26 EVALUATION PHASE OPTICAL TESTS

26.1 RESOLVING POWER TESTS

26.1.1 Introduction.

26.1.1.1 The reason for the popularity of this general method stems from the feeling that, artistic considerations aside, the function of an optical system is to give information as to the detail in an object which is usually quite some distance away. Short of looking at the actual detail of the type on which the instrument under test is to be used, it has seemed reasonable to use some sort of artificial but definite target. Since many targets of military significance have sharp edges, targets with sharp edges seem to make sense. The nature of optical system performance is such that the edges should occur in at least two orientations and these preferably at right angles to each other. This deceptively simple process culminates then in a statement as to how many lines per millimeter can be resolved on the film of a camera, or as seen by the eye in a visual device. Actually it makes more sense to talk about a limit of resolution in terms of lines per unit solid angle, etc.

26.1.1.2 It will pay us to look somewhat more closely as to why this apparently straightforward process is called "deceptively simple". To begin, we have the fundamental question of what kind of target are we going to choose as a representative sampling of the in-use object. The USAF has been using the target in Figure 26.1 for years, while the National Research Council of Canada (1) has been using annuluses on a dark background as shown in Figure 26.2 along with a sector target proposed by Nutling. The U. S. National Bureau of Standards until recently used a line target as shown in Figure 26.3 . This target and its applications were discussed in the reference cited. Recently NBS has adopted a new target and this is shown in Figure 26.4 .

26.1.1.3 In addition to these, other groups have chosen targets made up of letters or numbers or combinations of special symbols or objects (2) . To get informative as to the response of the optical system, at all angles, a target consisting of alternate black and white sectors has been used. (3) Apparently even the choice of the form of the target has been far from unanimous!

26.1.1.4 Let us look deeper. Even putting aside the question of form there is a considerable controversy over the contrast to be used between the dark and light portions. At least until the new NBS low contrast target came out, the British and Canadians were maintaining stoutly that the USAF high contrast targets were unrealistic as most of the objects photographed from an aircraft exhibited low contrast on the majority of days when photo-reconnaissance could be performed. We need not labor this point further except now we realize that not only the form but also the contrast is the subject of controversy.

26.1.1.5 With all this controversy the fact still stands that the system does have merit. Pestrecov (4) gave an excellent survey of the methods to date, and the serious student is referred to his work as regards relative merit of each. The particular claim for this technique is that it does give a single number that may be used to compare the performance of different lenses. The big question obviously is "granted it does give a figure by which to compare lenses but so do other techniques such as f- numbers, T- numbers, etc.", but does this really enable one to evaluate how a lens will perform in the field or does it merely tell how it would perform when photographing the very uninteresting lines and spaces on the test target? Unfortunately the answer to this question is not an unqualified "yes it does serve to state positively that this lens will be better than that in the field."

26.1.1.6 Having discussed these general ideas, let us now look at how the resolving power charts are actually used. As can be gathered from the above, different laboratories have their own techniques so we will sample three of the more common methods.

26.1.2 The NBS method.

26.1.2.1 Figure 26.4 shows the high and low contrast NBS charts. The dimensions of these patterns are given in the table below the charts. The contrast of the black on white is 1.4 while that of the black on grey is 0.20. The numbers on the chart "14, 20, 28", etc. refer to the number of lines/mm when this chart is used at a minification of 25X. The numbers refer to both the horizontal and vertical patterns whose linear extention

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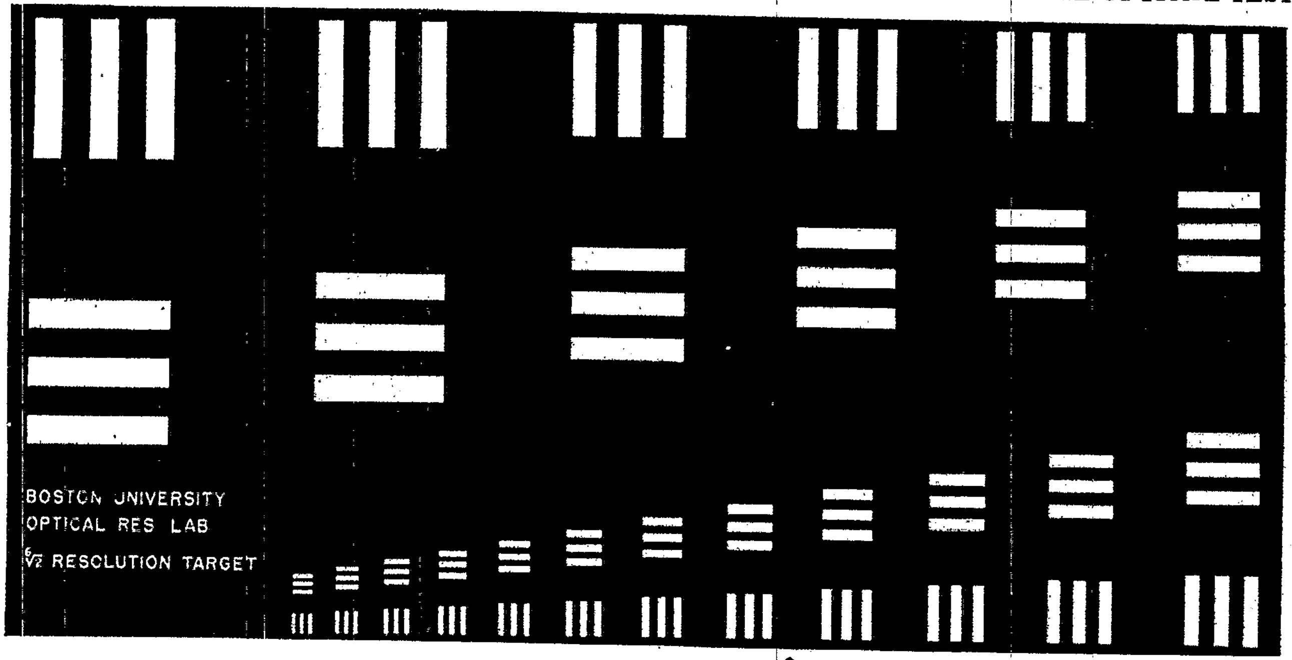
(1) Howlett, L. E., Photographic Resolving Power, Canadian Journal of Research, Vol. 24, Sec. A, No. 4, 15-40 (1946)

(2) MacDonald, NBS Circular 526, 51

(3) Jewell, A Chart Method of Testing Photographic Lenses, JOSA Vols. 2-3, Nos. 3-6, 52, (1919)

(4) Pestrecov, Photographic Resolution of Lenses, Photogrammetric Engineering, Vol. 13, (1947)

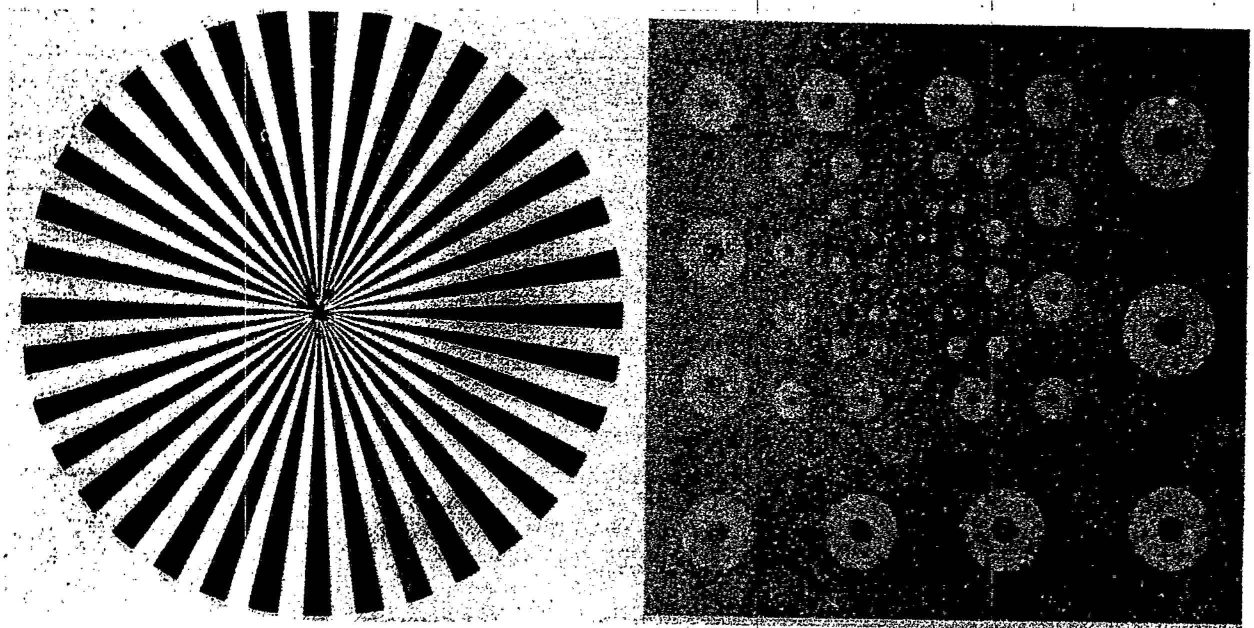


Calibration Sheet for B. U. O. R. L. $\sqrt[6]{2}$ Target

Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Conversion Figure	21.5	24.1	27.1	30.4	34.2	38.3	43.0	48.3	54.2	60.8	68.3	76.7	86.0	96.6	108	122	137	153

These are resolution values for a B. U. $\sqrt[6]{2}$ target of 1mm. width. To determine resolution for each unit in lines/mm for any size target, divide each figure listed above by the width of the target measured from the extreme edge of unit 1 (the largest) to the extreme edge of unit 6.

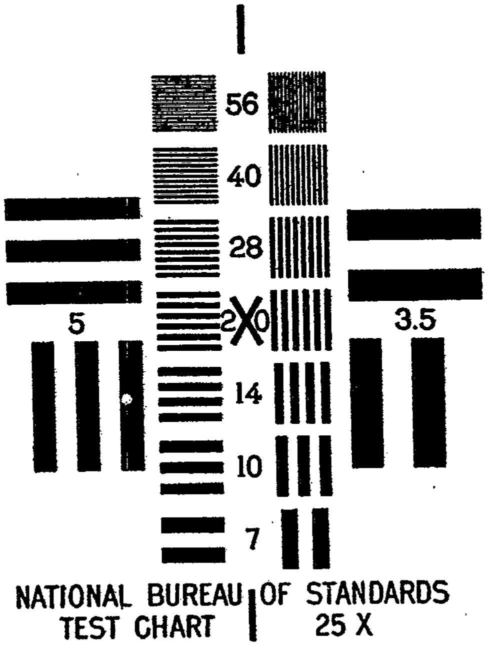
Figure 26.1 - The USAF resolving power target.



Sector target introduced by P. G. Nutting.

Canadian annulus target of 1.6:1 contrast ratio. The resolution values of the adjacent annuluses are in the $\sqrt[6]{2}$ ratio.

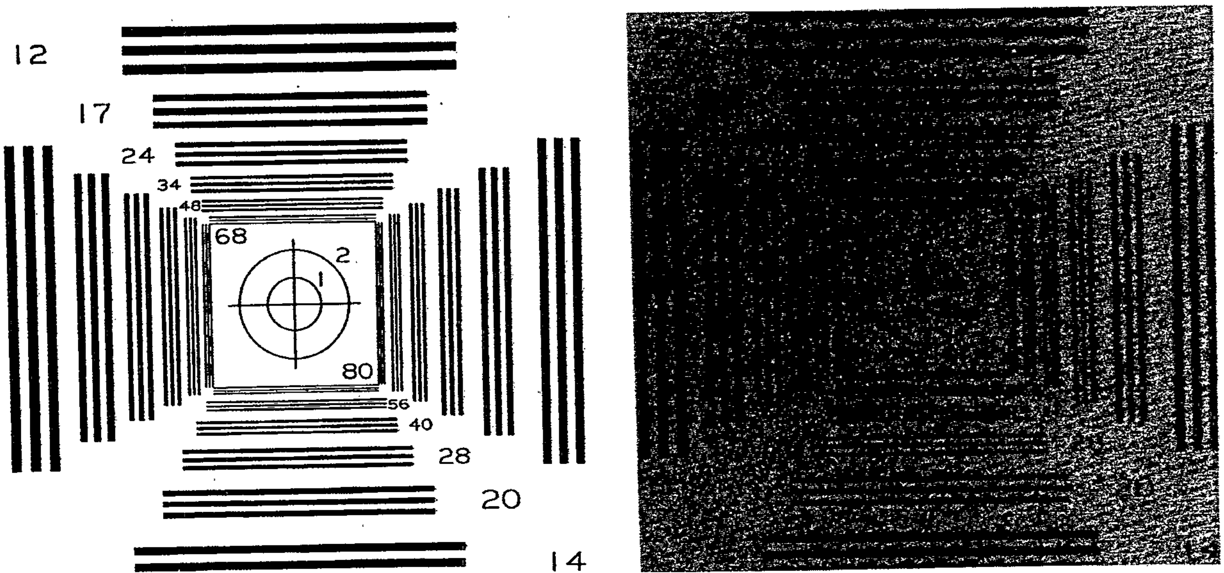
Figure 26.2 -The Nutting and Annuli resolving power targets.
(From Pestrecov's, Photographic Resolution of Lenses, Photogrammetric Engineering, Vol. 13, 1947)



This chart formed part of NBS Circular 428. The ratio of the line spacings in successive patterns of this chart is equal to $\sqrt{2}$. When the chart is photographed at the standard distance of 26f, the values of resolving power that can be measured with this chart range from 3.5 to 56 lines/mm.

Figure 26.3 - Old NBS resolving power target.

(A Test of Lens Resolution for the Photographer, NBS Circ. 428)



High-contrast N. B. S. resolution test chart

Low-contrast N. B. S. resolution test chart.

Pattern Number	80	56	40	28	20	14	68	48	34	24	17	12
Width of single black or white line	0.156	.233	.312	.446	.625	.893	0.184	.260	.368	.521	.735	1.042
Width of 3-line pattern	0.781	1.116	1.562	2.232	3.125	4.464	0.919	1.302	1.838	2.604	3.676	5.208
Width of space between patterns	0.781	1.116	1.562	2.232	3.125		0.582	.825	1.164	1.649	2.328	
Length of lines	18.0	19.6	21.9	25.1	29.6	36.1	18.0	19.6	21.9	25.1	29.6	36.0

Figure 26.4 -The new N.B.S. resolving power targets.

(Charts for Testing the Resolving Power of Photographic Lenses, F. E. Washer and I. C. Gardner, NBS Circ. 533(1953)) would run into the number. The chart used in this manner should be 26 focal lengths in front of the lens. The charts of course may be used both off as well as on axis. A common arrangement is to make a rack holding a series of the charts arranged in roughly the form of a square so that a photographic lens may be tested at all angles simultaneously. If the lens is to be tested visually, then it may be somewhat more desirable to reposition the test chart to the various angles of interest.

26.1.2.2 The observer after setting up the chart at the requisite distance determines which group is just distinguishable as three distinct lines and reports the corresponding number of lines/mm as the maximum resolving power of the lens at the given angle etc. Note that the measurement made in this way gives little or no information as to the response of the system to targets at fewer lines/mm.

26.1.2.3 Table 26.1 taken from NBS 533 shows the variation of resolving power of several hand held cameras. In this connection it is interesting to note the effect of using the high and low contrast targets. Inasmuch as we judge lines to be separated on the basis of contrast, it is important to note particularly Figure 26.5. The high contrast targets clearly may well be a more revealing as to what the actual resolution limits of the lens are. Further, the increased slope of the high contrast curve makes far more accurate measurements. Again we must warn that if the lens is to be actually used on low contrast targets, then we had better check it

Lens	EFL mm	F-number	Resolving power in lines per millimeter (angular separation from axis)									
			Tangential					Radial				
			0^o	5^o	10^o	15^o	20^o	0^o	5^o	10^o	15^o	20^o
A--	50	2	68	56	56	48	28	68	56	56	48	40
		2.8	68	68	68	68	56	68	68	68	68	56
		4	80	68	56	56	56	80	68	68	68	68
		5.6	80	80	68	68	80	80	80	80	80	80
		8	80	68	68	68	68	80	68	68	68	56
		11	80	80	80	80	68	80	80	80	80	80
		16	56	56	56	56	48	56	56	48	48	48
		22	56	56	48	48	48	56	56	48	48	40
B--	50	4.5	56	34	20	14	24	56	40	40	48	48
		5.6	56	28	17	20	34	56	40	40	56	56
		8	56	28	24	34	48	56	56	48	80	80
		11	56	34	34	34	56	56	56	56	80	80
		16	56	56	56	48	48	56	56	56	68	68
C--	85	2	68	68	34	17	--	68	68	48	34	--
		5.6	68	68	48	20	--	68	68	68	56	--
		11	68	68	48	24	--	68	68	68	80	--
D--	101	4.5	34	34	28	28	28	34	34	28	20	28
		5.6	40	34	28	28	28	40	40	28	14	28
		8	40	40	40	34	34	40	48	48	24	28
		11	40	48	48	40	40	40	48	48	34	34
		16	34	48	48	40	40	34	48	48	40	40
E--	101	4.5	28	28	24	12	7	28	34	34	28	20
		5.6	28	28	20	12	7	28	28	24	28	20
		8	34	28	24	17	14	34	34	34	28	28
		11	28	28	28	20	12	28	40	40	28	28
		16	34	34	28	17	12	34	40	40	28	20
		22	34	28	28	17	5	34	40	40	34	24
		32	34	28	24	17	12	34	34	34	34	28
F--			5.6	5.6	5.6	5.6	4.8	5.6	5.6	5.6	4.8	4.8

Table 26.1 - Resolving power at various apertures of several lenses of the type used on small hand-held cameras.

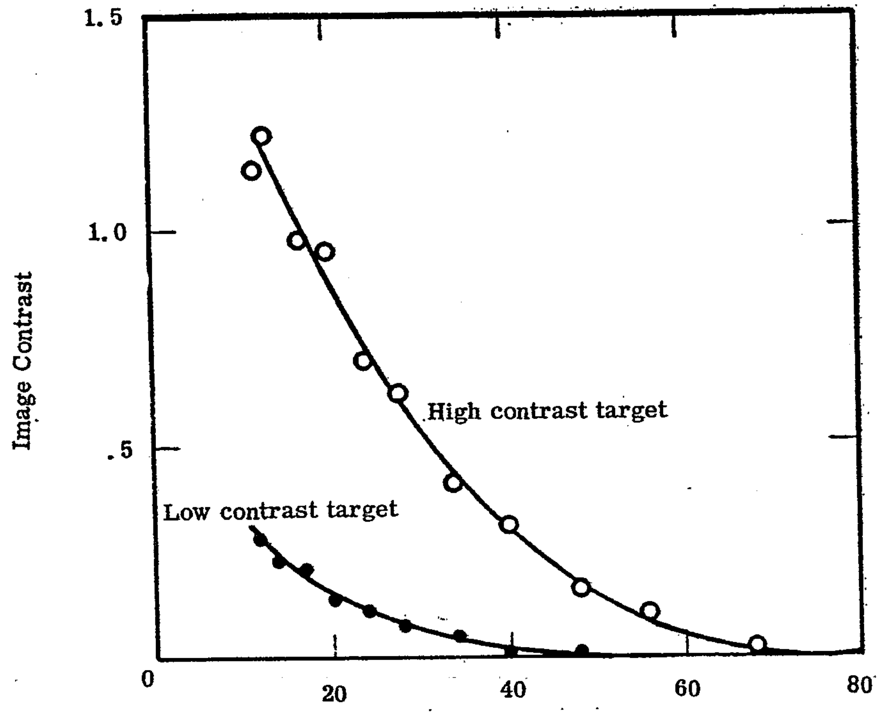


Figure 26.5-Variation of contrast in the image as a funtion of line frequency.

(Charts for Testing the Resolving Power of Photographic Lenses, F. E. Washer and I. C. Gardner, NBS Circ. 533(1953))

on low contrast targets. This also is shown clearly by Figure 26.5: if we want to resolve 50/mm at low contrast, then the lens examined is not suitable. If we want to resolve the same number of lines/mm at high contrast, then the lens might well be satisfactory. This is a crucial point in considering the usefulness of resolving power targets as evaluation tools.

26.1.2.4 Looking again at the question of visual optics such as binoculars, telescopes, periscopes, etc. we realize, as previously pointed out that here the most important characteristic is not lines/mm but rather lines/unit solid angle. We can also state this by saying that we are interested in the angular rather than the linear resolving power of the system. Tables 12 and 13 from NBS 533 enable the user to determine from the chart group just resolvable, the corresponding maximum angular resolving power for either the circles or the lines around them.

26.1.2.5 Care must be exercised in judging the resolving power of a visual system to be certain that the resolving power of the eye is taken into consideration. This means that the lines under study must all subtend an angle greater than that just resolvable by the eye--usually about 60 seconds of arc. This means then that the product of the resolving power of the target and the magnification of the system must be greater than, say 60 seconds of arc, if we are to obtain a true test of the resolving power of the system.

26.1.2.6 In this same connection, the resolving power of a sequence of optical systems is analogous to the effective bandwidth, or the effective rise time of a number of sequential amplifiers; the overall resolving power, Re , in terms of the resolving power of the individual components R_1, R_2 etc., is given approximately by

$$\frac{1}{Re} = \sqrt{\frac{1}{R_1^2} + \frac{1}{R_2^2} + \frac{1}{R_3^2} + \dots} \qquad (1)$$

26.1.2.7 The NBS chart, when used in this standard manner, will cover a range of 14 to 80 lines/mm. For systems having higher or lower resolving powers the targets may be moved closer or further away. In some instances it may be convenient, where systems capable of resolving several hundred lines/mm are repeatedly encountered, to avoid the long working distances involved in the method above and reduce the targets photographically. Should this be done, great care must be exercised to see that the resolving power of the film and copying camera are such as to not degrade the targets.

26.1.2.8 While the NBS charts were developed primarily for lens studies, they may also be used as a basis for compliance with certain government specifications, for example

Federal Specification:

GGG-G-501b	Goggles, eyecup, protective, impact-resisting (chippers', grinders' etc.).
GGG-G-511a	Goggles, eyecup, protective (welders).
GG-T-621	Transits, 1-minute; and transit tripods.

Military Specification:

MIL-O-13830 Ord	Optical components for fire control instruments; general specification governing the manufacture assembly, and inspection of.

Commercial Standard:

CS159-49	Sun glass lenses made of ground and polished plate glass thereafter thermally curved.

26.1.3 The U.S.A.F. resolution target.

26.1.3.1 Originally suggested at the Bureau of Standards and carried to its present status by the U.S.A.F. Photographic Laboratory at Wright-Patterson Air Base, the U.S.A.F. target was designed primarily to evaluate the performance of aerial camera lenses. While the use of this target is controversial, it is probably the most widely used of all at the moment. The following comments of A. Katz (5), then of Wright Field, are much to the point. They were made during a discussion following a paper by R. E. Hopkins.

> "In connection with the points raised by Dr. Pestrecov and in earlier papers, I notice that a number of people have been gleefully trying to kick the three-line resolution target to death. I want to point out again--and I have done this in other meetings--that it has served its purpose well. This purpose, simply stated, is to serially grade lenses in a manner that will correlate with their photograph-making rank. I have yet to be shown that our use of the three-line target in the judging of lenses to be used for aerial photography has led to any error, let alone consistent error."
>
> "Now we have lots of data, most of which is not neat and packaged. The exigencies arising with the working conditions in the Air Force are such as to effectively preclude the careful running of planned experiments. We substitute large numbers of airplane flights and tests, and after a number of years we come to pretty definite conclusions--by statistical osmosis, if you will. We know by now that when we get a lens that performs well in the laboratory (on the much maligned three-line high-contrast target) it will take high-quality photographs in the air on good days as well as bad days. The converse is also true. Laboratory test enable us to predict the quality of actual aerial photographs. I can't expect much more of a laboratory test. Let us not forget that it is only within the last 10 years that lens performance began to be specified in terms of resolution requirement over the field and that manufacturers began to use these tests, and it is only within the last couple of years that photointerpreters have begun to hear of lines per millimeter as a measure of performance."

26.1.3.2 The type of tests (6) to which Mr. Katz is referring are well demonstrated in a portion of a series of through-focus trials shown in Figure 26.6 on a 40-inch f/5 Baker telephoto aerial camera looking through a window of poor quality. The target was the standard high contrast U.S.A.F. target plus a low contrast version of same plus a two variable frequency high contrast targets first introduced by Washer and Rosberry (7). The target was distant from the camera some 35 focal lengths to minimize the effects of spherical aberration. The term "window" here refers to the glass covering the hole in the skin of the aircraft through which the aerial camera sees the ground. More or less comparable focal positions are shown side by side for ease of

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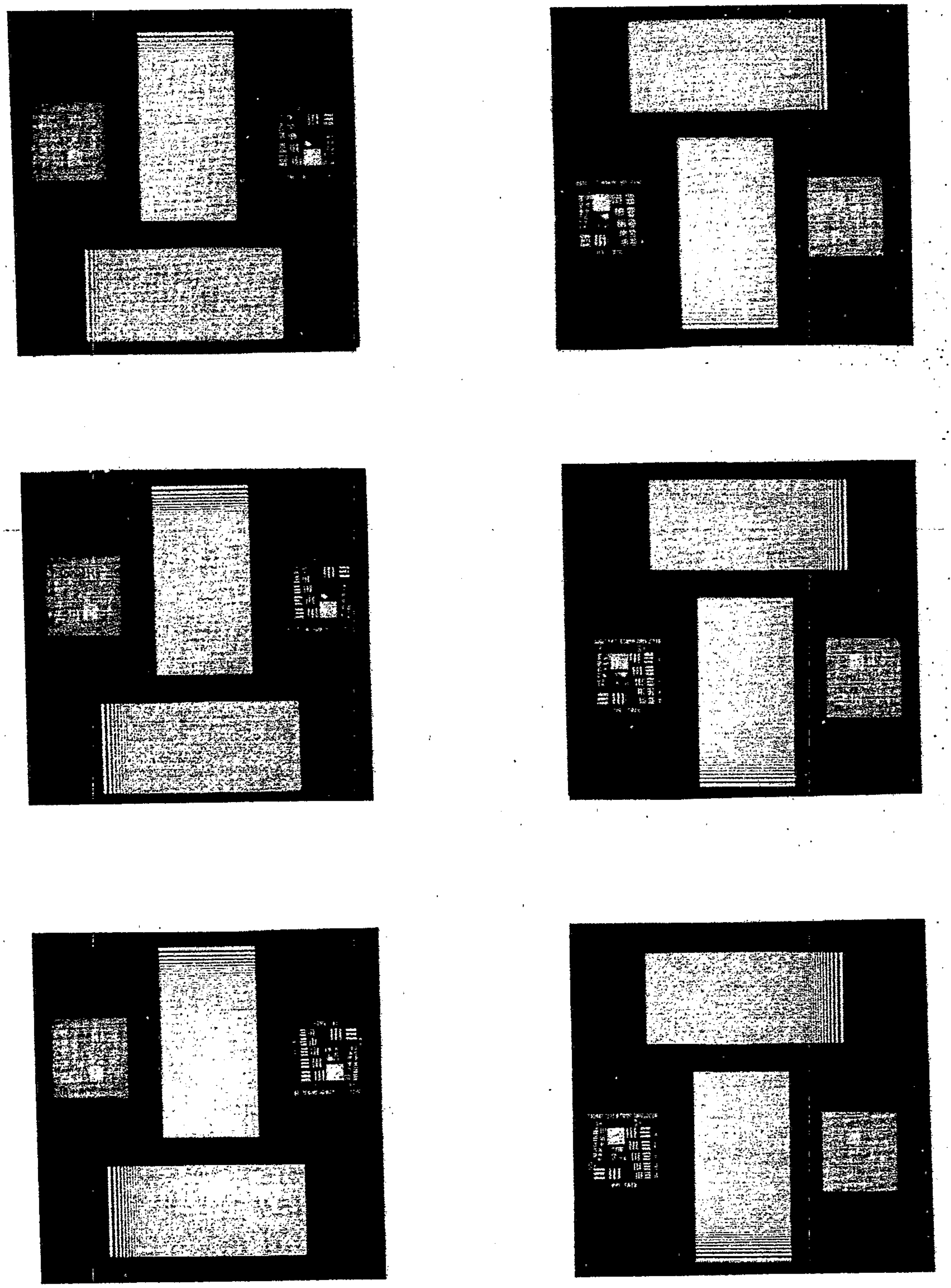
(5) NBS Circular No. 526, 200

(6) These tests were run by Mr. William C. Britton while at the Boston University Physical Research Laboratories and under a U.S.A.F. contract. Mr. Britton is now with Itek Corp.

(7) Washer and Rosberry, JOSA vol. 41, No. 9, 597, (1951)



window 45^{o} Obliquity

No Window

Figure 26.6 - Resolution target testing for Baker 40" f/5 telephoto aerial camera.

comparison. The focal settings was changed by .005" between successive exposures. The variation of the resolution limit with the high and low contrast targets is clear. The effect of off-axis aberrations is also clear.

26.1.3.3 Composite target tests such as these demonstrate the difficulty of deciding on which target, if any, to settle on to the exclusion of all others. In fact it is pretty generally the opinion of the "conservatives" that no one target gives all the information that is needed to fully evaluate a lens. Were a given optical system always to be used on exactly the same type target, that would give a one to one correlation between laboratory testing and field performance. It, thus, is the very versatility of optical systems that gives rise to our difficulty.

26.1.4 The Kinetic Definition Chart.

26.1.4.1 There was developed during World War II (8) and subsequently improved upon (9 and 10) a routine system for checking the resolving power of visual optical systems. The system is essentially a resolving-power target approach, but incorporates many features not formed in the spur-of-the-moment setups commonly found in laboratories. The targets employed, as well as plan and side view schematics are shown in Figure 26.7.

26.1.4.2 The apparatus derives the word "kinetic" from the motorization of some of its parts, but the term is misleading nonetheless. A glance at the charts will show that they are of constant line spacing but of various contrasts and situated in four positions. The ratio of lines/spaces is 1:1, and is essentially the chart first advanced by Foucault (11) in 1858. The variation in line spacing required to determine the resolving power of a system is effected by the optical reduction unit. This unit consists of four highly corrected microscope objectives of focal length 4, 8, 16, and 32 mm. By varying the distance from the target to the reduction unit by the adjustable space gauge shown in Figure 26.7, the lines/inch may be changed from coarse to fine.

26.1.4.3 There are several interesting aspects to the KDC Apparatus. One of these is the "artificial sky" which not only simulates (by varying its illumination) the sky against which many objects must be seen, but also the stray light found in most optical systems. This apparatus thus takes into account not only the low control of the object itself, but also the surround so important in retinal response. Incorporated into the KDC Apparatus is a standard telescope with an aperture that is variable. This very carefully constructed telescope is of superior quality and allows the observer, in effect, to set up a standard against which the test instrument is compared. Once again we see a recognition of the need for removing as far as possible the limitations of the particular observer's eye from the testing procedure. Here this is done by inclusion of an auxiliary telescope of such magnification that the limit of resolving power is determined by the instrument under test rather than the eye. The rest of the system is rather straightforward and all designed to give maximum ease of assessment to the observer.

26.1.4.4 The final report on the NBS chart or the USAF chart is the resolving power limit of the system. In this technique the final report is called the K.D.C. efficiency and is defined as follows:

$$\text{KDC efficiency} = \frac{\alpha_e}{\alpha_i \, Mi} \times 100 \qquad (2)$$

where

α_i = minimum angle resolvable using the instrument under test.
α_e = minimum angle resolvable with the eye alone.
Mi = magnification of the instrument under test.

Clearly then, this definition is not a statement of the resolving power of the instrument alone, but rather it is a comparison of the effective improvement the instrument affords over the eye alone.

26.1.4.5 The factors directly preportional to α_e and α_i are conveniently determined directly from the KDC apparatus as follows. With the auxiliary telescope in place (if it will be required with the instrument under test as previously explained) the observer adjusts the target-to-turret spacing until the target is just resolved and the K.D.C. scale (lower left of drawing, just above the reversing switch) is read. The pointer on this scale is coupled to the target holder. The instrument under test is then inserted in its proper place and the K.D.C. scale again read. The K.D.C. efficiency is now obtained from the equation:

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(8) NDRC Report (classified)
(9) Coleman and Harding, JOSA 37, 263, (1947)
(10) NBS, 526, 95, (1954)
(11) Foucault, Ann. de L'observation de Paris, 5, 197, (1859)

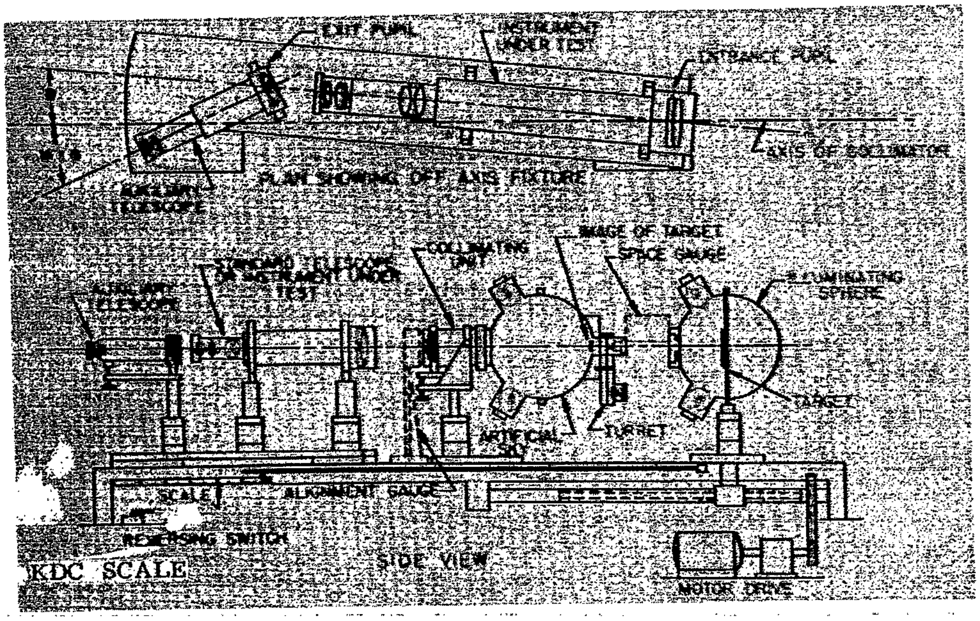
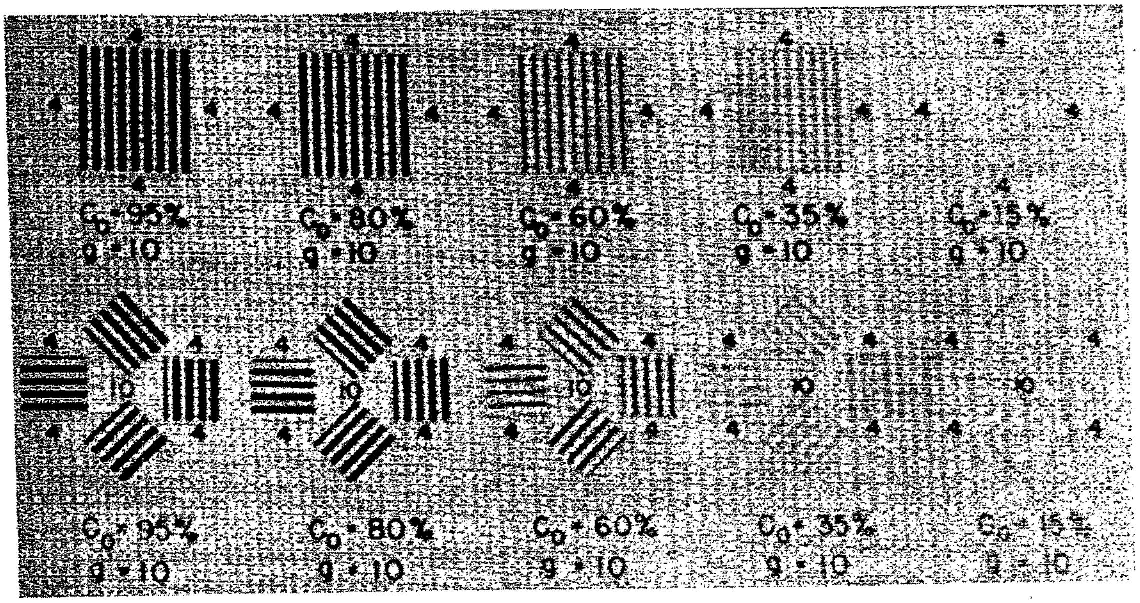


Figure 26.7 (a)- The KDC apparatus schematic.



Modified Foucault resolution targets.

$$C_o = \frac{B-b}{B} = \frac{\text{Reflectivity of white band} - \text{Reflectivity of dark band}}{\text{Reflectivity of white band}}$$

C_o = inherent target contrast: g = number of white bands per inch.

Figure 26.7 (b)- The KDC apparatus targets.

Figure 26.7 - The KDC apparatus and target charts.

$$\text{K.D.C. efficiency} = \frac{Xi}{MiXe} \times 100 \qquad (3)$$

where

Xi = K. D. C. scale reading with the instrument under test (and of course the eye)
Xe = K. D. C. scale reading with the eye alone
Mi = the magnification of the instrument under test.

If it is desired to compare a production instrument with the standard telescope, the K. D. C. reading taken with the standard telescope replaces Xe in the above equation.

26.1.4.6 There are many other uses of the K. D. C. apparatus but certainly its versatility and ease of manipulation recommend it when a large amount of work of this type must be done.

26.2 GENERAL DISCUSSION OF SINE WAVE TESTING

26.2.1 Introduction.

26.2.1.1 At about the time the controversy as to just what type of resolution target should be used was reaching its zenith, a paper given by Schade (12), an electrical engineer, brought to bear on the problem of optical system evaluation, the full resources of a completely different field viz., communication theory. While others such as Selwyn (13) and Duffieux (14) had preceded Schade in their investigations into this general area, there is little doubt in most minds that Schade (15) was responsible for focusing the attention of the optical world on the optical possibilities of this method.

26.2.1.2 It will be of interest to look briefly at Schade's original problem. Schade was studying the problem of optimizing the response of a television system starting with the optical pick-up in the studio through the electronic and electromagnetic systems to the final presentation on a kinescope in the home. His background here as an electrical engineer had taught him that one may study the response of an ordinary amplifier two ways (a) by feeding a single transient pulse to the amplifier and noting its response or (b) using sine waves of different frequencies and noting the phase shift and/or amplitude change as the sine wave signal passed through the amplifier. Fourier analysis shows that all the information contained in (b) is actually implicit in (a) but the transient is harder to use experimentally.

26.2.1.3 With the knowledge that this testing technique was a proven method, Schade in effect asked "why can't I do the same sort of thing for the optical part of the system? If I can do this, then I should be able to use the theories already developed for optimizing cascaded amplifiers." The question then arose as to what there was about an optical system that corresponded to the electrical sine waves. After the idea was conceived that the variation in intensity with angle as seen by the lens did indeed constitute a frequency, albeit a "spatial Fourier frequency" and not the frequency associated with $v = f\lambda$, the way was clear. There did remain then (and still does now) much theoretical work to do but at least the direction was indicated. The problem of translating the Fourier spatial frequencies into the temporal frequencies used in electronic amplifiers was easily solved by scanning techniques already under study in the sister field of flying spot scanner television.

26.2.2 Basic theory.

26.2.2.1 Inasmuch as this manual is not intended to develop all the pertinent theory but rather to acquaint the reader with possible methods, most of the details of the mathematical treatment will be omitted. The reader, however is invited to study closely the many excellent articles in this field. Some of these are in the following

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(12) Schade, A New System of Measuring and Specifying Image Definition: Symposium on Optical Image Evaluation, NBS, Oct., 1951. Proceedings published in NBS circuler 526, (1954).

(13) Selwyn, Theoretical Estimation of Combined Effects of Film and Lens on Resolution; RAE Report N. H. 698, April, (1940).

(14) Duffieux, L'intégrale de Fourier et ses Applications à L'optique, Besançon, Faculté des Sciences, (1946).

(15) Schade, Electro-Optical Characteristics of Television Systems, RCA Rev., 9:5-37, 245-286, 490-530, 653-686; (1948).

references: (16) through (23)

26.2.2.2 As indicated above and by Schade and Duffieux, an optical system may be considered as a two dimensional electrical filter. Further in electrical work we normally think in terms of amplitudes and at least in normal circuit work do indeed measure our signals by determining their amplitude. In optics, however, we cannot measure amplitude directly but instead measure intensity. A negative amplitude has no physical significance (although it can be interpreted as indicative of a 180° phase shift) for optics while it is a common and significant occurrence in electronics. As an aside we might note, however, that in the detection of electromagnetic radiation we can measure only power directly. The spatial frequencies to which we are referring are thus variations of intensity. This is an important point.

26.2.2.3 Let us assume that the coordinates in an object plane are denoted by ξ and η and in the image plane by x and y. The intensities in the object and image plane are then indicated by $O(\xi, \eta)$ and $i(x, y)$ respectively. We should note here that the terminology is not yet standardized and we are here following that of O'Neill (loc. cit. 16, p E-3). An object point $O(\xi, \eta)$ is then spread out into an image point $i(x, y)$, this "spread function" being denoted by $i(x, y)$. If we now apply this spread function to each point in the object, we can predict the appearance of the image by convolving the spread function with the object distribution according to equation (3).

$$i(x, y) = \int_{-\infty}^{+\infty}\int S(x-\xi,\ y-\eta)\ O(\xi, \eta)\ d\xi\ d\eta \qquad (4)$$

Assuming for the moment that this convolution is amenable to the techniques of the Fourier transform, we can do the same thing as (4) in the spatial frequency domain by utilizing equation (5).

$$i(\omega_x,\ \omega_y) = \tau(\omega_x,\ \omega_y)\ O\,(\omega_x,\ \omega_y) \qquad (5)$$

where $i(\omega_x,\ \omega_y)$ and $O(\omega_x,\ \omega_y)$ are the image and object expressed in terms of Fourier spatial frequencies and $\tau(\omega_x,\ \omega_y)$ is the so called "transfer function" of the system (for details see loc. cit. (16) p 232, et seq.)

26.2.2.4 Note clearly what has happened. We have replaced the convolution integral which is difficult to compute, by a product. The two equations of course say basically the same thing and their interrelationship is clearly seen by the more complete definition of the transfer function (loc. cit. (17), p26).

$$\tau(\omega_x,\ \omega_y) = \frac{1}{2\pi}\ \int_{-\infty}^{+\infty}\int S(x, y)e^{i(\omega_x x+\omega_y y)}\ dx\ dy \qquad (6)$$

Clearly we must be able to calculate, or otherwise determine, the spread function in (4) before the $i(\omega\ ,\ \omega\ )$ may be calculated theoretically. This is a sizeable task. It turns out, however, to be relatively simple to do it experimentally and this is effectively where the art stands at present. The technique, based on experimental determinations of $\tau(\omega_x,\ \omega_y)$ has lead to a new, although still controversial, method of evaluation of optical systems. Synthesis by use of this principle as a design method is still in its infancy.

26.2.2.5 Let us back off again and look at why equation (5) is so important an evaluation tool. The reason rests in part on the fact that the object and image are related in the spatial frequency domain by a multiplicative factor while in the spatial domain they are related by a complex summation. If we have two systems

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(16) Proceedings of Symposium on Communication Theory and Antenna Design AFCRC - TR-57-105 (ASTIA Document No. AD117067). While this symposium was aimed primarily at antenna designers, the organization of it was such that not only is the optics covered rather well by O'Neil and Parrent but also the basic mathematics and physical requirements are outlined in detail. One should note particularly the bibliography prepared by Parrent on Page M-1.

(17) O'Neil, Selected Topics in Optics and Communication Theory Itek Corp. (1958) Note - This publication has an exceptionally complete bibliography of work in this field.

(18) O'Neil, Publications of the Theoretical Optics Section, Itek Corp. (1958)

(19) Marechal, The Contrast of Optical Images and the Influence of Optical Aberrations, NBS Circular No. 526, p9, (1954)

(20) Elias, Optics and Communications Theory, JOSA, 43, 229, (1953)

(21) Hopkins, H. H., The Frequency Response of a Defocussed Optics System, Proc. Ray Soc (London), 321A, 91, (1955)

(22) Blanc-Lapierre, Upon Some Analogues Between Optics and Information Theory, Symposium on Microwave Optics, McGill University, (1953) - Proceedings published by Antenna Section Air Force Cambridge Research Center.

(23) Parrent and Drane, The Effect of Defocussing and Third Order Spherical Aberration on the Transfer Function of a Two Dimensional Optical System, Optica Acta, 3: (1956)

one of which clearly shows a better high frequency response than the other, we can be sure that this system will have the higher resolving power. Further the process of obtaining the sine wave response, or $\tau(\omega_x, \omega_y)$, will give (or usually does) the response at all frequencies and not only at the maximum resolvable condition as with the resolution target system. It is thus, theoretically, possible having $\tau(\omega_x, \omega_y)$ to predict the image for any object by the use of equation (5).

26.2.2.6 As might be expected, nothing is ever quite this rosy. Always there is the needle in the haystack or thorn in the rose. The difficulty here lies in the fact that the transformation from equation (4) to equation (5) presupposes that the optical system is perfectly linear and invariant over the object and image fields. Unfortunately this does not hold very well in poor systems. In good systems Linfoot and Fellgelt (24) have shown, however, that over the normal working field the assumptions are reasonably valid. A rather good discussion of the restrictions involved in making the jump from (4) to (5) has been given by Zucker (25) both for the case at hand, optical systems, and also for the allied problem-antennas. Much as it would be interesting to go into here more of the basic theory, the limitations of space require that we get on to the actual experimental techniques of measuring the transfer function and its applications. The interested reader will find the references given, however, replete with pertinent information. There are several methods of determining $\tau(\omega_x, \omega_y)$ or the equivalent, of which the following are representative only.

26.3 SINE WAVE TESTING WITH SINE WAVE TARGETS

26.3.1 The Schade system.

26.3.1.1 In Schade's original presentation, he demonstrated a system that, stripped to its basic features, was essentially that shown in Figure 26.8 wherein F represents a continuous film with a series of discrete sine wave targets. Each target was made by varying the intensity of the exciter lamp in a sound track camera sinusoidally with time while the film was moving at a constant rate through the camera. Sections of the film are shown in Figure 26.9 (26). P is a projector that allows the test pattern to be seen at any effective distance from the system under test, S. The light from S is focussed (usually with the aid of an auxiliary microscope) onto a scanning aperture, A. This aperture might be of any shape but usually it is most convenient to use a circle. Behind the aperture is a photomultiplier tube, PM, which feeds into a recorder, R.

26.3.1.2 In action then, the film moves through the projector producing a spatial frequency sine wave. The fact that the film is moving means that there will be a sinusoidally varying electrical signal from the photomultiplier tube. The sine wave response is then given simply by the ratio of this ac signal at a spatial frequency, N, to that which the system would give if the frequency were extrapolated to zero. In Schade's terminology $r_{\tilde{\psi}} = \tilde{\psi}_n/\tilde{\psi}_o$ where $r_{\tilde{\psi}}$ is the sine wave response. Typical sine wave response factor curves are shown in Figure 26.10. These response curves were taken from research done in this field by Shack (27) when at the National Bureau of Standards. Figure 26.11 from the NBS Report gives the variation of $r_{\tilde{\psi}}$ with focal position for a fixed spatial frequency while Figure 26.12 gives the variation of $r_{\tilde{\psi}}$ with focal position for a fixed color. Figures 26.13 and 26.14 show the variation of $r_{\tilde{\psi}}$ with spatial frequency for different colors. Note the negative amplitude in these figures. It is due to a 180° phase change. Schack's apparatus was much the same as Schade's but Shack used a scanning slit instead of a scanning pinhole.

26.3.2 The Lamberts system.

26.3.2.1 Lamberts (28) and Lamberts, Higgins, and Wolfe (29) have studied the sine wave response particularly in connection with their lens evaluation program at Eastman Kodak. The reader will find Lamberts' article particularly interesting as he not only discribes the basic theory very lucidly but also presents a rather novel variation on the fundamental method.

26.3.2.2 In the Schade method the scanning aperture is very small and usually circular or square. In the Lamberts system the scanning aperture is a long slit. By the use of the slit it is possible to replace a target whose intensity varies sinusoidally by a target with a variable area as shown in Figure 26.15. This type of target has also been used by Lindberg (30). The scanning slit is indicated by SS in Figure 26.16. It can be shown the light distribution in the image is given by,

$$F(x) = b_o + b_1 \ |A*| \ \cos(2\pi\gamma x - \phi) \tag{7}$$

Where b_o and b_1 have the meaning shown in Figure 26.15, and b_1/b_o is the "normalized amplitude" as discussed in Lamberts' article. γ is of course the spatial frequency and x is the shift of any particular aspect

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(24) Linfoot and Fellgelt, On the Assessment of Optical Images, Trans. Roy. Soc. (London) 247, (1955)

(25) Zucker, loc. cit. 5, p L-1

(26) Schade loc. cit. 1, p 233

(27) Shack, Investigations Into the Correlation Between Photographic and Photoelectric Image Evaluation, NBS Report No. 5483

(28) Lamberts, JOSA 48, 490 (1958)

(29) Lamberts, Higgins, and Wolfe, JOSA 48, 487 (1958)

(30) Lindberg, Optica Acta 1, 80 (1954)

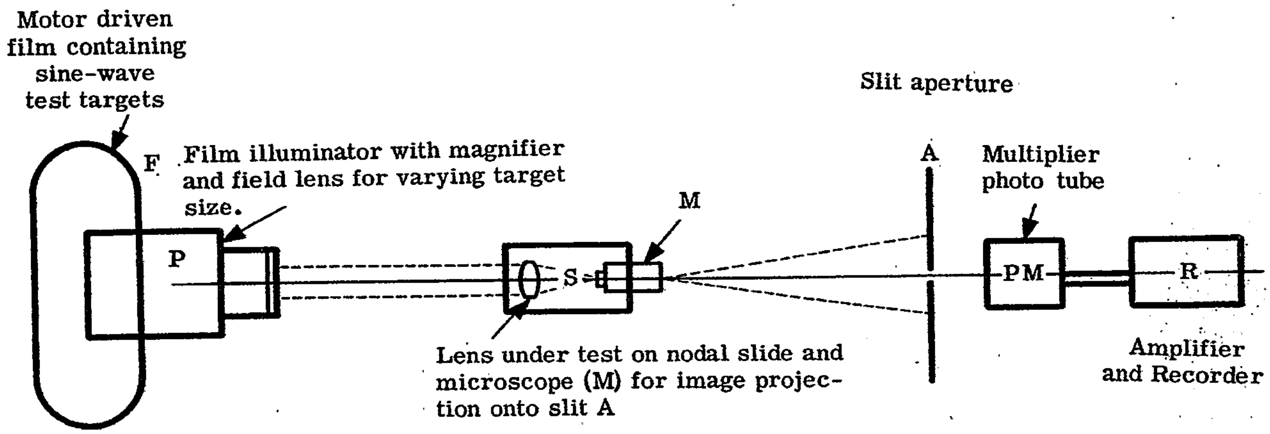

Figure 26.8 - The basic Schade system for determining the sine wave response of an optical system.

(Based on O.H. Schade's, Electro-Optical Characteristics of Television Systems, RCA Review, Vol. 9, 1948)

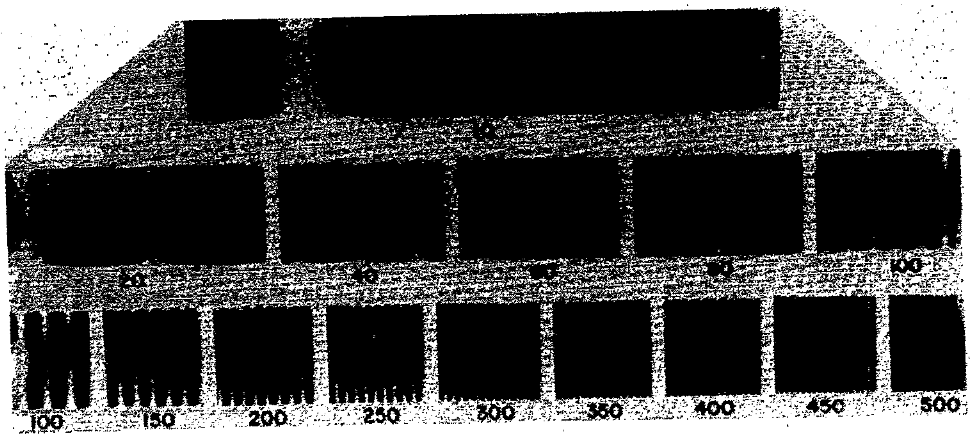

Figure 26.9 - Sine wave test targets.

(Based on O. H. Schade's, Electro-Optical Characteristics of Television Systems, RCA Review, Vol. 9, 1948)

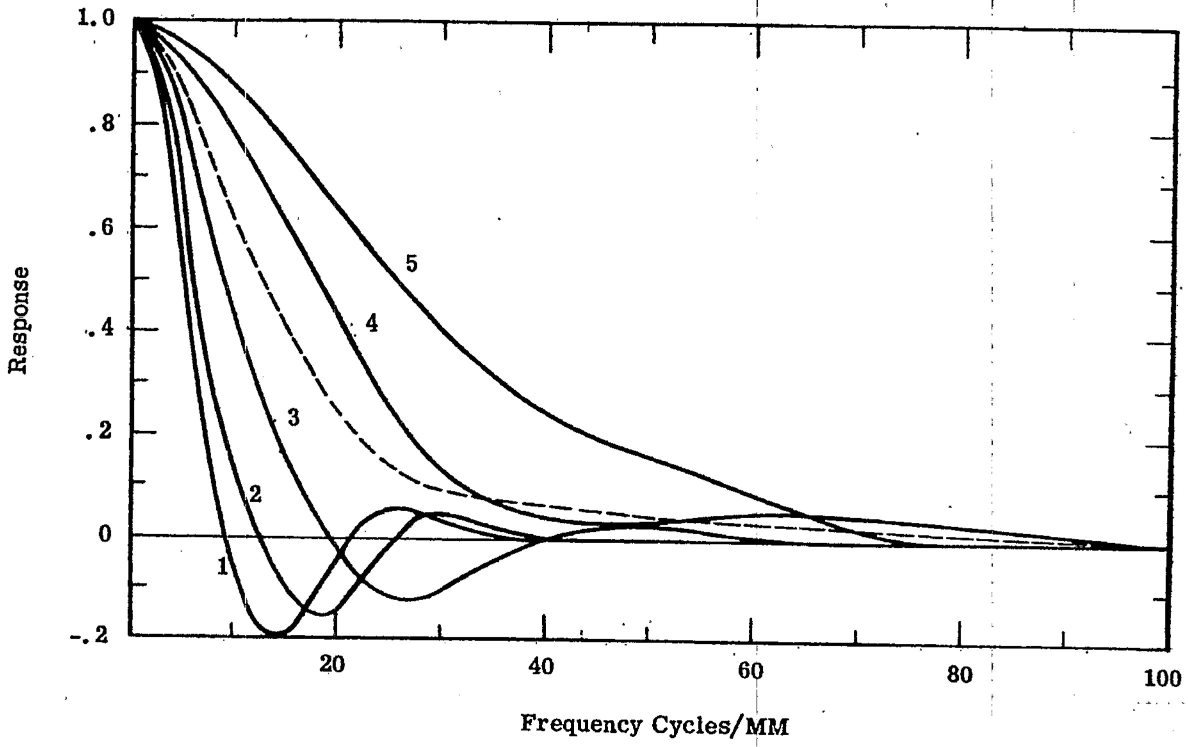

Figure 26.10 - Sine wave response factor vs line number (frequency) of lens A, .4 mm inside focus. In this and in following figures, curves numbered 1, 2, 3, 4 and 5 were obtained with Wratten filters 29, 25, 90, 16 + 60, and 45 respectively. The dashed curve was obtained with no filter. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

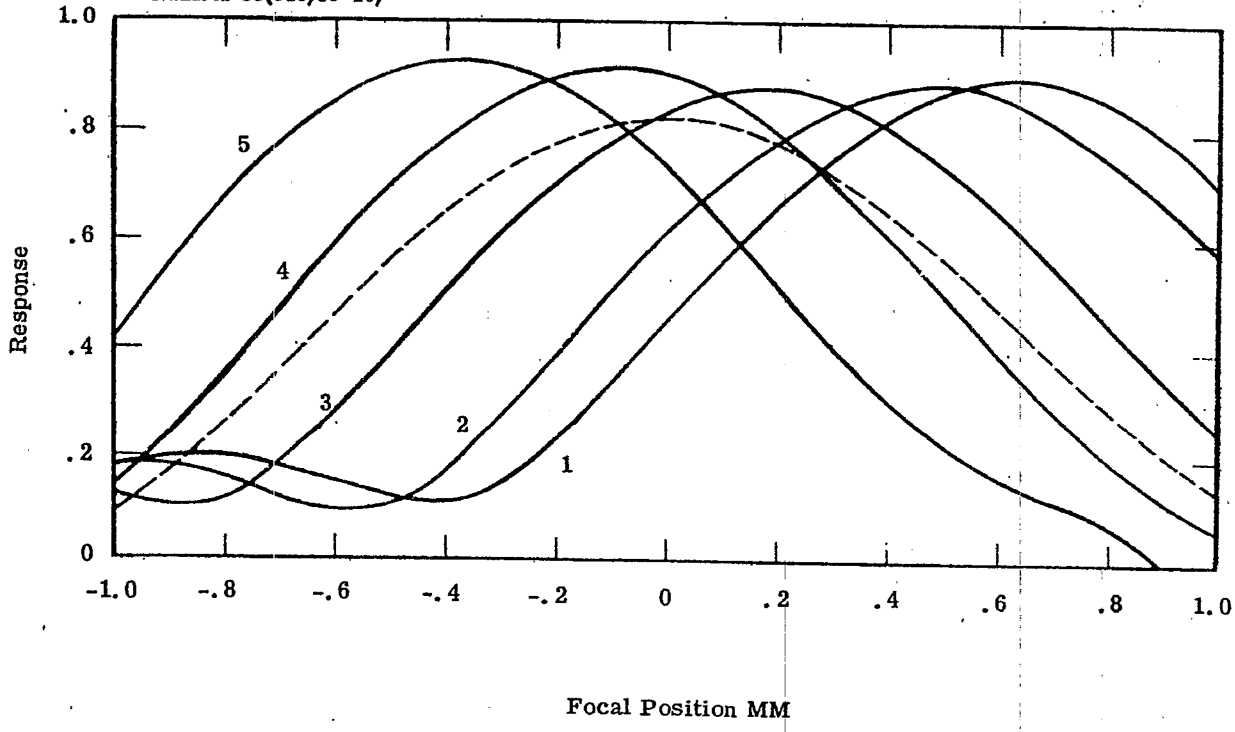

Figure 26.11 - Variation in response with focal position for lens A for different colors at a fixed frequency. The frequency chosen was 8 cycles per mm. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

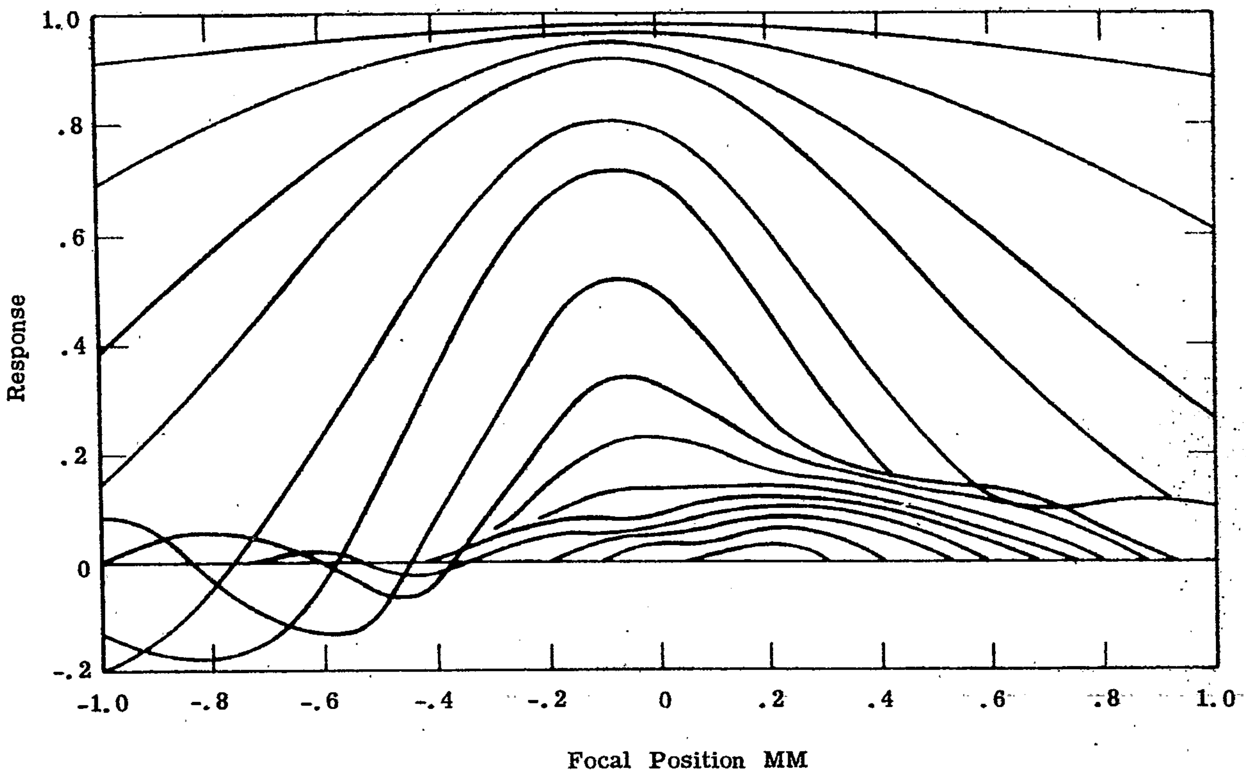

Figure 26.12- Through-focus response curves for lens A with filter 16 + 60.
(Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

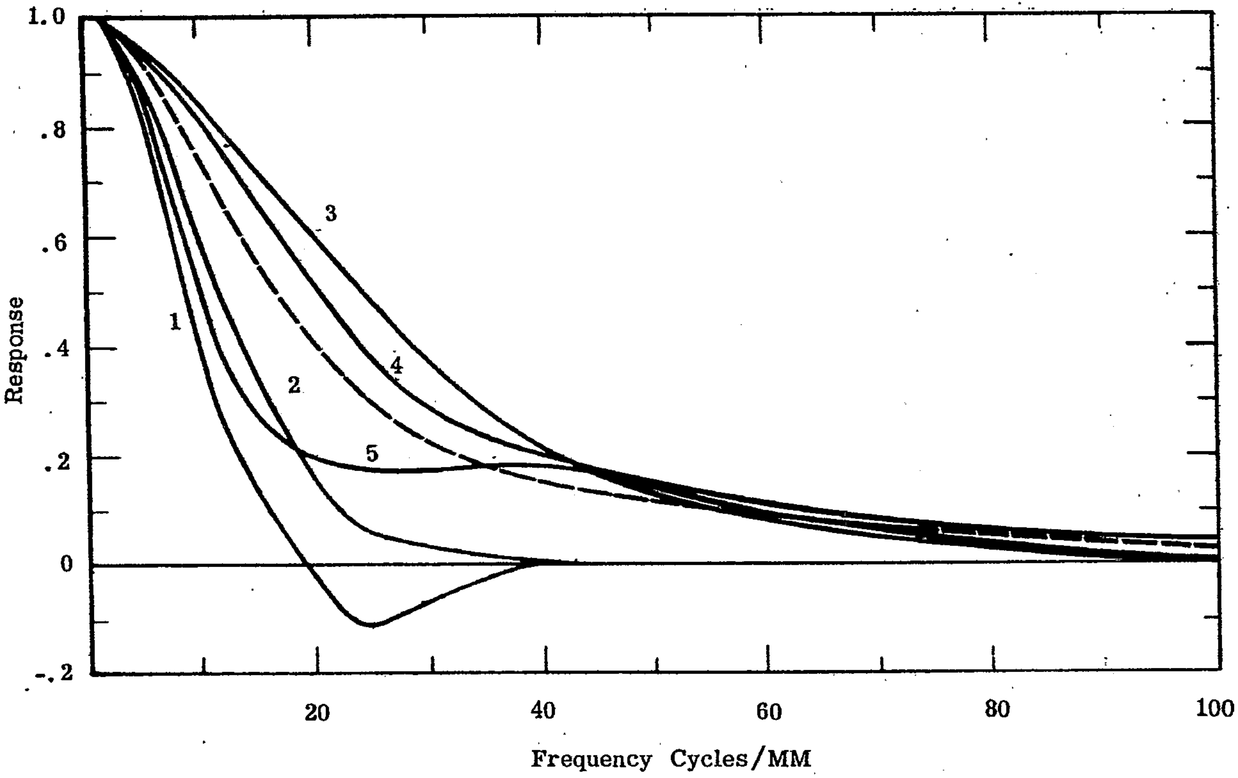

Figure 26.13- Frequency response of lens A at focus for various colors.
(Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

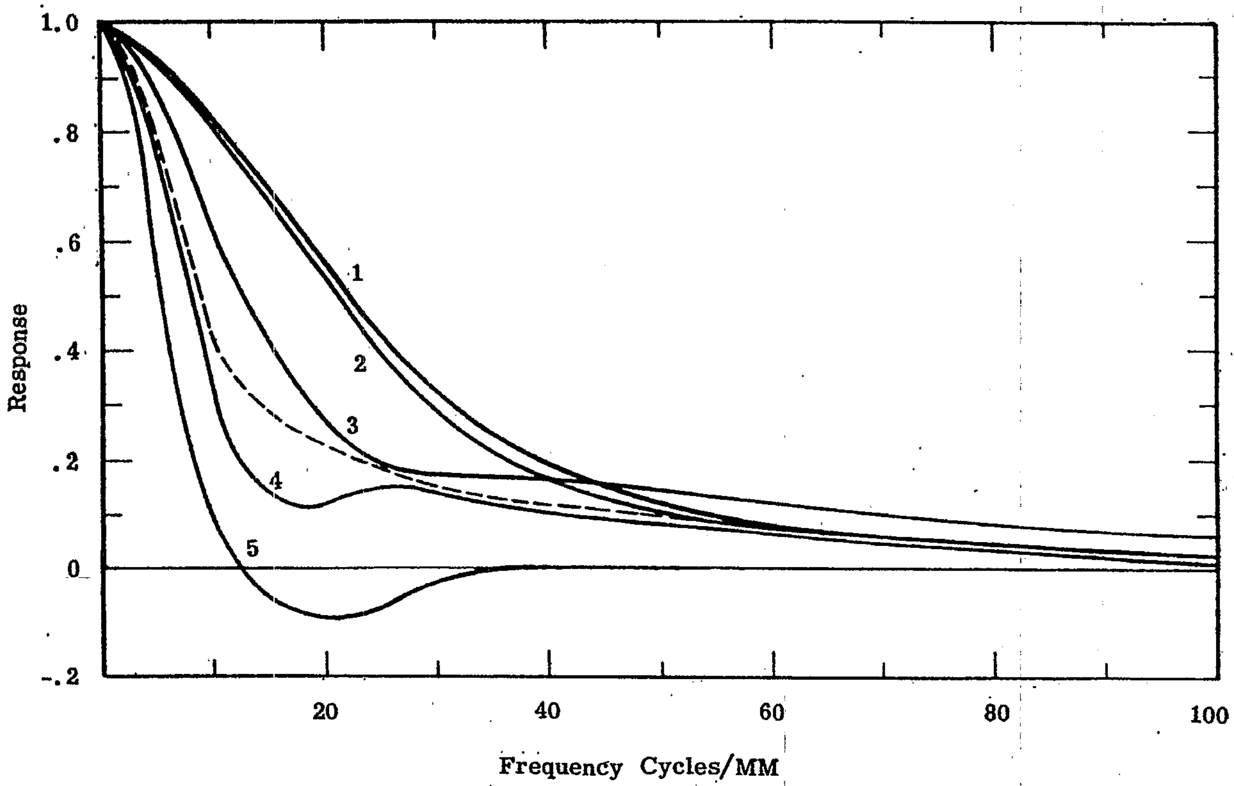

Figure 26.14- Frequency response of lens A .4 mm outside focus for various colors. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

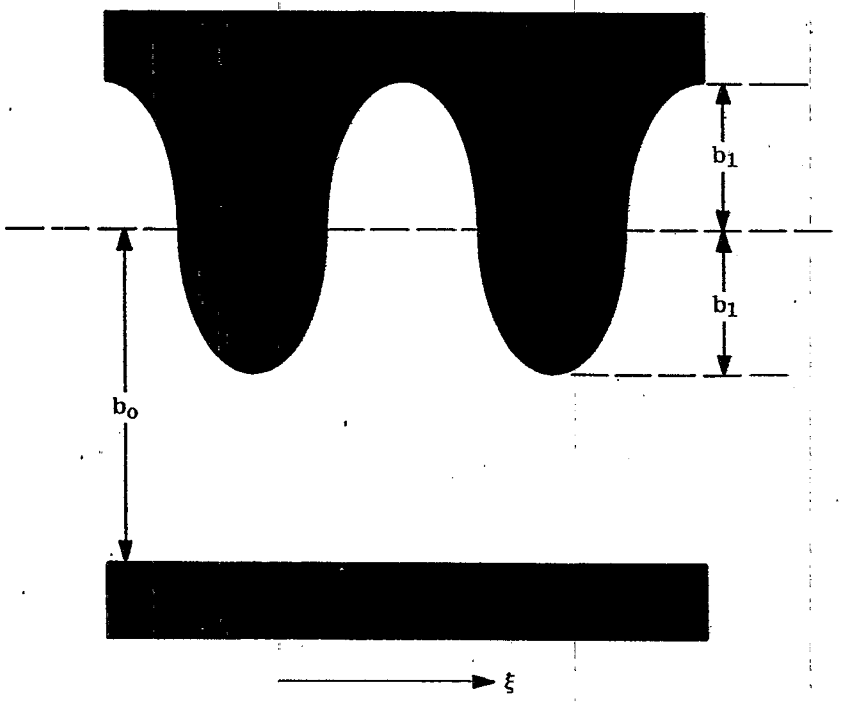

Figure 26.15- The Lambert's test object for measuring sine wave response. (Extracted from National Bureau of Standards Report No. 5483, Investigations into the Correlation between Photographic and Photoelectric Image Evaluation, R. Shack, under Air Force Contract Number 33(616)56-16)

of the target. ϕ is the spatial phase angle between object and image and is something not covered specifically in Schade's original work. A^* is the sine wave response previously defined. The lens bench used in the experiments set up to confirm the theory is shown schematically in Figure 26.16. TO is the test object, which for this work was either a slit (used to determine the spread function) or the target shown in Figure 26.9. L is the lens under test, with SS being the scanning slit, and P the photomultiplier and recorder ensemble. T is a tangent bar arranged to tilt the object and lens, L, when studying off-axis response.

26.3.2.3 The action of the system is similar to that of the basic method, and the reader may refer to the original article for further details. The reader should pay particular attention to the excellent discussion of the significance of the spatial phase angle, and the symmetric and asymmetric spread functions. Attention is also called to the discussion of the derivation of the spread function from the sine wave response. This is important when one remembers that in the introduction to this section the spread function was defined first, with the sine wave response introduced subsequently as a dependent variable. The fact that the one may be calculated from an experimental determination of the other bears out the statement made earlier about their relationship.

26.3.2.4 The significance of phase angle is pointed up in discussions of objects that represent coherent, incoherent, or partially coherent sources. Even with the simple systems checked by Lamberts the phase angle was a strong function of spatial frequency. Figure 26.17 shows both the normalized amplitude in percent (directly relatable to sine wave response via equation 7) and the phase angle as a function of spatial frequency in lines/mm for a certain lens.

26.3.2.5 Stephens (31) has recently indicated an interesting way of determining experimentally not only the cosine of the phase angle but also the sine of the phase angle. The advantage is that of increased precision for angles up to 45°.

26.3.3 The recording electronic lens bench of Herriott.

26.3.3.1 The recording lens bench we are about to describe is a long way from the first exploratory efforts in this field. Actually this lens bench is similar in purpose to the K.D.C. apparatus in that each was designed not so much to do research work as to check out large number of lenses routinely by their respective techniques. The target for this apparatus was first made by W. Herriott (32) and is shown in Figure 26.18. Note carefully that the spatial frequency varies continuously on the actual target with samples taken discontinuously along the length of the film to show the variation in the spatial frequency. The scanning slit is oriented vertically with respect this page. The target is on a 36 in. strip of 35mm film with 50 parallel opague tracks on 0.010 in. centers. The slit is a few microns wide and long enough to span most of the width of the 50 tracks.

26.3.3.2 In use the target film is wound around a drum inside of which is the light source and appropriate motors and clutches. Attention is called to the fact that the target does not directly present a sinusoidal variation of intensity to the optical system under test. The scanning slit, however, integrates the image over its length and the result is effectively the same as with the Schade system. The complete schematic layout of the system is shown in Figure 26.19.

26.3.3.3 In this method the sine wave response is measured by the contrast rendition which is defined as "$\frac{\text{image max - image min}}{\text{object max - object min}}$". Defined in this way, the result is independent of the contrast in the object, a point about which there was much discussion in connection with resolving power targets. The contrast rendition is plotted automatically as a function of spatial frequency. A typical recording showing the result of a through focus test is shown in Figure 26.20. (33)

26.4 SINE WAVE TESTING WITH SQUARE WAVE TARGETS

26.4.1 General discussion.

26.4.1.1 One of the problems involved in sine wave testing is the actual production of the sine wave targets themselves. This has proved to be a major problem, particularly so as the demands of the theorists got tighter and tighter. One method has already been outlined above. Other techniques have been developed (34 - 36) but the fact remains that it is still easier to make a square wave target than a sine wave target. The question has naturally arisen "can we not utilize the known Fourier sine wave content of a square wave to produce the equivalent of a pure multiple frequency sine wave target?" The answer is "yes" with some restrictions. If

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(31) Stephens, Computation of Achromatic Objectives, NBS, (1954)

(32) Herriott, W., JOSA 37, 472 (1947)

(33) Herriott, D., JOSA 48, 968 (1958)

(34) Kapany and Pike, JOSA, 46, 867 (1956)

(35) Kapany, Eyer, and Shannon, JOSA 47, 103 (1957)

(36) Kelly, Lynch, and Ross, JOSA 48, 858, (1958)

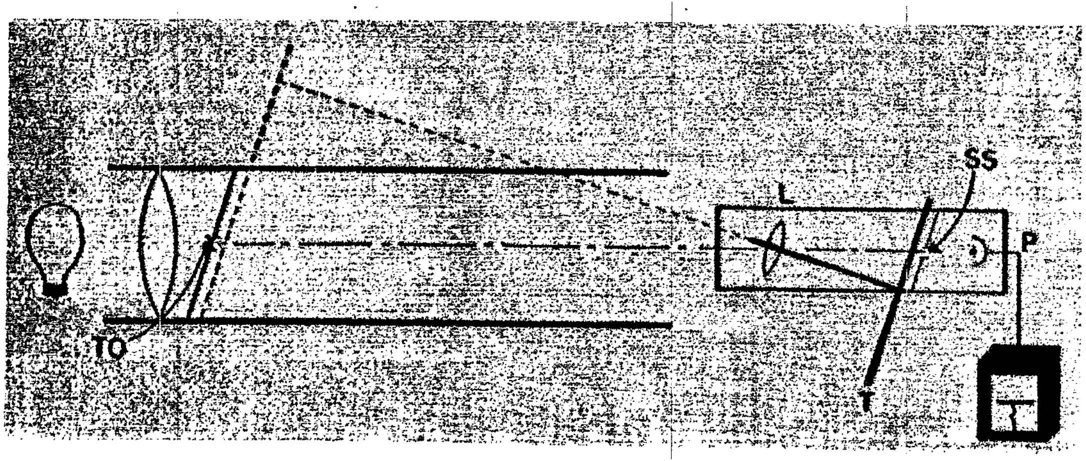
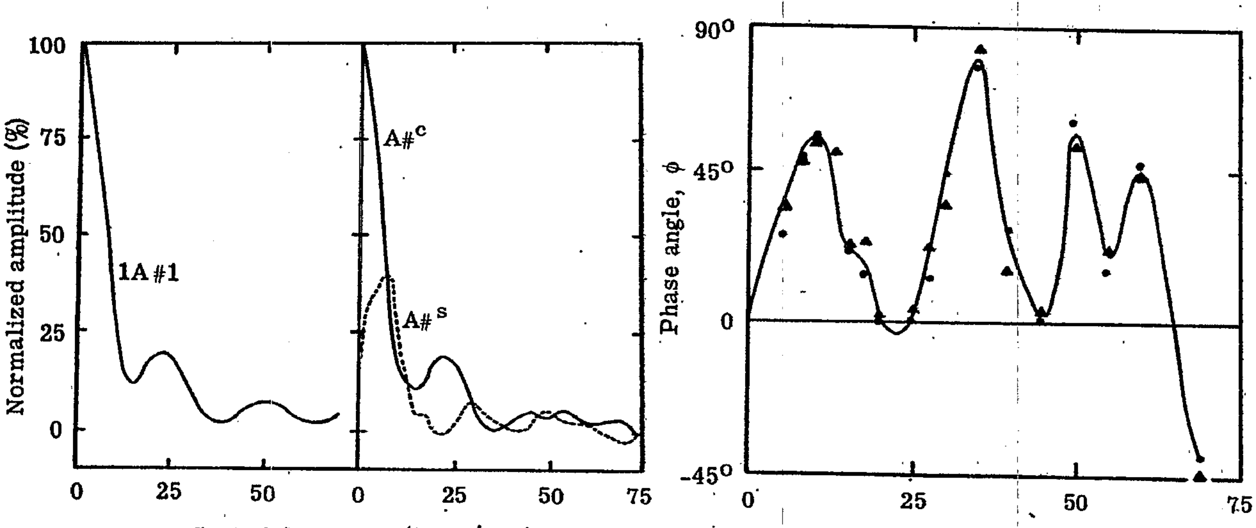


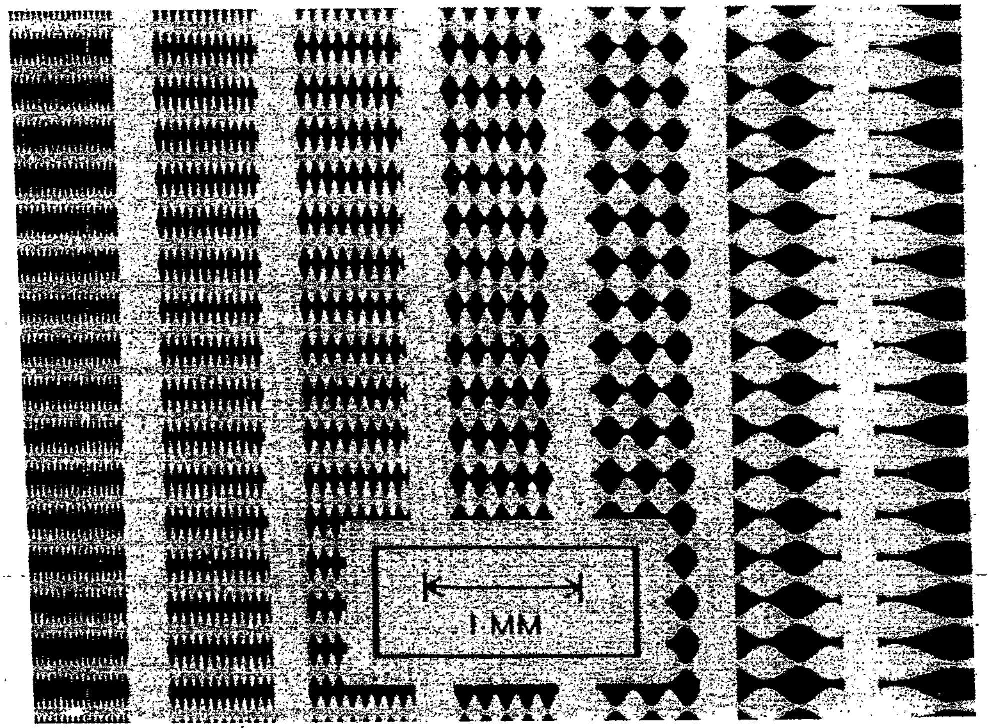
Figure 26.16 - Lambert's lens bench for determining sine wave response factors.



(a) Sine-wave response |A#| (left) and Fourier transfoms of it A#^{c} and A#^{s} (right) for a certain lens. A single sinusoidal test object was used to obtain |A#| and a double test object for A#^{c}; A#^{s} was computed from the other two.

(b) Phase angle as a function of frequency for the lens of fig. (a). The curve represents the mean of the two determinations ● and ▲.

Figure 26.17 - Normalized amplitude and phase angle as a function of spatial frequency.
(From Jour. Optical Soc. America, Lamberts 89, 1958)



Enlarged photographs at intervals along a sinusoidal target on which the frequency change is continuous.

Figure 26.18 - The Herriott continuous spatial frequency target for determining sine wave response. (From Jour. Optical Soc. America, W. Herriott 37; 472, 1947)

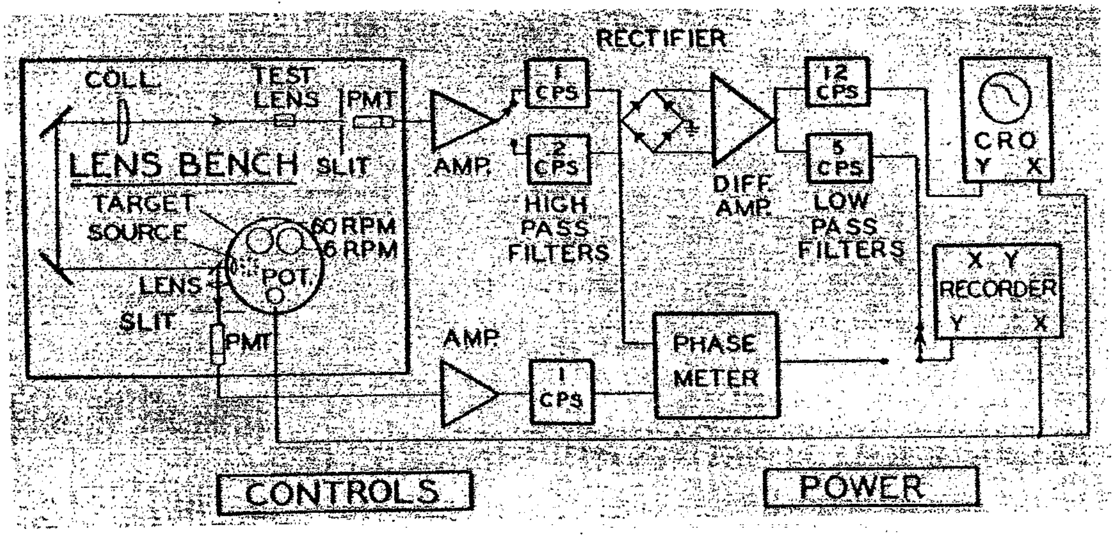


Figure 26.19 - Schematic diagram of the electronic system of the Herriott recording electronic lens bench. (From Jour. Optical Soc. America, W. Herriott 37; 472, 1947)

we suppose the optical system to accurately image a spatial frequency square wave such as a series of alternate dark and bright bars equally spaced, then the image may be represented as a spatial frequency series as,

$$F(x) = B_1 + \Delta B_1 \frac{4}{\pi} \left[\cos\left(2\pi n \frac{x}{\delta}\right) - \frac{1}{3} \cos 3\left(2\pi n \frac{x}{\delta}\right) + \frac{1}{5} \cos 5\left(2\pi n \frac{x}{\delta}\right) \ldots \right] \qquad (8)$$

Where x is the lateral coordinate and is defined as the width of a rectangle with the same area and height as the aperture flux distribution as shown in Figure 26.21 taken from Coltman (37). The square wave response factor is defined then as

$$r(n) = \frac{\Delta B_2/\Delta B_1}{B_2/B_1} \qquad (9)$$

and the sine wave response factor is defined as

$$R(n) = \frac{\Delta B_2/\Delta B_1}{B_2/B_1} \qquad (10)$$

26.4.1.2 It should be noted that there is a variation in the definition of response factors from author to author. This is clear if the reader will go back and check the definition of similar terms by Schade and D. Herriott. The end result in each case is essentially the same and one definition can be converted into another with no basic change in principle.

26.4.1.3 Coltman (38) shows that r(n) may be expressed in terms of the sine wave responses R(n) as given in equation (11).

$$r(n) = \frac{4}{\pi} \left[R(n) - \frac{R(3n)}{3} + \frac{R(5n)}{5} - \frac{R(7n)}{7} \ldots\ldots \right] \qquad (11)$$

solving for R(n) by successively subtracting series for $\frac{r(Kn)}{K}$ we can get,

$$R(n) = \frac{\pi}{4} \left[r(n) + \frac{r(3n)}{3} - \frac{r(5n)}{5} + \frac{r(7n)}{7} \ldots\ldots \right] \qquad (12)$$

The reader should see Coltman for the details. Suffice it to say that we have now expressed the sine wave response at a spatial frequency of n, the number of cycles in some unit distance. There are basically two ways of determining R(n). These will now be discussed.

26.4.2 The Coltman variable frequency square wave method.

26.4.2.1 The Coltman technique is similar in principle to the corresponding technique used in testing electrical amplifiers (39) with variable frequency square waves. Others such as Rosberry have also studied the method. It is usually found to be more trouble than it is worth to test electrical amplifiers this way, since if you have to vary the frequency of square wave, you might just as well vary the frequency of a sine wave and be done with it. In the optical case it is easier to vary the frequency of the square wave because spatial square waves can be made more easily than can spatial sine waves.

26.4.2.2 Coltman's method is similar, then, in principle to that discussed in Schade and Herriott's paper except for an analysis (40) that allows him to measure the sine wave response of the system by use of the more easily manufactured square waves. Not only is Coltman's article highly informative but it also gives an excellent discussion of the basis of the method and a specific example in the field of X-ray fluoroscopic work. Here the relative ease of studying systems in cascade by the sine wave method is shown and a discussion as to why sine wave targets are not used is given.

26.4.3 The fixed frequency square wave method.

26.4.3.1 Suppose that instead of variable spatial frequency square wave, we used a fixed spatial fequency square wave and get the higher frequency components by wave analysis of the electrical output of the photomultiplier tube. We now assume that the combination of scanning pinhole and associated photomultiplier circuit that transduces the spatial frequency to a temporal frequency spectrum in the image can be directly

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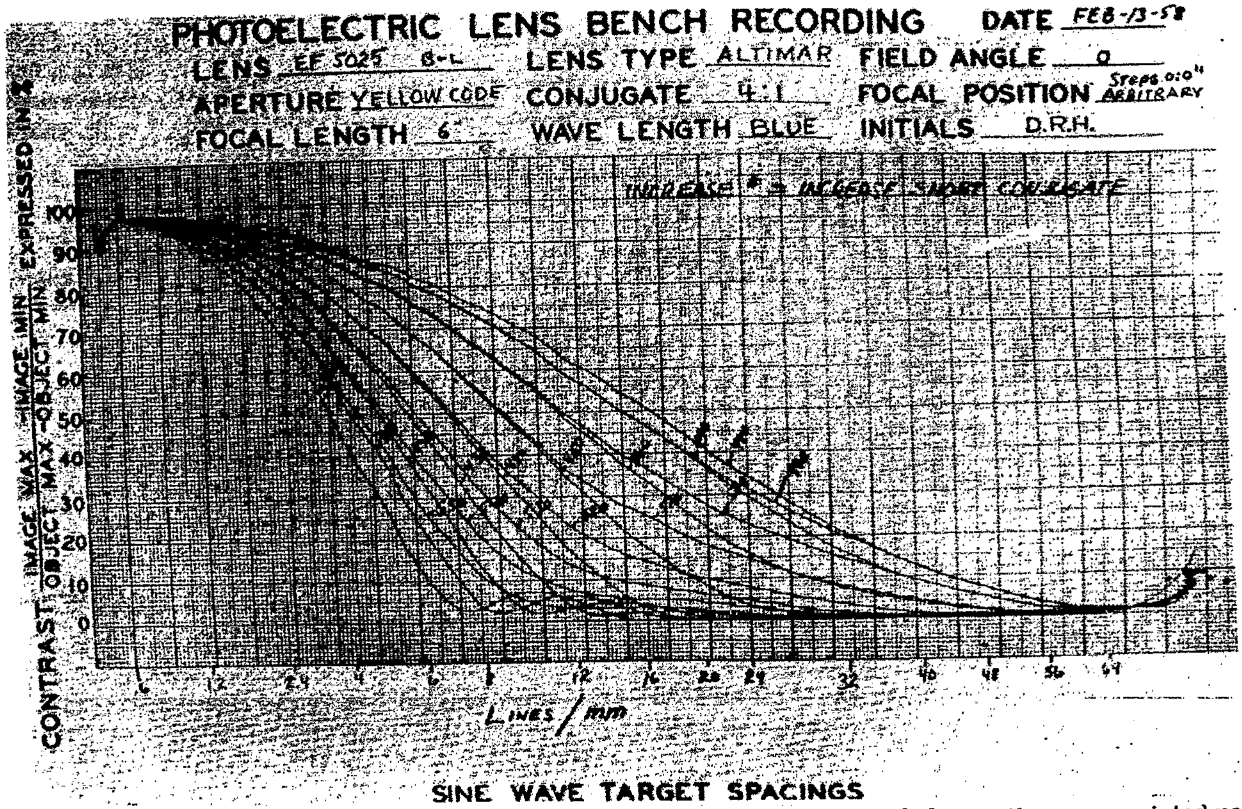
(37) Coltman, JOSA 44, 468, (1954)

(38) ibid.

(39) Rosberry, A Correlation Investigation Between Photoelectric and Image Analysis, NBS Report No. 5799

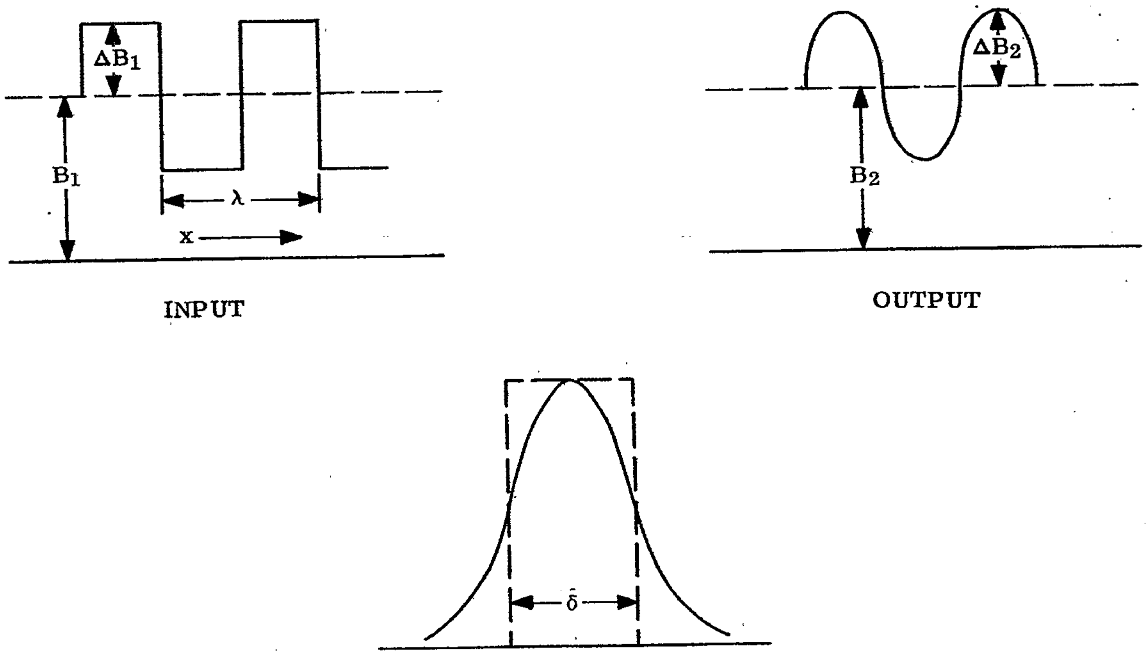
(40) Loc. cit.,



Curves of contrast rendition measured through focus and recorded directly on preprinted paper.

Figure 26.20 - Sample contrast rendition vs spatial frequency recording taken with the Herriott system.

(From Jour. Optical Soc. America, D. Herriott 48; 968, 1958)



APERTURE FLUX DISTRIBUTION

Figure 26.21 - Quantities used in the Coltman definition of response factors.

(From Jour. Optical Soc. America, Coltman 44; 468, 1958) related to the spatial frequency. Furthermore electrical tunable narrow band temporal frequency filters are standard items and have been for years. Therefore by feeding the output of the transducing element to a temporal frequency filter tuned to the fundamental of the square wave we can determine the sine wave response at that frequency. We then retune the filter to the next harmonic, record the output, etc. Account must be taken of the reduction in amplitude of the harmonics as given by the coefficients in equation (8).

26.4.3.2 It might occur to those versed in Fourier analysis that some other wave shape might be chosen such as a triangular wave that has all harmonics and not just the odd harmonics as in the case of the square wave. Such a wave could be used, but again the problem of production is such that it probably would be undesirable. Actually a single square wave target can suffice to cover almost any desired spatial frequency band - provided it is used with a minifying or magnifying system whose quality is far superior to that of the system being tested.

26.4.3.3 The difficulty involved with this method of square wave testing is that there is a phase shift associated with the tunable electrical filter. While there are ways to take this into account, they are rather complicated. Furthermore, it was assumed above that the percentage reduction of the amplitude as a function of frequency in the ideal image was known. This is true providing the detecting system is completely linear. For some systems, notably photographic ones, this may well not be so. Hence while we can get the sine wave response at any frequency, it may be difficult to relate it numerically to the response at other frequencies.

26.4.4 Automatic determination of power of an opthalmic lens by sine wave response.

26.4.4.1 In 1953 Gunter and Panetta (41) developed a method of applying the sine wave response criteria to the problem of automatically maintaining large aerial cameras in focus. The aerial camera aspects of the technique are of not so much interest to us here as is Gunter's definition of best focus used in connection with their analysis of the problem viz, "best focus is that point in image space where the spatial frequency response is an optimum within the bandwidth of information in which the observer is most interested." This definition is certainly a far cry from that usually found in optics and photography. It stems from the work of Schade rather than from that of the traditional treatments of Conrady etc.

26.4.4.2 Shortly thereafter Gunter (42), (43), (44) applied these same principles and this same definition of focus to the automatic determination of the power of opthalmic lenses. This was a research problem to see if the human factor could be removed in the routine inspection of opthalmic lenses. The women who customarily do this work are wont to get tired and their judgment varies. The first target was a square wave made by rotating a square cut gear as shown in Figure 26.22.

26.4.4.3 In the initial study the combination of SS and PM was moved along a lathe bed until the meter showed a maximum, the bandwidth of information having been selected by trial. Specifically this meant that a lens of say 2 diopters as judged by the eye was selected. This lens was placed in the test device and the temporal frequency filter adjusted until the meter output was maximum at a distance of exactly 50 cm. By checking with other standard lenses the variation of focal point as judged by the maximum meter response and the eye were shown to be well within commercial tolerances. A plot of meter response vs. focal position looked essentially the same as Figure 26.12.

26.4.4.4 The technique having been proved, the quest was now how to change the system so that a lens could be snapped into place and have the SS-PM unit automatically move so as to maximize the meter response i.e. move to the position of best focus. Important in the final system was a novel "square wave target" suggested by Hayes. The original square cut gear (seen from above) was modified as shown by the dotted lines in Figure 26.23 the hatched part of the gear remaining. The lens under test sees sequentially now edge a, b, a', b', a'', b'' etc. This is the equivalent of a square wave tipped at an angle of 45° in so far as L_x is concerned. The light from the source S was focussed midway between a and b so that the edges a and b appeared equally sharp to L_x. The action is simple. Referring to Figure 26.12 we see that near the peak the response curve as a function of focal length is quite symmetrical. An electronic circuit separated the responses from edges a and b and ordered a motor to adjust the position of the SS-PM unit until the responses were equal. The motor then stopped and the power of the lens was read directly from the scale. The system was easily more than sufficiently accurate. Modifications of the system were developed for specific purposes but the basic technique was unchanged.

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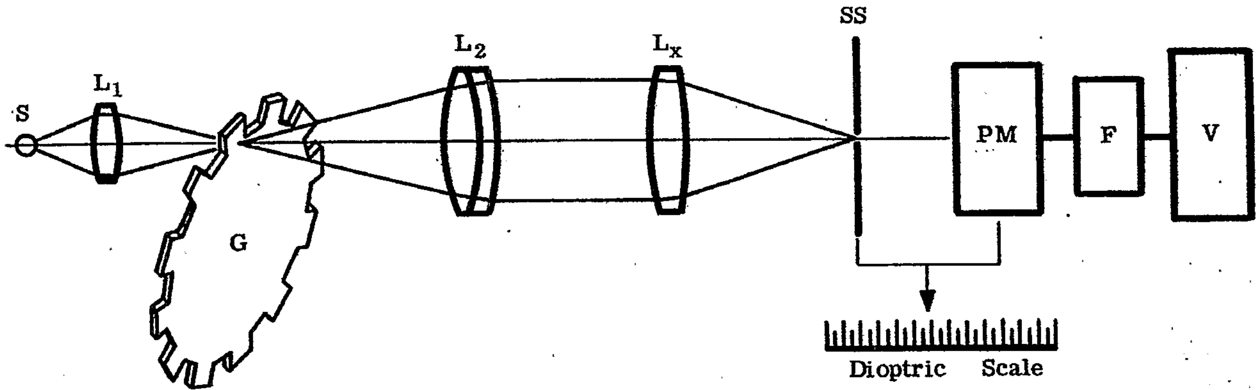
(41) Gunter and Panetta, An Automatic Electronic Focussing Device for Aerial Cameras, Boston University Optical Research Laboratory Technical Note 113, June, 1954

(42) Gunter, Whitney, Hayes. U. S. Patent 2897722, Electronic Lensometer.

(43) Gunter, Whitney, Hayes. U. S. Patent 2803995, Special Frequency Centering Device.

(44) Wing, Whitney, Hayes. U. S. Patent 2792748, Pyramid Centering Device.



S = source of light

L_1 = lens to focus light from S onto the teeth of G.

G = square cut gear rotated at 1800 rpm.

L_2 = a collimating lens.

L_x = the opthalmic lens under test.

SS = pinhole scanning aperture.

PM = photomultiplier tube and associated circuits

F = a tunable electric filter

V = voltmeter

Figure 26.22- Basic square wave system for studying Opthalmic lens power.

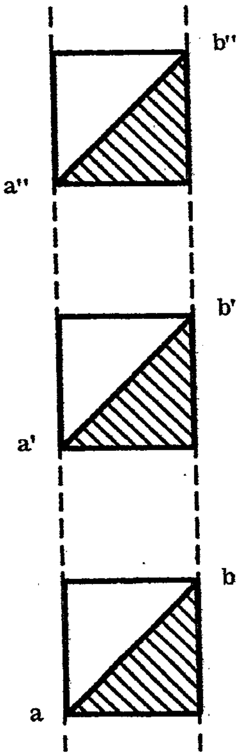


Figure 26.23- The Hayes target for the American Optical automatic lens power measurement system.

Custodians:
Army - U.S. Army Munitions Command
Navy - Bureau of Ships
Air Force - Middletown Air Materiel Area

Preparing activity:
Army - U.S. Army Munitions Command