# 24 DESIGN PHASE OPTICAL TESTS

## 24.1 INTRODUCTION

24.1.1 Uses. Optical testing methods are widely used in all branches of scientific and technical work. The basic techniques, or modifications thereof, enable some of the most sensitive and precise measurements man has ever known. Gage blocks may be measured to better than 0.0000001" with relatively simple interferometric apparatus while velocities of satellites hundreds or thousands of miles away may be measured with the Doppler shift techniques common to older astrophysics problems.

24.1.2 Related fields. It will be noted that from time to time reference will be made to work that has been done in the field of microwave antennas. This has been done in the belief that it will be very instructive to become acquainted with design techniques involving wavelengths that are frequently approaching a tenth of the radiating aperture. Further, the use of aberrations, interference, diffraction, and control of aperture illumination are discussed and demonstrated in a way frequently difficult at light optics frequencies. The very recent achievements in light optics where the aberrations are all reduced (save color) to the diffraction-limited stage is many years old in the microwave-antenna field. Microwave antenna designers borrowed heavily from older optical techniques and it is quite possible that a study of their efforts will be highly rewarding to the light-optics designer.

24.1.3 Methods and problems pertinent to optics.

24.1.3.1 While these methods cover a wide gamut, discussion in this section will be confined to a small sampling of the methods particularly suitable to the design, construction, and evaluation of visual optical systems. A few words regarding the origin of the testing problems which will be encountered will be in order.

24.1.3.2 The design of an optical instrument is obviously predicated on a need having been established. Sometimes the nature of this need is such that electrical and mechanical considerations may dictate, to a considerable extent, the physical shape of the optical system. However, even after this has been determined there still remains the problem of translating the customer's purely optical requirements into a form that is significant to the lens designer. Field of view, curvature of field, transmission over a given spectral band, distortion, etc. can be specified rather accurately and unambiguously. Questions, however, as to image quality and what figure of merit is to be used in deciding whether this or that design will most closely give the customer the information he seeks when he uses the instrument, raise problems that have yet to be solved completely. There seems to be more and more evidence of late that to phrase the problem in this way--viz. that the optical instrument be an "information handling system" -- is preferable to the more vague requirement that it be a system that forms a good image. Agreed, the former actually sounds more vague, but current effort indicates the above sentence is probably correct.

24.1.3.3 The postulating of a figure of merit implies that one must test proposed designs to see if they meet the assumed theoretical criterion. Once the design is firm, the optician takes over and now he must perform tests to see that his construction faithfully follows the prescription given to him by the lens designer. Here we must point out that there is another testing step necessary. The optician's job may be considered complete and accurate when the radii, edge thicknesses, center thicknesses, spacings, indices, etc. agree with the specifications handed down by the lens designer. The fact that the optician's work is presumably accurate does not, however, serve as a complete check on the usability of the system. It must be remembered that the designer used some theoretical criterion such as amount of energy in a point image, phase front or Seidel aberrations. The next step therefore is to see how well the constructed system lives up to his predictions in one or more of these respects.

24.1.3.4 There is little doubt that the ultimate test of any system is a field test under the original conditions imposed in the customer's specifications. A system can conceivably be excellent in the laboratory and yet be so sensitive to vibration that it is useless in the field. Further laboratory testing under simulated field conditions is therefore indicated; installation in field equipment being attempted only after the prototype has been tested thoroughly in the laboratory.

24.1.3.5 Here is another point that should be strongly raised. Granted that field tests are the ultimate in one sense, we should not lose sight of the fact that the nature of field tests frequently is such as to cloud the performance of the optical system by the introduction of parameters not basically a part of the problem. The writer clearly recalls airborne cameras yielding several hundred lines per minute resolution in the laboratory and only 20-30 lines per minute in the air. The trouble was definitely not with the camera or optical system but rather with the mechanical mounting in the plane. Some more or less absolute standard of perfection based on the customer's optical requirements is therefore mandatory. Tests in this category are extremely valuable. Resolving power, sine-wave tests, etc. fall into this category.

### 24.1.4 The testing program.

24.1.4.1 A consideration of the principles outlined above indicates that the complete testing program rather naturally falls into the following categories. It should be pointed out that many more types of tests are known in each category, but space permits only this limited sampling.

24.1.4.2 Testing during the design phase.

(1) Calculation of the Seidel Aberrations

(2) Calculation of the Spot Diagrams

(3) Determination of the phase front and perhaps the predicted diffraction by knowing the phase front and amplitude distribution over the aperture.

24.1.4.3 Testing during the manufacturing phase.

(1) Foucault Test

(2) Star Test

(3) Ronchi Test

(4) Interferometric Tests and/or determination of phase front.

(5) Measurements of curvature of field, astigmatism, transmission, field of view, front and back focal lengths etc.

24.1.4.4 Testing during the evaluation phase.

(1) Any or all of the tests in 24.1.4.3 above.

(2) Measurement of the resolving power.

(3) Measurement of the sine-wave response.

We will now proceed to discuss each of these tests.

## 24.2 CALCULATION OF THE SEIDEL ABERRATIONS

24.2.1 Object-image relationship. From a strictly theoretical point of view, an optical system may be said to be perfect if its response is "collinear" i.e. points are imaged as points, lines are imaged as lines, and planes are imaged as planes. A further qualification is required--namely that the definition just given applies strictly and only to an optical system where the magnification is unity for all image points. While such systems do have significance, most optical systems require either minifications (telescopes, field cameras, etc.) or magnification (microscopes, etc.). We therefore qualify the concept of collinearity by adding that magnification or minification may exist, but should be constant for all points in the image. The above definition, even with its qualifications, applies more to photographic than to visual optical systems because of the reference to a flat focal surface. While curved focal surface systems have been used in photography, they are rare because of the practical problems involved in film handling. Almost all photographic systems require a flat focal surface, i.e. a focal plane. For visual optics we may relax this requirement somewhat. Indeed the ideal system is one whose curvature of field matches that of the eye.

24.2.2 The importance of Seidel Aberrations. It has been found possible by Seidel (1) to express the deviation of an actual image produced by a system, from the theoretically perfect system by a series expansion. This series expansion was given previously in Section 8. The monochromatic deviations from the ideal flat focal surface collinearity are called aberrations and include spherical aberration, coma, astigmatism, curvature of field and distortion. To the extent, then, that this series expansion accurately depicts what happens to an image point, the calculation of these Seidel aberrations constitute a powerful first approximation in the design of an optical system. It is equally clear that the calculation of these aberrations may be considered as a theoretical test of such a system. The method of calculating these aberrations, and the detailed significance of each has been previously treated. The subject is raised here again to point out the use of these aberrations in the theoretical tests which may be applied to an optical system. The reader should refer to Sections 8-10 for more details. It should also be pointed out that these aberrations are strictly geometrical and that

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(1) Seidel: Astronomische Nachrichten, 43, 289-332 (1856).

two different systems may have the same aberrations and yet show quite different images due to the fact that the wave nature of light is completely ignored (except for the variation of index with wavelength).

24.2.3 Seidel Tolerances. The criticism sometimes levied is that it is pointless to design a system on the basis of purely geometrical optics because of the neglect of interference etc. To our knowledge no optical system has been designed, at least in recent years, without reference to the wave nature of light. Frequently this is done by explicitly placing tolerances on the aberrations by reference to the Rayleigh (2) stipulation that the maximum path deviation from a given object point to a given image point be not more than $\lambda/4$. Discussions of this may be found in Conrady (3) and Martin (4). These optical tolerances are:

For primary marginal spherical,

$$\text{permissible primary } LA' = 4\lambda/N' \sin^2 U'_m \tag{1}$$

For primary zonal spherical, (assuming $LA' = 0$)

$$\text{permissible } LZA' = 6\lambda/N' \sin^2 U'_m \tag{2}$$

For primary Coma ($\text{Coma}_s$)

$$\text{permissible Coma}_s = \pm \lambda/2N' \sin U'_m \tag{3}$$

For focal range,

$$\text{Focal range} = \lambda/N' \sin U'_m \tag{4}$$

For astigmatism, Ast's

$$\text{permissible Ast's} = \lambda/4N' \sin^2 U'_m \tag{5}$$

For curvature of field,

$$\text{permissible } X' = \text{focal range} = \lambda/N' \sin^2 U_m \tag{6}$$

Note: $U'_m$ is the angle between the ray and the axis, $N'$ is the index of refraction in image space; and $\lambda$ is the wavelength of the radiation.

24.2.4 Use of the Seidel Tolerances. One should use these tolerances with exceeding care particularly with high speed systems. This occurs because the focal range allowed by the $\lambda/4$ path difference criterion is assumed small compared with the actual focal length. Secondly the field angle is assumed sufficiently small so that $\sin^2\left(\frac{U'_m}{2}\right) = 1/4 \sin^2 U'_m$. One should further regard these tests as representing a theoretical arbitrary standard which may be too tight or too loose in special circumstances. For fast systems (microwave antennas are a good example outside of the field of visual optics), the tolerance on spherical aberration as computed from (1) is too loose--usually by a factor of 4 or more. The tolerance on coma is too loose for many visual systems where the coma may be the most serious aberration and every attempt should be made to reduce it sensibly to zero. The astigmatic tolerance is usually too tight, and a lens may be expected to produce good results even if the astigmatic tolerance is exceeded by a factor of 2.

24.2.5 Conclusions. The subject of Seidel aberrations from purely geometrical optics is considered here in conjunction with tolerances imposed by physical optics because they have been the prime standards against which lenses were compared until the relatively recent present. Most lenses still are designed on this basis today although there are some who think that sine-wave response calculations may replace them in future years. In conclusion we may say that the reduction of aberrations to within, or at least close to, the stipulated tolerances is a necessary but not sufficient condition that to assure a lens so constructed will perform well. Actually the reduction of the aberrations to the specified limits results in a wavefront that is sensibly spherical in image space. The true image, however, involves amplitude as well as phase, and the Seidel aberrations give no explicit information regarding amplitude.

## 24.3 THE SPOT DIAGRAM

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(2) Lord Rayleigh, Collected Papers, vol. 1, pp. 415-459.

(3) Conrady, Applied Optics and Optical Design, pp. 136, 395, 434 et seq., Dover, (1957).

(4) Martin, Technical Optics, vol. 1, p. 139, Pitman, (1948).
also Jacobs, Fundamentals of Optical Engineering, 443, McGraw Hill, (1943).

24.3.1 Introduction. In the past, the labor involved in doing any but the simplest of ray tracing was such that relatively few rays were traced in the actual lens design process. With the advent of electric desk calculators, it became possible to trace more rays in the same time. As a result tracing rays out of the meridional or tangential fan became more common. It was not until the relatively recent present that the designer was freed of this time limitation by the development of the high-speed, electronic-computing machinery. It is now possible to trace hundreds of rays in the same time it took to trace just a few some years ago. This has resulted in lenses being designed much more carefully than ever before. The aberrations determined by tracing rays as just discussed are definitely an approximation that is very good under some circumstances, but the usual Seidel third-order aberrations are frequently misleading: higher order aberrations sometimes being dominant.

24.3.2 Aspherics. Another factor brought into being recently is the use of aspheric surfaces. Desk calculators or no, tracing through aspheric surfaces can be a monumental task when done by hand. There is ample evidence, however, that freed from the restriction of purely spherical surfaces, the designer can almost always do a far better job with aspherics than he can with spherical surfaces.

24.3.3 Development and limitations. One of the first testing techniques that took full advantage of the power of the large computers was that evolved by Herzberger (5) and later by Hopkins (6) and was called the "spot diagram." In essence the entrance pupil is divided into equal areas, and a ray is traced through the center of each area--the assumption being that the energy represented by each ray is the same. The intersection of these rays with an assumed focal plane was a spot, hence the term "spot diagram." The more compact this spot, the more nearly perfect was the lens judged to be by the standards of geometrical optics. This is discussed in Section 8. We should thus clearly realize that this technique is restricted to non-diffraction limited systems. In this connection we should also realize that while most optical systems today are not diffraction-limited, there is a growing class of high precision systems widely emphasizing aspherics where the only aberration left is color, and where the performance is almost an order of magnitude better than it was ten years ago. For such systems, the spot diagram can serve only as a rough first approximation. The vast majority of visual and photographic optical systems are aberration-limited rather than diffraction-limited so the spot diagram is still a powerful tool.

24.3.4 Techniques. There are basically two techniques for getting a spot diagram. In one the required number of rays is actually traced, and the intersection points with the assumed focal surface are plotted. In the other a relatively small number of rays is plotted, and the intersection coordinates of the others are obtained by an interpolation and extrapolation process developed by Herzberger. It should be noted here that the interpolation process does more than just give the intersection points. Via the series expansion required for the interpolation it also gives a set of terms not unlike those of Seidel. The difference is major, however, in that the Seidel aberrations work particularly well near the axis while the "Herzberger aberrations" fit well over the entire aperture. Space does not permit us to go more deeply into this use of spot diagrams, but the reader is encouraged to refer to Herzberger's articles on this subject (5), (7), (8).

24.3.5 Examples. Those interested in this subject are also urged to obtain National Bureau of Standards Report No. 5640 entitled "Numerical Analysis of a 6" f/3.5 Aerial Camera Lens (006BC035 - 15)". This report by Stavroudis and Sutton shows clearly the extent to which the spot diagram testing is currently employed. Not only are the spot diagram shown for various assumed focal plane positions and angles of obliquity, but also the values of vignetting, distortion, chromatic aberration, energy distribution, and resolving power are derived for this lens directly from the spot diagrams. It is interesting to note the excellent correction that seems to have been achieved in this lens. For full aperture the diameter of the Airy disk is 4.0 microns. If we inspect the following table, Table 24.1, taken from Stavoroudis report, we see that 80% of the total points fell within a circle on axis whose diameter was 3.93 microns. For an aberrationless system theory indicates there will be 83% of the total energy within the Airy disk. The close agreement between theory and spot diagram prediction indicates the excellence of the design, at least for on axis work. In another series of experiments Stavroudis and his colleagues at the National Bureau of Standards calculated the spot diagram of a completed lens. The comparison of the spot diagrams and corresponding actual photographs for two given positions is shown in Table 24.1.

## 24.4 PHASE FRONT CALCULATIONS

24.4.1 The spherical wavefront. It has been pointed out that the Seidel Aberrations, when they are fully corrected, result in a spherical wavefront converging on the image point. Modern computing machinery has enabled the designer to calculate directly the wavefront and thus determine not only the phase errors over the aperture but where the focal point should be placed.

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(5) Herzberger, J. Opt. Sec. Am 37, 485 (1947).

(6) Hopkins, J. Opt. Sec. Am 44, No. 9, 692-698 (1954).

(7) Strong, Concepts of Classical Optics, Appendix L by Herzberger, p. 537, Freeman (1958).

(8) Herzberger, Optical Image Evaluation, National Bureau of Standards Circular No. 526, U. S. Gov't. Printing Office (1954).

| % Total points | 0° μ | 7° μ | 11° μ | 14° μ |
|---|---|---|---|---|
| 10 | 0.674 | 3.66 | 4.88 | 4.02 |
| 20 | 1.22 | 6.54 | 12.2 | 13.2 |
| 30 | 1.69 | 14.0 | 25.6 | 27.2 |
| 40 | 1.97 | 24.7 | 43.7 | 45.7 |
| 50 | 2.43 | 37.8 | 66.2 | 67.6 |
| 60 | 3.05 | 53.3 | 89.3 | 92.2 |
| 70 | 3.71 | 71.7 | 118. | 119. |
| 80 | 3.93 | 97.8 | 149. | 148. |
| 90 | 6.08 | 132. | 187. | 189. |
| 100 | 12.1 | 247. | 266. | 311. |

Focal length = 5.972460

Plane of best focus at -0.042 mm

Table 24.1- Energy Distribution 006 BC01515.

The table gives the diameters of the smallest circles containing specified percentages of the total number of points in each of the four spot diagrams at the plane of best focus. The common center of the circles for a given spot diagram was taken where the density of the points appeared greatest.

The diameters are listed in microns to three significant figures. Note that the diameter of the Airy disk for a perfect lens as a full aperture of f/3.5 is 4.0μ. Dr. R. N. Wolfe of Eastman Kodak Co. Research Laboratories made a similar series of experiments in 1947 in conjunction with some of Herzberger's early work in this field (9). The subject has been extensively investigated as regards automatic data reduction by Goetz and Woodland (10) at IBM. Miyamote (11), Keim and Kapany (12) as well as many others have studied this very interesting optical test.

24.4.2 Technique. There are many ray tracing programs that will give this information. The one developed by Feder(13) is offered here. Again the techniques of using this method of testing are varied but the following one is typical. See Figure 24.2. Three or more rays are traced from plane PP through the entrance pupil, the optical system into image space. The entrance pupil is EE. Frequently among the rays of interest are the upper rim ray (U), principal ray (Pr), and lower rim ray (L). A point B' on the principal ray in image space is picked arbitrarily and, from the ray tracing data, the optical path length BB' is determined. From the ray tracing data for rays U and L as well as those originating at other points (frequently zonal) such as D and F, optical path lengths equal to BB' are laid off along the rays. The termination points C', D', F' and K' are then marked and the curve passing through them constitutes the equiphase front in the plane of the paper. The deviations from a perfect circle (or sphere in three dimensions) are clear and corrections may be made as necessary.

24.4.3 Applications and limitations. This phase front technique has long been used in the design of microwave antennas because of the optical simplicity (generally speaking) of such systems. It is particularly useful in optical design as the phase front may be determined experimentally by long established techniques. This gives the designer an immediate check on how well the optician has fulfilled the prescription given to him. It should be pointed out that the diffraction pattern may now be determined, providing the amplitude distribution over the front is known. In some cases it is simpler to use basically the same technique but actually determine the phase variation over the exit pupil. The amplitude distribution over the exit pupil is determined and the diffraction pattern calculated as before. The possibility of varying the amplitude over the aper-

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(9) Herzberger, J. Opt. Sec. of Am. 37, 485 (1947).

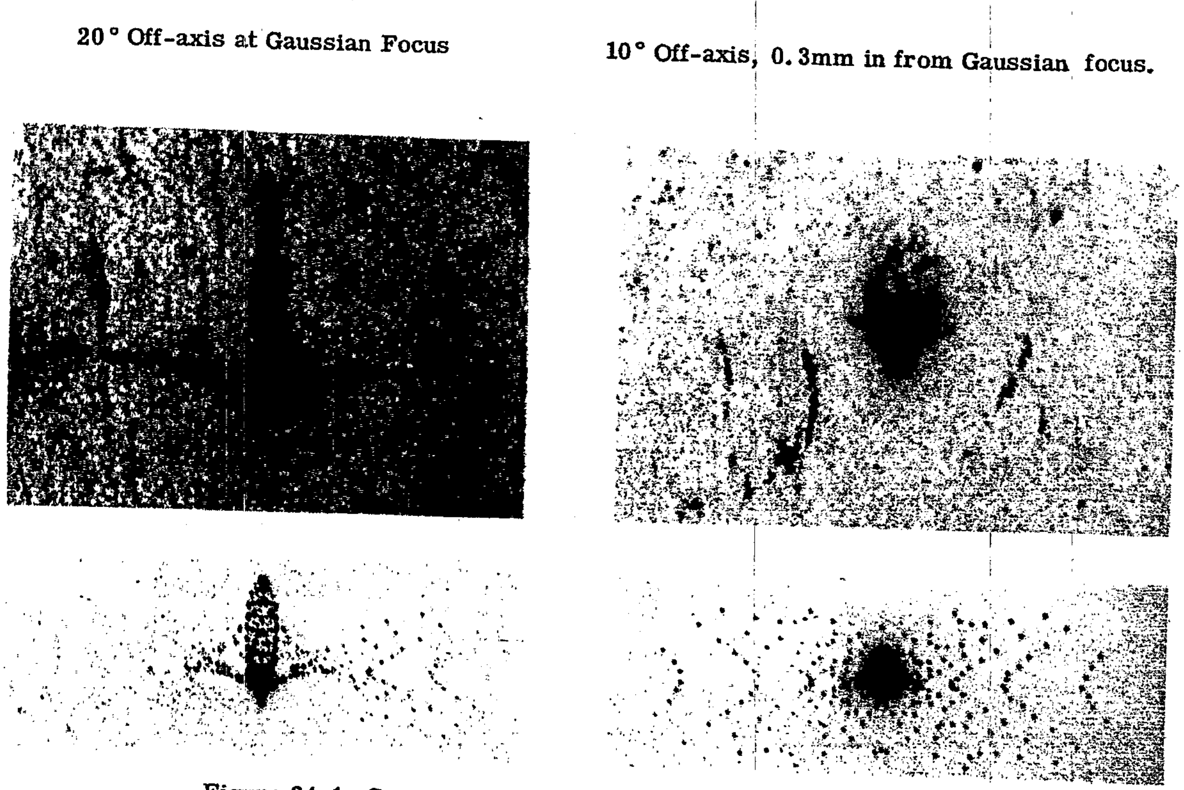
(10) Goetz and Woodland, J. Opt. Soc. of Am. 48, 965 (1958).

(11) Miyamoto, J. Opt. Soc. of Am. 48, 57, (1958); 48, 567 (1958), and 49, 35 (1959).

(12) Keim and Kapany, J. Opt. Soc. of Am. 48, 351 (1958).

(13) Feder, J. Opt. Soc. of Am. 41, 630 (1951).

ture by control of aperture shape, variation of transmission, or illumination with radius has been known for some years. A few of the efforts in this direction are the work of Conder and Jacquinot (14) in spectroscopy, the work of Osterberg and Wilkins (15) with microscope objectives, and the work of Silver (16) on tapered illumination of microwave antennas.



20° Off-axis at Gaussian Focus

10° Off-axis, 0.3mm in from Gaussian focus.

Figure 24.1- Comparison of spot diagram and actual photograph.

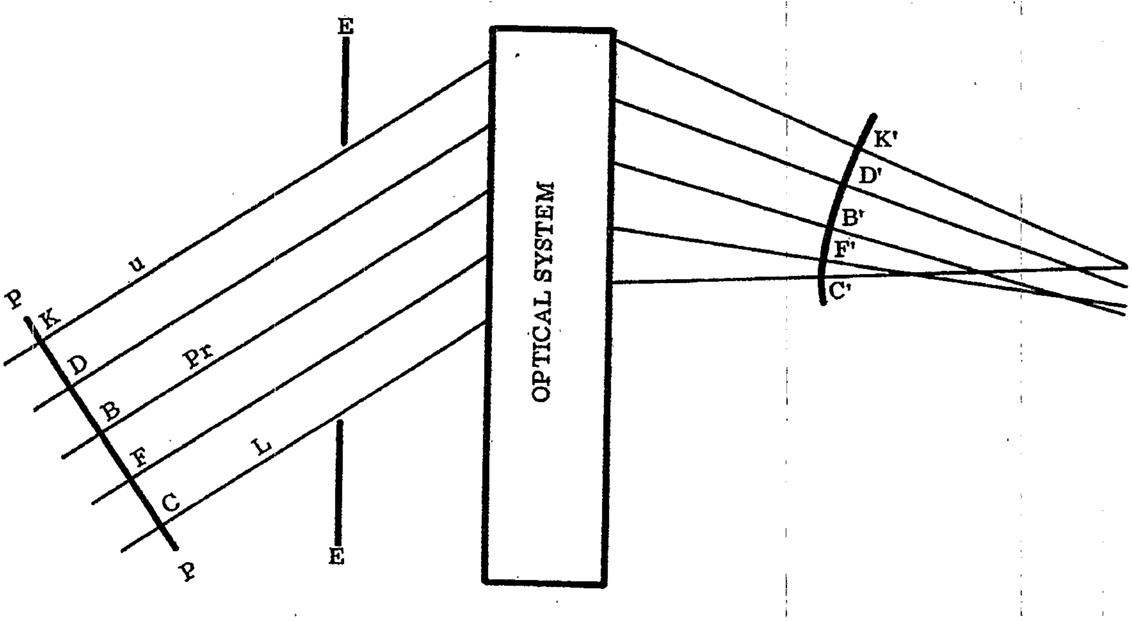


Figure 24.2- Determination of constant phase front from ray tracing data.

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(14) Conder and Jacquinot, "Méthode pour l'observation des radiations de faible intensite au voisinage d'une raie brillante" Compte Rendus de l'Académie des Sciences (Paris), 208, 1639, (1939).

(15) Osterberg and Wilkins, "The Resolving Power of a Coated Objective," J. Opt. Soc. of Am. 39, 553 (1949).

(16) Silver, "Microwave Antenna Theory and Design" 187, McGraw Hill (1949).