# 22 INFRARED OPTICAL DESIGN

## 22. 1 INTRODUCTION

22. 1. 1 General. The basic principles of optical design for the infrared region are the same as those for visible and ultraviolet light. The differences arise mainly from the nature of the materials which must be used, and from the operational and environmental requirements of most of the current applications.

## 22. 2 INFRARED OPTICAL MATERIAL

22. 2. 1 Image converter tube. Reflecting and refracting materials suitable for use at the various infrared wavelengths have been discussed in Section 16. In particular, reference was made to the publication by Ballard, McCarthy, and Wolfe, tabulating information on currently available materials. (Development work is active in this field, and the designer should keep abreast of the situation with appropriate journals and other sources of possible information on new materials). Only general comments on materials will be made here, although they will be extended somewhat in the subsequent portions of this section. Radiation in the 0.8 to $1.2\mu$ region is used with night vision devices employing image converter tubes, such as the "Sniperscope". These are ordinarily "active" devices. That is, they are used to look at objects which are illuminated by infrared light from a source which is under control of the user. The light source is usually a tungsten lamp or carbon arc covered by a filter which absorbs the visible light while passing the infrared. The effective wavelength range results from the combination of the spectral characteristics of the source, of the filter, and of the photo-sensitive cathode of the image converter tube.

22. 2. 2 Infrared imagery. In infrared use, an objective similar to a photographic objective forms an image, in infrared light, on the photocathode of the tube. The illuminated areas of the cathode emit electrons which are accelerated and focussed on a fluorescent screen at the opposite end of the tube, thus forming a visible image which can be viewed by the user with the aid of a magnifier.

22. 2. 3 Glass for infrared usage. Ordinary optical glass transmits satisfactorily in this region and is always used. However, the dispersion characteristics of the several types become much more nearly alike in the infrared than they are in the visible and, as a consequence, much stronger powers of crowns and flints are required to obtain achromatization. For example, a doublet with a 100mm. focal length, made of light barium crown (1.5725/57.4) and dense flint (1.6170/36.6), and achromatized in the visible, will have a crown with a 36mm. focal length and a flint with a focal length of 57mm. If the same glasses be used to achromatize the doublet in the region from $0.8\mu$ to $1.2\mu$, the crown must have a focal length of 19mm. and the flint 23mm. The chromatic aberration of a single crown lens in this region is approximately two-thirds to three-quarters of that for a lens of the same glass from the C line to the F line in the visible, and some slight advantage can be taken of this fact. It is still true, however, that in using a basic lens type, e.g., a Petzval, it is sometimes necessary to replace a single crown lens by two, in order to avoid the high-order aberrations which would otherwise result from the strong curves necessary for achromatization.

22. 2. 4 Optical glass infrared absorption. Radiation in the region from the visible to about $3.5\mu$ is within the range of usefulness of lead sulfide cells. Ordinary optical glass begins to absorb slightly at about $2.0\mu$, and the absorption becomes very great at approximately $2.6\mu$ to $2.7\mu$, depending on the type. Consequently, ordinary glass cannot be used for systems requiring performance beyond $2.7\mu$. Although the usefulness of lead sulfide cells extends to $3.5\mu$ or beyond, it may happen that the combination of (1) the spectral characteristics of the source, (2) any filters in the system, (3) the intervening medium such as air, and (4) the lead sulfide cell itself, will produce a situation under which only a very small proportion of the response of the cell would be lost by using ordinary glass. In such a case, it is worth while to consider carefully whether the loss of a slight amount of response is sufficient to outweigh the advantages of using ordinary glass. It is well to study a number of glasses in this connection, since there is some variability in transmission from type to type, remembering that the flints as a class, transmit slightly better than the crowns.

22. 2. 5 Materials suitable for wavelengths beyond $2.7\mu$. The materials available and suitable for use at wavelengths beyond approximately $2.7\mu$ have, for the most part, refractive characteristics quite different from those with which the designer works in the visible and the near infrared. Indices of refraction range from approximately 1.35 (lithium fluoride) to 4.1 (germanium). The range of dispersion characteristics is even more striking. With ordinary glass, the ration of $\nu$ values available for achromatization is limited to about 2.4:1 or less. In the infrared, this range may run as high as 46:1, the value for a positive silicon element and a negative element in the $3.5 - 5.5\mu$ region. In spite of this great range of values of optical constants, it usually turns out that for reasons not primarily optical, only a few media are available for a given application. The designer must make his selection from those few, and determine

by trial which combination allows him to get the best correction within the limits of allowable complexity of the system. For this reason, among others, reflecting systems are frequently used. The reflectors are completely achromatized, of course, and have nearly constant reflectance over a large range of wavelength. Refracting elements are used with the reflectors to control aberrations.

## 22. 3 ENVIRONMENTAL REQUIREMENTS

22. 3. 1 Current applications. Currently, most designs of infrared optical systems are intended for use by the Military, and therefore the systems must meet Military requirements for serviceability after long exposure to adverse environmental conditions. In particular, many infrared systems are to be airborne, and must meet stringent requirements for compactness and lightness of weight, as well as the ability to withstand rapid changes in temperature and humidity without damage. These requirements place more rigid limitations on the choice of materials, and on the elaborateness of system, than are encountered in other applications.

22. 3. 2 Choice of materials. The choice of material for the windows of airborne equipment is one which must be based primarily on considerations of this nature. These windows are sometimes flat, but are more frequently dome-shaped, since they are used with scanning equipment. Being exposed on the exterior of the vehicle, they must be resistant to the variations of temperature and humidity to be expected in service. The material must be hard enough to withstand excessive scratching; for example, from dust kicked up during take-off and landing. Particularly when used in supersonic vehicles, the material must be able to withstand the thermal shock resulting from friction with the atmosphere. When heated from such friction, it must not radiate much energy in the infrared region in which the optical system is operating; otherwise a false signal may be generated or a true one obscured. Of course, a material radiates only in the region in which it absorbs (See Section 16) and therefore, ordinarily a material transmitting well enough to be considered for use as a window will not give trouble in this respect. However, because of the scarcity of suitable materials, it is sometimes necessary to consider those which do have slight absorption in the region of use and the possible effect of such unwanted radiation must be considered. In this connection, it is important to know the transmission characteristics of the material at elevated temperatures, since these characteristics may differ significantly from those at ordinary temperatures.

22. 3. 3 Size limitations. The window material must be obtainable in pieces large enough for the intended use. This requirement frequently rules out a number of otherwise promising materials. Some attempts at getting around this difficulty have been made by using segmented windows made up of a number of small pieces, and by replacing domes by polyhedrons made up of small, flat pieces. Such structures do not seem to have been generally satisfactory, however, probably because of the difficulty of providing adequate strength in combination with freedom from excessive obscuring by the supporting framework.

## 22. 4 OPERATIONAL REQUIREMENTS

22. 4. 1 Detection of infrared. To be useful, the infrared optical system must feed the energy it collects into a photosensitive device of some type. Fundamentally then, the design of the complete infrared instrument requires similtaneous consideration for the optics, the photosensitive device, and the associated equipment (usually electronic in nature), with respect to the performance requirements to be met. With respect to this discussion, photosensitive devices will only be described for the purposes of orientation.

22. 4. 2 Classification by instruments. Infrared devices are customarily classified by systems engineers as "image forming", or "non-image forming". An "image forming" device is an instrument whose output is a visual pictorial display of the field viewed by the device. An example is the "Sniperscope" previously mentioned. A "non-image forming" device is an instrument whose output is a signal, which is usually electrical. An example is an instrument giving information of the presence of a target of some nature in a particular portion of the field of view. This classification, while logical from the systems engineer's point of view, is not always very significant to the optical designer, since the optical systems of many "non-image forming" devices must actually have an optical image somewhere in the system in order to permit the location of a target within a particular portion of the field of view. Similarly, some "image-forming" devices, which depend on scanning procedures, require optical systems which simply condense the energy from a small field onto a photocell.

22. 4. 3 Classification by wavelength range. For the optical designer, a more useful classification of infrared devices is by the wavelength range which the instrument utilizes. For the purpose of this discussion, the range of the near infrared will include the region from $0.75\mu$ to $3.0\mu$ and beyond the near infrared will include the region from $3.0\mu$ to $1000\mu$.

## 22.5 THE NEAR INFRARED REGION

22.5.1 Current applications. There are three types of commonly used systems which work in the near infrared; that is, in the region from the visible to about 1.3μ. All three are image formers, both in the systems sense and in the optical sense. They are the infrared photographic process, the image-converter tube systems, and the triggered radiation system. Ordinary optical glass is suitable at these wavelengths and the optical design is quite similar to that for photographic objectives, within limitations discussed in 17.7.2

22.5.2 Infrared photography. Infrared photography uses plates or films similar to those of photography with visible light, except that the emulsions have been sensitized by the addition of infrared-sensitive dyes.

### 22.5.3 Infrared image converter systems.

22.5.3.1 The system using an infrared image converter tube has an optical objective which forms an infrared image on the photosensitive cathode of the image converter as shown in Figure 22.1. The cathode emits electrons into the space within the tube, the rate of emission from a given area being proportional to the intensity of illumination of the area. The electrons are focused by electrostatic or electromagnetic means, in order to form an image on an electron-sensitive phosphor at the other end of the tube. The phosphor emits visible light, and the image can be seen by viewing the phosphor with the eye, usually with the aid of a magnifier.

22.5.3.2 Specifications for the components of the system are determined by a compromise between the desired performance characteristics and the state of the tube-maker's and the optical designer's arts. There are usually rather stringent requirements for compactness and portability. The instrument must operate as a telescope of a certain power, usually unity or greater. It is desirable to have a fast objective, since this increases the range at which the instrument is effective. Given the desired field angle, the size of the cathode of the tube determines the focal length of the objective, or vice versa. The electrostatic tubes (which are the sort usually used in this country) operate at a magnification less than unity. The sub-system consisting of objective and tube can then be considered as having an equivalent focal length equal to the focal length of the objective multiplied by the magnification of the image tube. (Both the objective and the electrostatic tube invert the image, so an erect image is presented on the phosphor.) For example, if the system is to have 1-1/2X power, the objective has a focal length of 50mm, and the image converter has a magnification of 0.7X; then the focal length

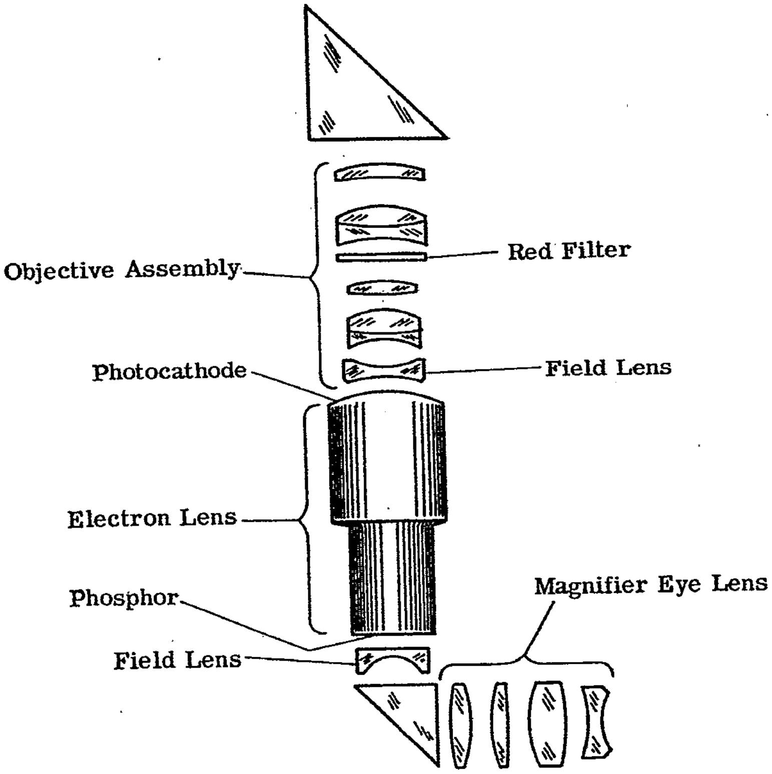


Figure 22.1- Optical schematic of an image converter system.

of the front system is 0.7X . 50mm = 35mm, and the magnifier must have a focal length of about 23mm.

22.5.3.3 The photocathode usually must be rather sharply convex toward the incident light, since the electronic, as well as the optical system, has field curvature. Thus, its curvature is of the opposite sign to that necessary to match the natural curvature of field of a refracting objective. For this reason, a strong negative field lens is employed in front of the cathode. The required power is such that it may be difficult to obtain the required correction for curvature with a single lens without getting total internal reflection of the light from the outer portions of the field; the work must then be divided between two elements. In addition to correcting the curvature, the field lens introduces coma, astigmatism and distortion, since it is not located exactly at the image. The coma and astigmatism must be balanced in the main part of the objective.

22.5.3.4 The electronic system of the image tube produces strong pincushion distortion in the image on the phosphor. This distortion, together with that of the objective and field lens, is dealt with, if at all, in the magnifier through which the phosphor is viewed.

22.5.3.5 The viewing lens system must be considered as a magnifier rather than an eyepiece. (See Section 13) Since the phosphor is a self-luminous surface, it emits light in all directions, and there is no natural exit pupil such as is present in an ordinary viewing telescope. This is an advantage in that there is more freedom to position the eye of the observer than would be the case with the presence of an exit pupil. However, it results in the necessity of correcting the magnifier for spherical aberration and coma, in order to avoid weird distortions and blurrings which can occur if the observer's eye does not happen to be exactly on axis. (This correction can be much cruder than necessary in an objective, since the pupil of the eye accepts only a portion of the bundle from a given object point at any one instant).

22.5.3.6 The conditions of use are such that it may be advantageous to use one or more aspheric surfaces in the magnifier. Aspherics of sufficient precision can be made by processes suitable for mass production. Such a magnifier is shown schematically in Figure 22.2. The magnifier consists of an eye lens and a field lens. One surface of the eye lens is aspherized to correct for spherical aberration. The bending of the lens is chosen to minimize coma. The field lens is aspherized to compensate for the pincushion distortion at the phosphor. (The aspheric may be given a slight power on axis to facilitate fabrication.)

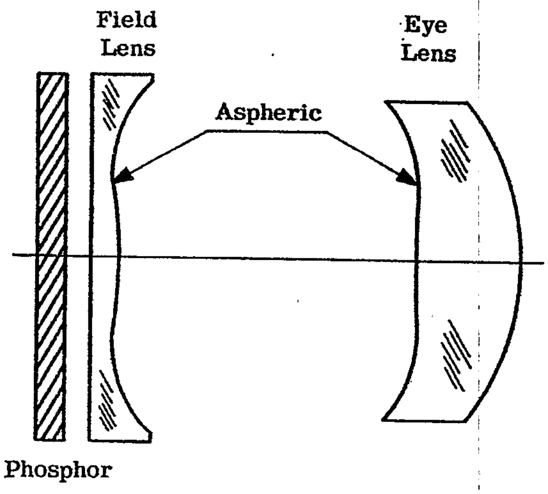


Figure 22.2- Aspheric Magnifier.

### 22.5.4 The triggered radiation type.

22.5.4.1 Instruments of the this type depend on the ability of some phosphors to store energy when irradiated by short wave radiation, and to emit it as visible light when triggered by irradiation with infrared. The short-wave radiation may be ultraviolet (or visible) light, or that from a bit of radioactive material. An objective forms the infrared image of the scene being viewed on the phosphor. The various parts of the phosphor emit visible light in proportion to the intensity of the infrared radiation.

22.5.4.2 In this system, it is necessary to provide optical means of inverting the image, since the system would otherwise perform like an inverting astronomical telescope. This may be done by using a lens or a prism erecting system, either preceding the phosphor (thus working in the infrared) or between the phosphor and the eye. (Other ingenious means have been used in special designs).

## 22.6 THE INTERMEDIATE AND FAR INFRARED REGION

### 22.6.1 Distinguishing characteristics.

22.6.1.1 Design in the region beyond the near infrared has two main distinguishing characteristics, in addition to the necessity of using materials other than optical glass. One is the limitation on image quality. The other is the limitation imposed by the combination of the performance requirements and the characteristics of the types of energy detectors which must be used in many applications.

22.6.1.2 Aside from these limitations, the design requirements are much as they are in other wavelength regions, and the designer must be prepared to deal with requirements quite similar to those found in other optical designs.

### 22.6.2 Limitation on image quality.

22.6.2.1 Since the diffraction limit of resolving power (see 16.28) depends on the wavelength of the light being used, the best image quality obtainable from a source with a given aperture is much poorer in the infrared than in the visible. Taking 0.56$\mu$ as typical of the visible region, the resolution at 3$\mu$ is five times as coarse, and at 10$\mu$ is eighteen times as coarse as in the visible. Since there is no gain to be achieved from improving the correction beyond the point at which the Rayleigh criterion is satisfied, the designer may stop his work with a residue of aberration which it might be well worth while to remove if he were working in the visible region. He may also have to warn the proposer of the system of the limited resolving power which can be obtained.

22.6.2.2 Currently the resolution requirements of most systems are even coarser than the limit which would be imposed by diffraction. However this is not always the case, even at present, and as the infrared art develops it is likely that there will be many more requirements for performance near the diffraction limit.

### 22.6.3 General functions of the optical system.

22.6.3.1 As an aid in discussing the relation of the energy detector to infrared optical design it is worth while to review the functions of the optical instrument in general terms.

22.6.3.2 Every optical instrument is designed to obtain information concerning the radiation characteristics of a portion of space. This portion of space is called the field of view of the instrument. It may be, for example, the crater of an arc, as in emission spectroscopy; a volume of space, as in absorption spectroscopy or in an infrared search system; or a surface, as in a slide projector. The radiation may or may not originate in the field; that is, the field may or may not be self-luminous. In emission spectroscopy and pyrometry the field is self-luminous. In absorption spectroscopy and with active viewing systems it is not self-luminous. In missile guidance systems a part of the field, the target, is self-luminous, while the background light for the most part originates outside the field of view. The importance of the distinction between self-luminosity and non-self-luminosity is only secondary. More basic is the question of whether it is possible to control the nature of the radiation, either in the design of the equipment or during its use.

22.6.3.3 One important function of the optical system is the rejection of radiation from outside the field of view. The field stop in many instruments is an embodiment of this function. In other cases, as in certain types of scanning systems, the limitation of the field is obtained by more elaborate means.

22.6.3.4 The type of information to be obtained from the field depends on the application. In spectroscopy, the object is to obtain a measure of intensity as a function of wavelength, without regard to the portion of the field of view from which the radiation comes. In spectroscopic systems for on-stream process control, the object is, in addition, to present this information, or a portion of it, as a function of time. In image-forming systems, the object is to have rather detailed information concerning radiation intensity as a function of position throughout the field. If the image-forming system is of the "color-translation" type, at least some information about the spectral distribution of intensity at each point of the field must also be provided. In infrared search systems, the object is to know, from moment to moment, the presence and location within the field of small areas, or targets, having radiation characteristics slightly different from those of the remaining background portions of the field.

### 22.6.4 Detector characteristics.

22.6.4.1 The manner in which the information is obtained from the incoming radiation, and in fact, subject to the over all requirements for the instrument, the precise nature of the information, depends greatly on the kind of energy detector which can be used.

22.6.4.2 To the optical designer, the energy detector is the last surface of his system, which must receive and absorb all the useful energy collected by the system. The nature of the detector and its associated equipment imposes limitations on his choice in bringing this about, and also on the minimum of light-gathering power which he must build into the optical system. Several characteristics of the detector are worthy of discussion.

22.6.4.3 For the purposes of this discussion, the detector may be considered as a figurative "black box" with an input, the radiation, and an output, usually an electrical signal in infrared devices. Ordinarily it is more useful to consider the input as flux, or flux per unit area of the detector, rather than as total energy. The flux may be expressed in watts, or in some other unit of power. As will be seen in the following paragraph, it is frequently necessary to consider the distribution of the flux as a function of wavelength. The output is measured in appropriate units, in megohms for example if it is the change in electrical resistance of the cell due to the incident radiant power. The responsivity of the cell is the output per unit input; in our example the number of megohms change in resistance per watt of input. (The term sensitivity is sometimes used for what is here called responsivity, but the word sensitivity has also been employed for a number of other concepts, so its use will be avoided altogether in this discussion.)

22.6.4.4 Two qualifications must be put on this concept. In the first place, the output per unit input may depend on the wavelength of the radiation. Thus to characterize a detector adequately it is necessary to give its responsivity as a function of wavelength; and to predict its response, the distribution of the incident power as a function of wavelength must be known. As a class, the detectors known as photoelectric detectors are highly wavelength dependent. The thermal detectors as a class have substantially constant responsivity regardless of wavelength, and the spectral distribution of the radiation can be ignored.

22.6.4.5 The second qualification arises from the fact that the output may not be strictly a linear function of the input, even allowing for spectral effects. Many detectors show saturation effects when strongly irradiated. Frequently the detector is operated under conditions such that the response is substantially linear. In other cases the concept of responsivity must be modified suitably.

22.6.4.6 Another important characteristic of the cell is called its detectivity. It is a measure of the smallest input, or smallest change in input, that can be reliably detected. All detectors have a random output, not related to the input, known as noise. As a rule of thumb, the increment of input necessary to produce an increment of output equal to the noise may be taken as the minimum detectable input. When expressed as power, this is known as "noise-equivalent power". The larger the noise-equivalent power, the poorer is the detector for small inputs. The reciprocal of the noise-equivalent power is called the detectivity. (The word sensitivity has sometimes been used to mean the detectivity.) Detectivity depends on the type of detector, on the way it is made, and on the environmental and electrical conditions under which it is used. Ordinarily the detectivity is improved by keeping the detector area small.

22.6.4.7 When the input is suddenly changed, the output does not change instantaneously, but takes a finite time to adjust to the new level. The time constant is a measure of the time required for such an adjustment. It is important in predicting the response of the detector to short bursts of radiation, and in determining its suitability for use in scanning systems and in other systems in which the input is made to vary at high frequency. As a class, thermal detectors have much larger time constants than photodetectors.

22.6.4.8 In choosing the size of the detector it is to be remembered that flux density, rather than total flux on the detector, is the criterion of the amount of output which will be obtained. For example, if two square lead sulfide cells of similar characteristics be operated under similar conditions, but the area of one is twice the area of the other, then the outputs of the two will be equal when the flux per unit area on the two is the same, although the total flux on the larger is then twice that on the smaller. Sometimes a large cell may be operated under conditions not practical with a smaller one so as to produce a higher output at a given flux density (for example, a photocell of large area can safely be operated at a higher bias voltage than a small one), but in many cases this is not practical (for example, the voltage available for biasing may be limited) and flux density is the criterion of the output obtainable. In general this is an advantageous situation, since it is usually possible to use a more compact optical system to produce a given flux density on a small area than on a large one. Within limits, small detectors have better detectivity than large ones of the same kind. As a consequence in critical situations where detectivity is an important characteristic small cells are used. In photoconductors, dimensions of a few tenths of a millimeter are common.

22.6.4.9 The responsivity of a detector may depend somewhat on the way the flux is distributed over its surface. There may be local "hot spots" which are more responsive than the rest of the surface. Since the photoconductive cells are used with a bias voltage, the response depends on the way the radiation falls with respect to the points at which the leads make contact with the detector. For this reason it is desirable to plan the optical system so that the non-uniformity will not be a disadvantage. This ordinarily means that the detector is not placed at, or immediately adjacent to, an image plane, but rather at an image of the entrance aperture of the system.

### 22.6.5 Target detection and location.

22.6.5.1 An important class of problems is exemplified by airborne infrared search systems. In applications of this sort it is necessary to have a large field of view, and to detect and locate small, weak targets which are at great distances from the optical system.

22.6.5.2 There may be large amounts of radiation in the field besides that coming from the targets which it is desired to detect. Such radiation is called "background". The problem thus is one of distinguishing the radiation of the target from that of the background. It is necessary to take all possible advantage of differences between the target and the background. One important difference lies in spectral distribution, the target radiation usually having a peak wavelength different from that of the background. (The spectral distribution must be evaluated at the optical system. The intervening atmosphere is in effect a part of the field of view, and its absorbing and scattering characteristics must be taken into account.) Contrast between target and background is further increased by the use of optical filters to absorb as much as practical of the radiation at wavelengths at which the background is stronger than the target. (For a discussion of infrared filters see Ballard et al, loc. cit.) Choice of the type of detector depends in part on its having adequate responsivity in the spectral region near the peak wavelength of the target.

22.6.5.3 The technique known as spatial filtering is frequently used to take advantage of the dimensional differences between the target and other sources of radiation likely to be in the background. For a discussion of spatial filtering see, for example, Aroyan. *

22.6.5.4 Detection and location of the target within the field of view is accomplished by dividing the field into elements by some means and observing either the difference in flux between an element and adjacent ones, or the change in flux in each element with time. Since the intensity difference between target and background is small, the detector must be chosen and used so as to have good detectivity, and the optical system must have a large aperture to insure that the difference between target and background can be recognized by the detector.

22.6.5.5 An attractively simple scheme for providing the necessary subdivision of the field uses an objective which forms an image of the field at its focal plane. In the plane of the image is placed a rotating opaque plate carrying a set of small apertures so arranged that at any instant a single aperture is transmitting light from some small portion of the image, and during a single rotation of the plate the whole image is scanned. A condenser system placed behind the image plane collects the radiation and brings it to the detector. (The condenser is usually designed to form an image of the aperture of the objective on the detector.) The rotation of the plate can be related electrically to the output of the detector so that the system as a whole recognizes the portion of the field from which radiation is being transmitted at any instant. Attractive though it is, this simple scheme is rarely adequate because of the simultaneous requirements of large field and wide aperture. The inadequacy is not due entirely to lack of ingenuity on the part of the optical designer, but results from a fundamental limitation on the light-receiving ability of a small surface.

### 22.6.6 Receiving ability of a surface as a limiting factor.

22.6.6.1 The method used in the following analysis of the light-receiving ability of a surface is old, though it does not seem to be so well-known as desirable. See for example Drude. ** The method applies to any surface through which all the useful energy must pass, and thus its conclusions apply to focal surfaces as well as detector surfaces.

22.6.6.2 Suppose the instrument to be confronted by a black body, the surface of which is at least large enough

---

*Aroyan, G. F. "The Technique of Spatial Filtering." Proc. I.R.E. 47; 1561-68; Sept. 1959.

**Drude, Paul, "Lehrbuch der Optik," Leipzig, 1900. English trans., "The Theory of Optics," N.Y. and Dover, 1959 to fill the whole field of view of the instrument. That is, it is large enough so that any ray which enters the optical system and passes into the surface whose light-receiving ability is being investigated can be considered to have originated in the surface of the black body. It is convenient though not necessary to assume that the black body is infinitely distant from the instrument. Assume that the black body is at some fixed, uniform temperature.

22.6.6.3 Suppose that the surface whose light-receiving ability is being investigated is the surface of a black body which is at the same temperature as that of the external black body. It follows from the second law of thermodynamics that, regardless of the nature of the optical system, the internal surface cannot receive from the external one an amount of flux greater than that which the internal surface itself is radiating, for otherwise the system would be acting as a self-operating heat pump. In most systems the flux received by the internal surface from the external one will be considerably less than the maximum, due to the finite aperture of the system, absorptions within the system, etc.

22.6.6.4 Let $I$ be the number of watts per unit solid angle radiated by either surface per unit area of the surface in the direction normal to the surface. This quantity is determined by the black body temperature, and for our purposes is to be considered constant. Let $A$ be the area of the surface whose light-receiving ability is being investigated, and $n$ the index of refraction of the medium on that side of this surface on which the light is incident. (The detector surface may be exposed to air, for example, or it may be in optical contact with glass, light reaching the surface through the glass.) Let $F_s$ be the total flux radiated by the surface into the medium with which it is in contact. Then it can be shown that

$$F_s = \pi I n^2 A \tag{1}$$

(Drude, loc. cit.) Then $F_s$ also represents the upper limit of the flux we can hope the surface would be able to receive from the external black body.

22.6.6.5 As an example of the significance of this result, suppose that a system is contemplated which has an objective aperture 150mm. in diameter, at the front of the system. It is desired to have the objective focus the energy from a 20° field (i.e., from 0° to 10° off-axis) on an image plane, and then to condense the light on a photocell. It is desired to determine the minimum focal plane area and minimum cell area. It is first necessary to write down the expression for the flux received at the aperture from the 20° field. For purposes of later discussion this formulation will be made more elaborate than would otherwise be necessary. Let

$$\overline{A} = \text{the area of the aperture} = 75^2 \pi \text{ sq. mm.}$$

22.6.6.6 It can be shown that the element of flux $dF$ received at the aperture from that elemental portion of the external blackbody which lies in a direction making an angle $\alpha$ with the axis of the system (i.e., with the normal to the aperture) and which subtends a solid angle $dw$ as seen from the aperture is

$$dF = I \overline{A} \cos\alpha \; dw.$$

Here $I$ has the same value as in (1) since both the internal and the external blackbody are at the same temperature. The quantity $\overline{A} \cos\alpha$ is the projection of the area of the aperture in the direction of the source. From the point in the external blackbody on the axis of the system the aperture appears circular. From points farther away from the axis the aperture appears foreshortened, so that the effective area is only $A \cos\alpha$. Let

$$B(\alpha) = \overline{A} \cos\alpha$$

and name

$B(\alpha)$ the effective aperture function.

Then

$$dF = I \; B(\alpha) \; dw.$$

22.6.6.7 It is convenient to consider the elementary part of the external blackbody as being the annulus included between the two cones corresponding to field angles of $\alpha$ and $\alpha + d\alpha$ respectively. Then

$$dw = 2\pi \sin\alpha \; d\alpha$$

$$dF = 2\pi I B(\alpha) \sin\alpha \; d\alpha \tag{2}$$

$$F = 2\pi I \int_0^{\alpha'} B(\alpha) \sin\alpha \; d\alpha . \tag{3}$$

Here $\alpha'$ is the maximum value of the field angle from which radiation is to be collected.

22.6.6.8 In this example, since $B(\alpha)$ is known, (3) can of course be written

$$F = 2\pi I \bar{A} \int_0^{\alpha'} \cos\alpha \sin\alpha \, d\alpha .$$

$$= \pi I \bar{A} \sin^2 \alpha' , \qquad (4)$$

or substituting for $\bar{A}$ and $\alpha'$,

$$F = \pi^2 I \, 75^2 \sin^2 10° . \qquad (5)$$

The second law and equations (1) and (5) then require

$$F \leq F_s ,$$

whence

$$\pi^2 \, I \, 75^2 \sin^2 10° . \leq \pi I n^2 A$$

and

$$A \geq 75^2 \, \pi \sin^2 10°/n^2 = 533/n^2 . \qquad (6)$$

Thus if the focal surface must be in air, so that $n = 1$, we must expect its area to be greater than $533 \text{mm}^2$. Similarly, if the detector be optically immersed in a medium having an index of refraction $n = 2.2$, its area must be greater than $110 \text{mm}^2$. A detector having this area would have too poor a detectivity in many applications.

22.6.6.9 The treatment in the example can be generalized. In the example it was tacitly assumed that the system had rotational symmetry about the optic axis. Even if this is not the case it is still convenient to assume a set of spherical coordinates centered at some point in the optical system, using $\alpha$ for the polar angle and $\beta$ for the azimuth angle. Consider an elementary portion of the external blackbody which lies in the direction $(\alpha, \beta)$ and subtends a solid angle $dw$ as seen at the optical system. Consider a bundle of rays starting from the element of blackbody. When the rays reach the system, some will pass in and reach the focal plane and ultimately the detector. The others will be excluded by the various apertures of the system. The cross-sectional area of that portion of the bundle which does eventually reach the detector may be called the effective aperture of the system for the direction $(\alpha, \beta)$. We denote it by $B(\alpha, \beta)$, which may be called the effective aperture function of the system.

22.6.6.10 The flux collected by the system from the element of the blackbody is then

$$dF = I \, B(\alpha, \beta) \, dw \qquad (7)$$

(we assume for the present that absorption and similar losses are negligible) and the total flux collected by the system is

$$F = I \int_\alpha \int_\beta B(\alpha, \beta) \, dw \qquad (8)$$

the integral being taken over the whole field, i.e., over all directions $(\alpha, \beta)$ for which $B(\alpha, \beta) \neq 0$. It follows from the second law of thermodynamics, and from (1) and (8) that

$$\int_\alpha \int_\beta B(\alpha, \beta) \, dw \leq \pi n^2 A . \qquad (9)$$

22.6.6.11 It is important to note that in (9) the radiation intensity, I, of the blackbody has cancelled out, and (9) is a condition on the characteristics of the system itself, regardless of the nature of the actual radiation field with which the system may be confronted. The expression on the right hand side may be taken as representing the maximum light-receiving ability of a surface of area A. The equation gives a fundamental limit to the combination of field coverage and aperture which can be achieved with a surface of area A.

22.6.6.12 When the system does have rotational symmetry we can proceed as in the example, considering $B(\alpha, \beta)$ to be a function of $\alpha$ only, and obtaining (3) which with (1) yields

$$2 \int_0^{\alpha'} B(\alpha) \sin\alpha \, d\alpha \leq n^2 A \qquad (10)$$

### 22.6.7 Practical limits and techniques.

22.6.7.1 Of course practical difficulties prevent attaining the limiting light-receiving ability. To do so would require bringing in rays at all angles of incidence to the surface up to 90°. This is difficult to accomplish because of the precision required in constructing and focussing, and because of the resulting sizes and shapes required for the optical components. As a rule of thumb, something like 1/2 to 3/4 of the limiting light-receiving ability can be utilized. The former requires angles of incidence of 45° or greater at the surface; the latter, 60° or more. Consequently in practice one is limited to

$$\int_\alpha \int_\beta B(\alpha,\beta)\ dw \leqq k \pi n^2 A \qquad (9')$$

or

$$2 \int_0^{\alpha'} B(\alpha)\ \sin\alpha\ d\alpha \leqq k\, n^2 A \qquad (10')$$

wherein k may be 1/2 to 3/4. Losses by absorption and reflection in the system must of course also be taken into account.

22.6.7.2 As another example, suppose it is desired to monitor at 120° field, and that a detector area of $0.25\text{mm}^2$ is chosen as having optimum detectivity under the expected operating conditions. Assume further that from knowledge of expected targets and backgrounds an aperture of at least $4400\text{mm}^2$ is considered necessary to insure that the target will be recognized by the detector. (This aperture area is that of a circle 75mm in diameter). It is desired to use a simple scanning system of the type described in 22.6.5.5. The designer must decide whether he can do this.

22.6.7.3 It is evident at once that although in use the scanning plate permits only a small portion of the field to be viewed by the detector at any instant, nevertheless the system must be designed so that, if the plate were removed, the detector would view the whole field at once. That is, the light-receiving ability of the detector must be made to cover the whole field at the full aperture.

22.6.7.4 Without attempting to decide for the moment how it can be accomplished, assume that $B(\alpha)$ is to be made constant for the whole field of view, and equal to $4400\text{mm}^2$. Then equation (10') becomes

$$8800 \int_0^{60^\circ} \sin\alpha\ d\alpha \leqq 0.25\, k\, n^2$$

or

$$8800\ (1 - \cos 60^\circ) \leq 0.25\, k\, n^2 .$$

Assuming k may be taken as 3/4, this becomes $4400 \leq 0.1875\ n^2$ .

22.6.7.5 If the cell is to operate in air, or a vacuum, $n = 1$, and the inequality obviously is not satisfied. The system as proposed cannot possibly meet the requirements. The scheme would have to be abandoned and another adopted which permits the detector to be employed in a system with adequate aperture but with a limited field.

22.6.7.6 There are various ways of doing this. A simple one consists essentially of building a system of adequate aperture and suitably small field, and then pointing it rapidly first at one portion of the field and then another. Thus each part of the large field is observed intermittently, although not continuously. The pointing is usually done by rotating mirrors. Another scheme causes the detector to scan the image at the focal plane of an objective by moving it about in the focal plane. The difference between this scheme and the one originally suggested lies in the fact that here the whole light-receiving ability of the cell is operative on the small portion of the field being viewed at the moment, while in the former only a small part was used at any instant, the rest being blocked off by the opaque portions of the rotating plate. Still other schemes use an array of cells to scan the image, or a mosaic of cells fixed in position in the image plane, or a combination of these methods. See Figure 22.3. A complete discussion of choice of an optimum system is too lengthy to be included here.

22.6.7.7 In equations (9), (10), (9') and (10') the square of the index of refraction, $n^2$, appears on the right hand side. This is indicative of the fact that the light-receiving ability of a detector surface is increased if it is in optical contact with some medium other than air. From analogy with oil-immersion microscope objectives, it is customary to say that the surface is immersed in the medium. The increase in light-receiving ability is analogous to the increase in numerical aperture of a microscope objective obtained by using an immersion system. The surfaces of the glass or crystal on which the cell is formed must be so disposed as to permit light from the field of view to strike the cell at all incidences from zero up to very high angles. If the substrate is simply a plane parallel plate in air, for example, the critical angle of refraction in the glass will limit the light-receiving power to that which the cell would have in air. If the cell is placed at the center of a hemisphere of the glass, however, the full light-receiving power can be used.

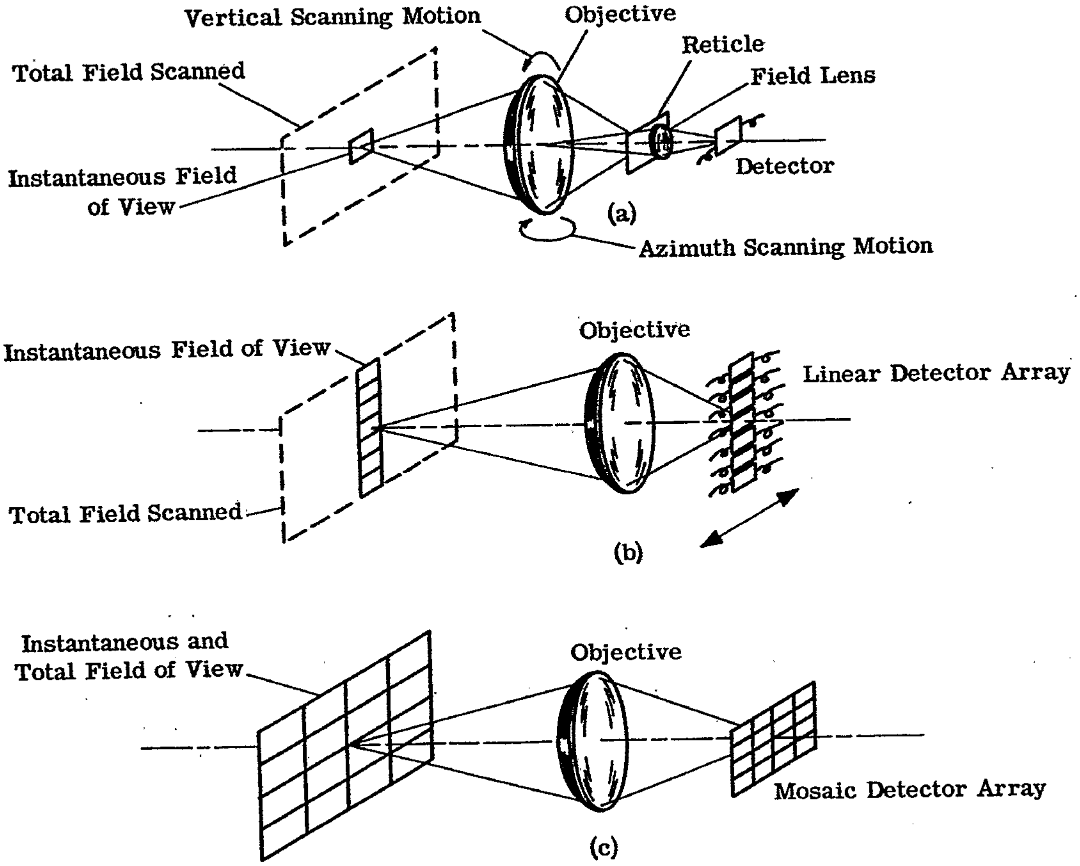
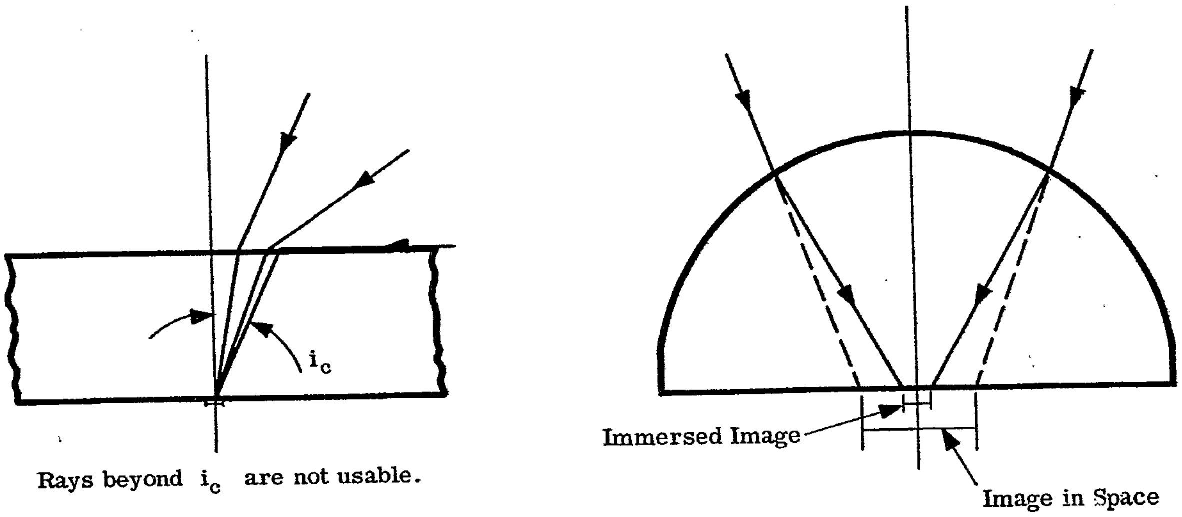


Figure 22.3 - Examples of basic scanning systems.



Rays beyond $i_c$ are not usable.

Figure 22.4- Illustration of advantages of an immersed detector.

22.6.7.8 The benefits of cell immersion are often overlooked, and the design of a system is thereby made harder. However, as the possibilities become more widely appreciated, cell makers are giving more attention to the problems of producing immersed cells. These problems are difficult, because in addition to being a suitable material for supporting the cell, the substrate must also transmit the desired radiation. The higher the index, the greater the increase in light-gathering ability. Strontium titanate, for example, has an index of 2.2, and increases the light-receiving ability of the cell by a factor of 4.8.

## 22.7 SUMMARY AND CONCLUSION.

22.7.1 Advantages and disadvantages. In infrared work the designer may meet problems as varied and complex as those encountered in the visible part of the spectrum. Since the laws of reflection and refraction are the same at all wavelengths, the same basic design principles are used in both regions. The most important differences to which he must become accustomed arise from the natures of the available optical materials on the one hand, and from the requirements of some of the currently important infrared applications in the other. In addition he must remember that the resolving power obtainable with a given aperture is poorer than in the visible, due to the longer wavelength of the light. Out to about $2.7\mu$ he may use ordinary glasses, but must allow for the fact that the dispersion characteristics of the several glass types are more nearly alike than in the visible. Beyond $2.7\mu$ he must use materials whose characteristics may vary widely from those of optical glass, sometimes favorably and sometimes not. He will want to use reflecting systems more frequently than in the visible. The use of many infrared devices for military applications, particularly airborne ones, adds requirements of ruggedness, resistance to adverse environmental conditions, compactness and lightness of weight to the optical ones. Most such devices are part of instruments which are complex combinations of optics, mechanics and electronics, and the choice of the basic optical characteristics is only part of the process of choosing the optimum design parameters to meet the performance goals for the whole instrument. The designer needs to know enough about the characteristics of the whole instrument and the inter-relationships of its parts to be able to contribute intelligently to the decisions in the choice of parameters. Especially, he needs to know something about the energy detectors used in the infrared, and how their limitations of responsitivity and detectivity limit his design.