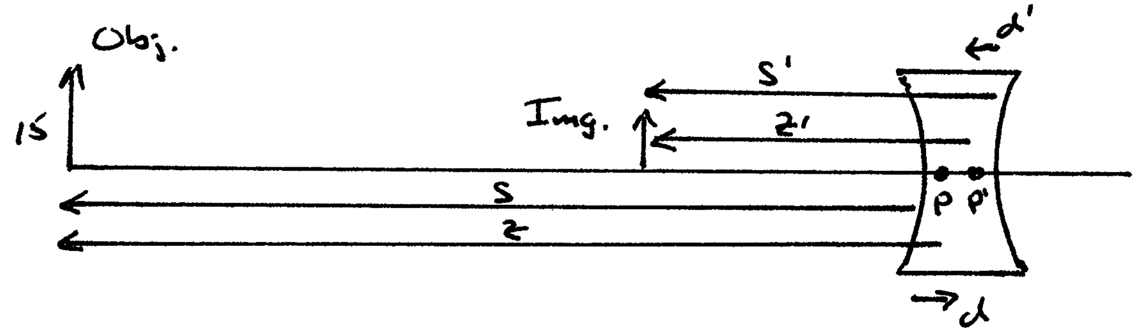
3) (15 points) A ±15 mm high object is 150 mm to the left of the front vertex of a thick lens in air. The lens specifications are:

R1 = -100 mm R2 = 50 mm
t = 10 mm n = 1.55

Determine the focal length of the lens.
Determine the image size and the image location relative to the rear vertex of the lens.
Is the image erect or inverted? Real or virtual?

NOTE: Use Gaussian Reduction and Gaussian Imaging for this problem. Cascaded imaging may not be used (you may not image through one surface and then use this image as an object for the other surface).

Biconcave Lens:



$\phi_1 = (n-1)/R_1$ $\phi_2 = (1-n)/R_2$

$\phi_1 = -0.0055/mm$ $\phi_2 = -0.011$

$t = 10 mm$ $\tau = 6.452 mm$

$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 \tau = -0.01689/mm$

$f = -59.21$ mm

Continues...

$d = \frac{\phi_2}{\phi}\tau = 4.20 \text{ mm}$ $d' = -\frac{\phi_1}{\phi}\tau = -2.13 \text{ mm}$

$S = -150 \text{ mm}$

$z = S - d = -154.20 \text{ mm}$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} = \frac{1}{-154.20} + \frac{1}{-59.21}$$

$z' = -42.78 \text{ mm}$ (left of P')

$S' = z' + d' = -42.78 \text{ mm} - 2.13 \text{ mm}$

$S' = \underline{-44.91} \text{ mm}$ (left of V')

$\underline{\text{Virtual}}$!

$$m = \frac{z'}{z} = \frac{-42.78 \text{ mm}}{-154.20 \text{ mm}} = 0.277$$

$h = 15 \text{ mm}$

$h' = m h = \underline{4.16 \text{ mm}}$

$\underline{\text{Erect}}$!

Focal length = -59.21 mm

Image: Real [✓ Virtual] [✓ Erect] Inverted

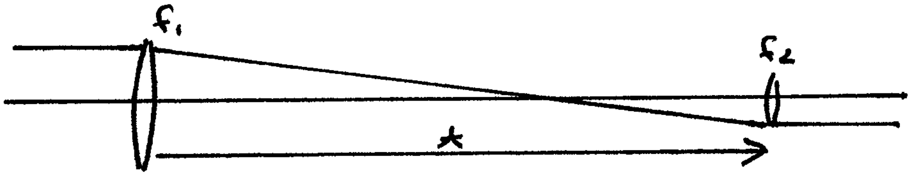
Image size = ± 4.16 mm Located 44.91 mm to the L of the rear vertex.

4) (20 points) An afocal system consists of two positive thin lenses in air. The focal length of the first lens is 200 mm and the magnitude of the system magnification is 0.2.

$$|m| = 0.2$$

$m < 0$ for two positive lenses

a) Complete the design of the system.



$m = -\frac{f_2}{f_1} = -0.2$ $f_1 = 200$ mm

$f_2 = 40$ mm

$t = f_1 + f_2 = 240$ mm

$f_2 =$ 40 mm t = 240 mm

b) The second lens has a diameter of 10 mm, and the second lens serves as the system stop. Determine the EP and XP locations and sizes for the system.

NOTE: Use Gaussian Methods for this problem. No Raytrace is permitted.

EP: Since the second lens is the stop, the EP is the image of the second lens through the first lens into object space. Light R → L

$n = n' = -1$

$z_{STOP} = t = 240$ mm

$\frac{-1}{z'_{EP}} = \frac{-1}{z_{STOP}} + \frac{1}{f_1}$ $f_1 = 200$ mm

Continues…

$z'_{EP} = -1200$ mm (left of L1)

$$m_{EP} = \frac{z'_{EP}/n'}{z_{STOP}/n} = \frac{-1200\,mm}{240\,mm} = -5.0 = \frac{1}{m}^{*} = \frac{1}{-0.2}$$

$$D_{EP} = |m_{EP}|\, D_{STOP} = \underline{50\,mm} \qquad D_{STOP} = D_{L2} = 10\,mm$$

*Note that D_{EP} could be found using the magnification of the afocal system

XP: The second lens is the stop and the XP.

$$D_{XP} = D_{STOP} = D_{L2} = 10\,mm$$

Entrance Pupil: 1200 mm to the L of the first lens. D_{EP} = 50 mm

Exit Pupil: 0 mm to the — of the second lens. D_{XP} = 10 mm

c) A cube of dimensions 10 mm x 10 mm x 10 mm is imaged by this optical system. What is the volume of the image of the cube?

Assume that the cube is a wire grid so that obscuration, the index of refraction of the cube and transparency are not issues.

Object Volume $= 1000\,mm^3$

- The lateral dimensions of the cube are reduced by m. The face of the cube is reduced by m^2.
- The longitudinal thickness of the cube is reduced by $\bar{m} = m^2$.
- The volume of the cube is reduced by $\underline{m^4}$!

$m = -0.2 \qquad \bar{m} = 0.04$

$m^4 = 0.0016$

$V' = m^4 V = 1.6\,mm^3$ Volume = 1.6 mm^3

5) (15 points) An optical system in air is comprised of two thin lenses:

L1: $f_1 = -100$ mm
L2: $f_2 = 50$ mm
$t = 50$ mm

An object is placed 300 mm to the left of the first lens. The object size is ± 10 mm. Use paraxial raytrace methods to determine the system focal length and the location and size of the image.

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the image size must be determined from a raytrace. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Gaussian imaging methods may not be used for any portion of this problem.

	∞ Obj	L1	L2	F' Image			
Surface	0	1	2	3	4	5	6
f		-100	50				
$-\phi$		0.01	-0.02				
t	∞/300	50					
Focal Ray				75.0			
y	1	1	1.50	0			
u	0	0.01	-0.02				
Image Location				83.33			
y	0	3.0	5.0	0			
u	0.01*	0.04	-0.06				
Image Size							
$\bar{y}$	10	10	15.0	-1.666			
$\bar{u}$	0*	0.1	-0.2				
y							
u							

* arbitrary A value of u = 0.1 can be used, but it produces large ray heights.

Continues...

Method of Solution:

Focal Length: Trace a ray parallel to the axis.

$u_0 = 0.0$ $y_0 = 1$ mm

$u' = -0.2$

$$\phi = -\frac{u'}{y_0} = 0.02/\text{mm}$$

$$f = \frac{1}{\phi} = 50 \text{ mm}$$

While not required, the BFD is found to be 75.0mm.

Image: Trace rays from the axial object point and from the top of the object.

$y_0 = 0.0$ $\bar{y}_0 = 10$ mm

$u_0 = 0.01$ (arbitrary) $\bar{u}_0 = 0.0$ (arbitrary)

The image is found when the axial ray crosses the axis

$s' = 83.33$ mm (right of L2)

The image height at that location:

$h' = -1.666$ mm (inverted)

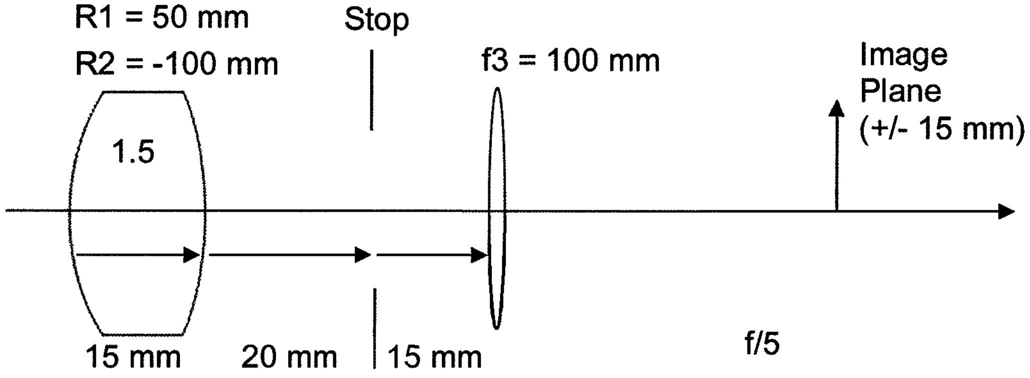
f = 50.0 mm

Image Location: 83.33 mm to the R of L2; Image Size = ± 1.667 mm

(inverted)

6) (30 points) The following diagram shows the design of an optical system that is comprised a biconvex thick lens in air, a stop, and a thin lens in air. The index of refraction of the thick lens is 1.5.

The system operates at f/5. The object is at infinity.
The maximum image size is ± 15 mm.



Determine the following:
- System focal length.
- Back focal distance
- Entrance pupil and exit pupil locations and sizes.
- Stop diameter.
- Angular field of view (in object space).

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the FOV must be determined from the chief ray. Raytraces must be done on the raytrace form. Be sure to clearly label your rays on the raytrace form. A method of solution explaining your procedure and calculations must be provided. Gaussian imaging methods may not be used for any portion of this problem.

Your answers must be entered below. Be sure to provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.)

Entrance Pupil: 47.54 mm to the R of the first vertex. D_{EP} = 10.85 mm

Exit Pupil: 17.65 mm to the L of the thin lens. D_{XP} = 7.80 mm

Stop Diameter = 6.62 mm

System Focal Length = 54.27 mm Back Focal Distance = 21.30 mm

FOV = ± 15.4 deg in object space

	$\vec{}$ Obj		EP		R1		R2		Stop		f_3		XP		F' Image
Surface	0		1		2		3		4		5		6		7
~~C~~ R			–		50		-100		–		f = 100		–		
t						15		20		15					
n		1.0		1.0		1.5		1.0		1.0		1.0		1.0	
-φ			–		-0.01		-0.005		–		-0.01		–		
t/n				-47.54		10		20		15		-17.65		38.95	
Potential Chief Ray															
$\tilde{\bar{y}}$			0		-2.9		-2.0		0		1.5		0		
$n\tilde{\bar{u}}$				0.061		0.09		0.1*		0.1*		0.085			
u															
Potential Marginal Ray															38.95
$\tilde{y}$	1		1		1		0.9		0.61		0.3925		0.7177		0
$n\tilde{u}$		0		0		-0.01		-0.0145		-0.0145		-0.018425		-0.018425	
u															
Extended Potential Chief Ray															
$\tilde{\bar{y}}$			0		-2.9		-2.0		0		1.5		0		3.311
$n\tilde{\bar{u}}$		0.061		0.061		0.09		0.1		0.1		0.085		0.085	
u															
Marginal Ray - Scale Factor = 5.43															
y	5.43		5.43		5.43		4.89		3.31		2.13		3.90		0
nu		0		0		-0.0543		-0.0787		-0.0787		-0.100		-0.100	
u															
Chief Ray - Scale Factor = 4.53															
y			0		-13.14		-9.06		0		6.80		0		15.0
nu		0.276		0.276		0.408		0.453		0.453		0.385		0.385	
u															

* arbitrary

Continues...

Provide Method of Solution:

Set up the raytrace header. The thin lens may be entered as a focal length and converted to power.

EP/XP Location: Trace a potential chief ray starting at the center of the stop. The pupils are located where this ray crosses the axis in object/image space.

$R1 \rightarrow EP = 47.54$ mm (Right of R1)

$F3 \rightarrow XP = -17.65$ mm (Left of F3)

Focal Length: Trace a potential marginal ray parallel to the axis in object space ($\tilde{y}_0 = 1$ $\tilde{u}_0 = 0$). The rear focal point F' is located where this ray crosses the axis.

$XP \rightarrow F' = 38.95$ mm

$BFD = (F3 \rightarrow XP) + (XP \rightarrow F') = -17.65\text{mm} + 38.95\text{mm}$

$BFD = \underline{21.30\text{mm}}$

$\phi = -\frac{\tilde{u}'}{\tilde{y}_0}$ $\tilde{u}' = -0.018425$

$\phi = 0.018425$

$f = f'_R = 1/\phi = 54.27$ mm

Extend the potential chief ray to the image plane (F').

Continues

Entrance Pupil: The system operates at f/5

$f/\# = f/D_{EP} = 5$ $f = 54.27$

$D_{EP} = \underline{10.85\ mm}$

$r_{EP} = 5.43\ mm$

Pupil/Stop Sizes: Scale the potential marginal ray to the proper $r_{EP} = 5.43\ mm$

Scale Factor $= 5.43mm/1.0mm = 5.43$

Ray heights at the stop and XP:

$r_{STOP} = 3.31\ mm$ $D_{STOP} = 6.62\ mm$

$r_{XP} = 3.90\ mm$ $D_{XP} = \underline{7.80\ mm}$

FOV: Scale the potential chief ray to the required image height at 15 mm (from the current value of 3.311 mm)

Scale Factor $= \frac{15mm}{3.311\ mm} = 4.53$

$\bar{u}_0 = 0.276$

$HFOV = \tan^{-1}(\bar{u}_0) = 15.4°$

$FOV = 30.8°$ or $+/- 15.4°$

Continues...