19. OPTICS FOR MISSILE TRACKING

19.1 INTRODUCTION

19.1.1 Functions. The main function of optical missile tracking instrumentation is to determine, with precision, the location and trajectory of missiles and satellites. A second important function is to observe the physical appearance and orientation of the space object, and its alterations over short periods of time. In addition, the instrumentation must provide a permanent record of the object's flight for study and later analysis of the trajectory.

19.1.2 Problems.

19.1.2.1 As can be seen from the preceeding paragraph, the problems of tracking and recording the object's flight are closely related to those in astronomy, particularly to those encountered in the observation of planets and planetary detail. However, in missile tracking, additional problems are encountered, since the objects to be observed are not precisely known with respect to location and trajectory. In the proper solution to the problems involved, many contradictory requirements exist, and the correct choice must be made among these requirements for a design necessary to fulfill any particular specification. These requirements are listed as follows:

(1) The field of view must be wide enough so that the missile image is picked up.
(2) The relative aperture must be high enough to see and record the image under the prevailing lighting and atmospheric conditions.
(3) The physical aperture must be large enough so that the Rayleigh limit does not apply to the detail desired.
(4) The focal length must be long enough to have sufficient detail appear in the final image.
(5) The recorded images must properly follow one another fast enough to determine the trajectory, and/or to disclose the physical condition of the missile.

19.1.2.2 A few remarks are in order concerning these points. The requirement for wide field (1) is so contradictory to all the rest that separate instrumentation in the form of tracking telescopes are needed for picking up the object. In almost all cases, two tracking telescopes are used, as shown in Figure 19.1, one for azimuth and one for elevation and each with its own operator. The function of these operators is to keep the

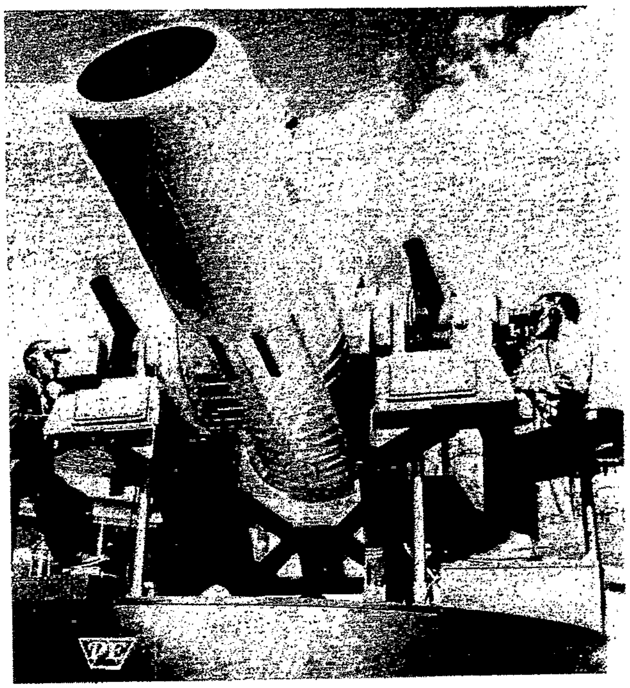


Figure 19.1- Tracking telescopes used with ROT1.
(Courtesy of Perkin Elmer Corp.)

space object close enough to the cross lines so that it will be within the much narrower field angle of the main optical system. This main optical system is completely photographic in nature, so that the final information is recorded on film. Also recorded on the same film, are certain necessary data, particularly an accurate time signal, so that the data from at least two separated stations may be correlated to determine the trajectory of the object by means of triangulation. If errors are to be minimized, a third station may be employed, and suitable data reduction and evaluation means, by high speed computers, used to obtain the trajectory of the object to a high degree of accuracy.

19.1.3 Scope. In this section, the main optical photographic system, which may be either refractive or reflective, will be the basis of the discussion. In addition, emphasis will also be given to those particular optical devices which have been used to obtain desirable properties. The relative merits of refractive and reflective systems will not be gone into here, except to say that for large aperture systems, the arguments are all in favor of the reflective system. In the region of smaller physical apertures, i.e., up to 12 inches diameter, the advantages of greater stability, greater depth of focus, and superior diffraction pattern of the refractive system may overcome the much lower size, weight and freedom from color aberrations of the reflective systems. At larger apertures, however, the balance is so decidedly reversed that the reflective systems have enjoyed an almost complete monopoly in the first half of the twentieth century in the field of astronomy. More advanced designs are now being proposed and constructed in which combined reflective and refractive systems will be of increasing importance, as will be seen from some of the systems described herein.

19.2 REFRACTIVE SYSTEMS

19.2.1 Correction of secondary aberrations. The problems of designing purely refractive systems, of high performance over a relatively small field angle, has been treated extensively in Section 1 of this volume, as has the necessity of correcting for primary spherical aberration, coma, and axial color. Off-axis aberrations, such as astigmatism, distortion, and lateral color, are usually of less importance for a narrow field angle, although they must not be entirely neglected. However, when the focal lengths become large, i.e., in excess of 12 inches, and the relative apertures are of the value of f/8 or lower, great consideration must be given to the correction of the so-called secondary aberrations, the most important of which are secondary spectrum, zonal spherical aberration, and sphero-chromatism.

19.2.2 Secondary spectrum. The methods of handling this troublesome aberration have been discussed in Section 11. For the relatively large apertures we are concerned with in this discussion, the three-glass method leads to fairly high curvatures and impractically thick crown elements. This is also true in combining ordinary crown glass with the crown-flints, where the difference in the Abbe constants is relatively small. It is hoped that glass will eventually be manufactured which will help to alleviate this troublesome aberration. Some years ago, the writer of this section was able to achieve a fairly decent paper design, for secondary color correction, by combining dense barium crown glass with one of the Eastman glasses (EK-320) which gave a higher Abbe spread. However, success was frustrated by the inability to obtain the latter glass, at that time the only available of its type, in better than low-grade C quality. Presently, however, manufacturers are claiming better quality for their lanthanum equivalents, and the situation may change. In addition, other designers are now finding that the later KZF glasses of Schott show considerable promise in this regard and it is imperative therefore, that the designer keep abreast of the newest glass types.

19.2.3 Use of the air space. We turn now to a discussion of the two remaining secondary aberrations namely, zonal spherical aberration and sphero-chromatism, together with a qualitative explanation as to how these aberrations may be minimized or eliminated. With the very long focal lengths involved in the missile tracking problem, either of these can be large enough to ruin the image on the axis. Therefore, the solution to the control of these aberrations is through the judicious use of the air spaces in the interior of the optical design. While these aberrations are of particularly high importance in the design of missile optics, the same considerations apply for other applications throughout the entire field of optics, and a thorough discussion of the corrective properties of the air space is very much in order.

19.2.4 Zonal spherical aberration.

19.2.4.1 In order to understand how the air space may be used to correct for zonal aberration, a short discussion on the nature and origin of the zonal bulge is in order. The spherical aberration of a single surface is given by the expression

$$Sph = a_1 y^2 + a_2 y^4 + a_3 y^6 + \dots\dots\dots, \qquad (1)$$

where y is the height of the ray above the axis at the surface, and the a's are constant and all of the same sign. The first term is the primary or "Seidel" term and the remaining ones are of so-called higher order.

19.2.4.2 If we now plot the spherical aberration of a single refracting surface, it will look like the solid line in Figure 19.2. The dotted line is the Seidel contribution and since the latter is restricted to the first term in equation (1) it will be a pure parabola. The interval between the solid and dotted lines is due to the presence of higher order terms.

19.2.4.3 Now consider how the zonal bulge originates in so simple an optical design as the corrected doublet. Such a doublet (crown leading) is shown in Figure 19.3. Surfaces 1 and 3 are undercorrecting and relatively weak in curvature as compared to surface 2 which is overcorrecting. It is to be expected that 1 and 3 surfaces will have largely Seidel contributions, while the contribution of the second surface will be relatively rich in higher order terms. If we bend the doublet so that the marginal aberration is zero, we show the sum of the spherical aberration of the undercorrecting surfaces 1 and 3 in Figure 19.4 (a), and that of the overcorrecting surface 2 in (b). The requirement for corrected marginal aberration dictates that $OM_1 = OM_2$. However, the preponderance of Seidel aberration in Figure 19.4 (a) results in a near parabola for the curve, while the presence of higher orders in Figure 19.4 (b) gives a more extreme type of curve as shown, with a milder parabola (dotted extension) plus higher orders. Adding abscissas such as along the line XX to give the final result shown in Figure 19.4 (c) reveals a spherical aberration curve with characteristic zonal bulge, with a maximum of undercorrection at approximately 0.7 of full aperture.

19.2.4.4 Figure 19.5 shows a single positive lens with a marginal ray M and a zonal ray Z. This lens has a large amount of spherical undercorrection and thus the M ray crosses the axis at F_m and the Z ray at F_z. The two rays intersect at the point P. If we set the entrance height of the Z ray at 70 percent of that of the M ray, this ratio is very nearly maintained when the rays leave the lens. However, this percentage ratio gradually increases towards the right and finally reaches 100 percent at the point P, actually reversing beyond this point. If we consider this lens as the first of the doublet shown in Figure 19.3 the negative element of the doublet is located so that the zonal ray Z traverses it at very nearly the original 70 percent height. However, if we allow an air space to exist between the elements as shown in Figure 19.6, the negative overcorrecting element is located at a position where the Z ray is at a height in excess of the original 70 percent value as related to the marginal ray. It thus is acted upon by the negative lens at a point higher than its original assigned value in the aperture, and receives a trifle more overcorrection than it would have received were it a height of 70 percent of the marginal ray. This overcorrection of the zonal ray can be adjusted to reduce or completely eliminate the zonal bulge, or even reverse it, if desired. This control of the zonal bulge obviously depends upon the amount of undercorrected spherical aberration produced by the positive lens and of

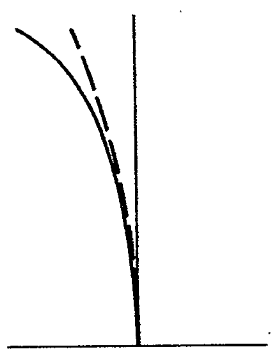


Figure 19.2- A plot of spherical aberration of a single refracting surface.

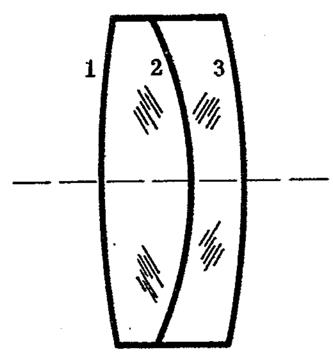


Figure 19.3- Corrected doublet-surfaces 1 and 3 undercorrecting, surface 2 overcorrecting.

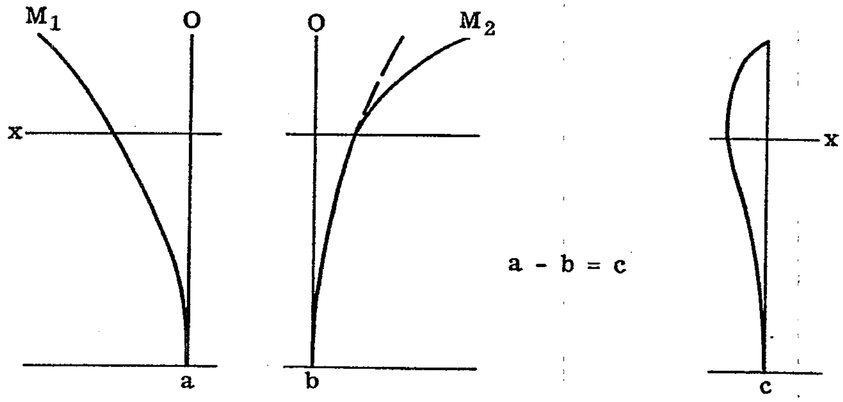


Figure 19.4- Plots of spherical aberration for the lens in Figure 19.3, with marginal aberration equal to zero.

the size of the air space.

19.2.5 Sphero-chromatism.

19.2.5.1 If the cemented doublet in Figure 19.3 is designed to eliminate spherical aberration for a wavelength in the middle of the spectrum, e.g., sodium D, we will discover that the C (red) rays will show considerable undercorrected spherical aberration, while the F (blue) rays will exhibit overcorrected spherical aberration. If the doublet is color-corrected to bring the paraxial C and F rays together, the spherical aberration curves will appear as in Figure 19.7 (a). Also, a better overall color correction will result if the paraxial rays are allowed to be slightly undercorrected, so that the two curves will intersect at approximately the 0.7 zone, or even a little higher as shown in Figure 19.7(b). The cause of this change in spherical aberration with wavelength will be made clear by the following discussion.

19.2.5.2 The refracting power of the doublet is very nearly the same for red (C), yellow (D), and blue (F). However, this refracting power is the sum of those of the positive and negative elements. If the lens is spherically corrected for yellow (D), the positive and negative elements will have certain refracting powers. For red (C) light, the individual refracting powers will be decreased (the sum remaining the same) because of the lower index of refraction. Conversely, for blue (F) light the elements' refracting powers will be increased. The nature of these changes are such, as to have the doublet spherically undercorrected for red (C) light and overcorrected for blue (F) light.

19.2.5.3 The air space can be used to correct this aberration. In Figure 19.8, white light entering a positive single element is refracted by the lens so that the F ray is bent more than the C ray. If the negative overcorrecting element is placed in contact with the positive element, no benefit is achieved. However, if the air space is present as illustrated in Figure 19.9, the F ray strikes the negative element at a lower height than the C ray and thus is subject to less overcorrection by this element. In this way, the naturally spherical undercorrection for red (C) and overcorrection for blue (F) can be neutralized or even reversed by increasing the air space. The amount of the correcting effect depends on the size of the air space and the angular interval between the (C) and the (F) ray upon emerging from the positive element. This angle depends upon the refracting power of the element and the dispersion of the glass.

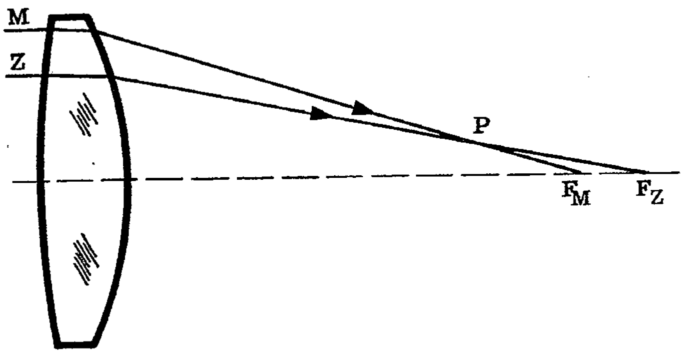


Figure 19.5- Single positive lens used in doublet of Figure 19.3.

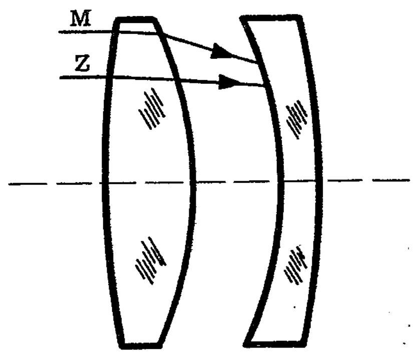


Figure 19.6 - Clark lens, illustrating air spacing to increase over-correction of zonal ray.

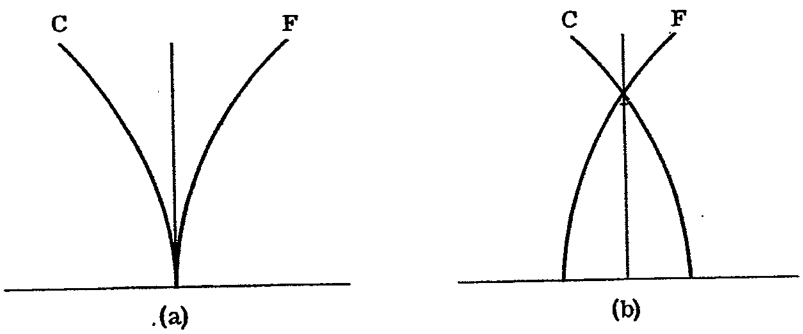


Figure 19.7- The doublet used in Figure 19.2 corrected for spherical aberration and color.

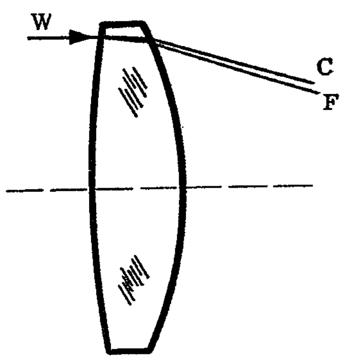


Figure 19.8- Refraction of monochromatic light by the positive element of the doublet shown in Figure 19.3.

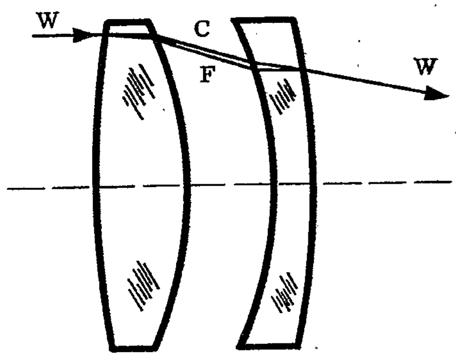


Figure 19.9- Refraction and correction of monochromatic light by the Clark lens shown in Figure 19.6.

19.2.5.4 The air spaced doublet shown in Figures 19.6 and 19.9 is generally known as the Clark lens, produced by Alvan Clark and his successors in the last half of the nineteenth century in the form of large aperture astronomical refractors of exceptional performance. However, there are several objections to the Clark lens. It is extremely difficult to correct both for zonal spherical and for sphero-chromatism with one and the same air space. Further, while in a cemented or contacted doublet the highly curved contact face is of low power, the separation into two elements means that the original contact face has become two strong opposing surfaces of high power. The result is that the axial adjustment and centration become quite critical and the lens become difficult to mount, adjust and maintain. These objections can be overcome by splitting the positive element into two parts and attaching one of these to the negative element as shown in Figure 19.10. Excellent performance with this combination can be achieved up to much higher apertures than is possible with either cemented or contracted doublets. See Section 11. The splitting up of the positive power into two elements means that all of the curves can be made quite mild, and additional degrees of freedom are available to the designer. The designer may choose the relative powers of the two positive components and, also, can investigate the possibility of making these components of different glass types, particularly, with respect to the dispersion factor.*

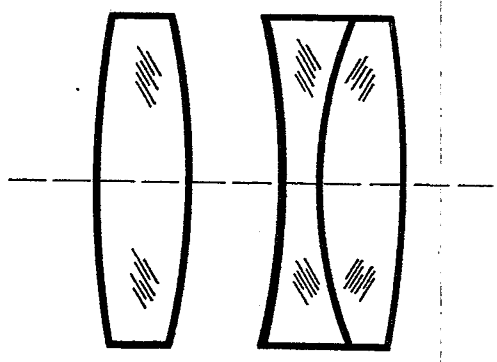


Figure 19.10-Modification of the Clark lens.

19.2.5.5 The lens form shown in Figure 19.11 has incorporated these principles into a simple and effective design used on one of the smaller theodolites. The actual construction has been used for a 40 inch f/6 lens and for a 24 inch f/5.6 lens. The front lens grouping incorporates the general principles discussed in the preceed-

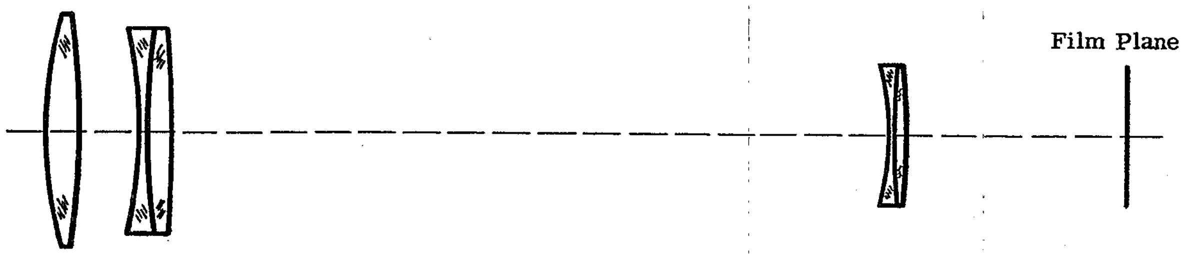


Figure 19.11- Optical layout, 24" high resolution lens.

* JOSA Vol. 42,7 pg. 451, S. Rosin.

ing paragraph. The rear negative doublet neutralizes the Petzval curvature and astigmatism of the front group so that high resolution is maintained over a 70 mm. film format. The spherical aberration curves for all visible colors are straight lines perpendicular to the axis, without any zonal bulge whatsoever (see Figure 19.12). This may be contrasted with the more usual type of curve such as shown in Figure 19.7b.

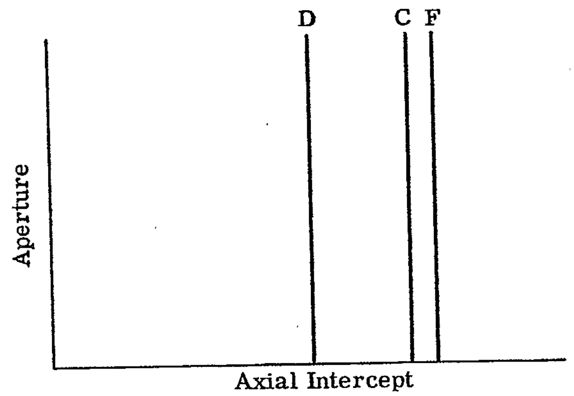


Figure 19.12- Spherical aberration curve for design shown in Figure 19.9.

19.3 REFLECTIVE SYSTEMS

19.3.1 Evolution of the reflective system.

19.3.1.1 The science of astronomy, based on the telescope as it is, discovered early in the twentieth century that the refractive objective had reached the limit of its development. To carry the physical apertures to ever higher values required the use of the reflective system, of which the principal converging element is the reflecting mirror. In this section, a few examples of advanced reflective designs will be given, but in order to properly orient the reader, and to enable him to effect his own designs for particular requirements, a short history of the evolution of reflecting systems and a discussion of some of their fundamental properties will be given.

19.3.1.2 The simplest of all of reflective systems is the concave mirror as shown in Figure 19.13. Rays from a distant object are converged by the mirror to a focus (F). If a film were placed at (F), an image at this point, which is located on the axis half-way between the instantaneous center of curvature (C) at the axis of the

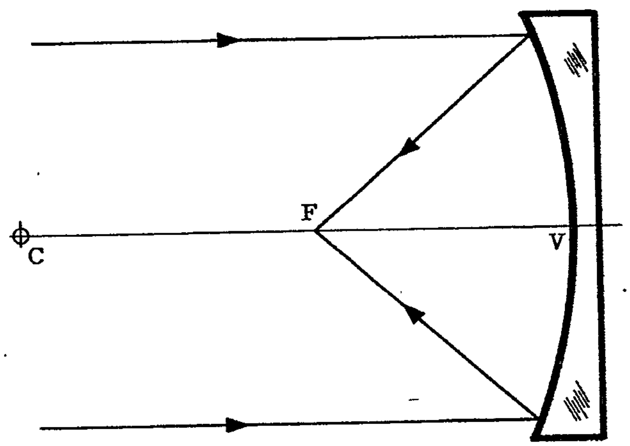


Figure 19.13- A concave mirror.

mirror and the vertex (V) of the mirror itself would be obtained. The simplest of all such mirrors is a portion of a sphere. However, a spherical mirror gives a very poor image at (F), since it is afflicted with a large amount of spherical aberration. This may be verified by ray trace, or by reference to the Seidel expression for spherical aberration. In order to sharpen the image at (F) in Figure 19.13, an aspheric surface for the reflector must be used, and this surface will be a paraboloid with its focus at (F).

19.3.1.3 The great astronomical instruments of the first half of the twentieth century fall into two main classes; the simple paraboloids, the greatest of which is the giant 200 inch diameter mirror at Mt. Palomar, and the Schmidt telescope.

19.3.2 The Newton system.

19.3.2.1 There is one serious mechanical disadvantage to the basic arrangement of Figure 19.13. If an eyepiece for visual observation, or a photographic plate is positioned at F, the center of the beam would be seriously interrupted. To overcome this difficulty, Newton, as early as the seventeenth century, proposed a plane mirror (M), to be positioned as shown in Figure 19.14, to bring the focus outside the beam where observation could be made. If a photographic motion picture film is placed at (F) in Figure 19.14, the recording apparatus can be made as large as necessary. A number of successful missile tracking devices have been constructed in accordance with this arrangement. It will be noted that the beam is partially obscured by the mirror (M). All reflective systems, except for the unimportant off-axis parabola, are characterized by this hole in the pupil, whether it is caused by a photographic plate or a mirror.

19.3.2.2 While it is true that a paraboloidal mirror forms a perfect image on the axis of the system, there remain important limitations. As has been discussed, physical optics and the finite wavelength of light impose a limitation on the resolving power of all optical systems. A good rule for this limitation is the simple equation

$$R = \frac{4.5}{a}, \qquad (2)$$

where R is the resolving power in seconds of arc, and a is the mirror aperture in inches.

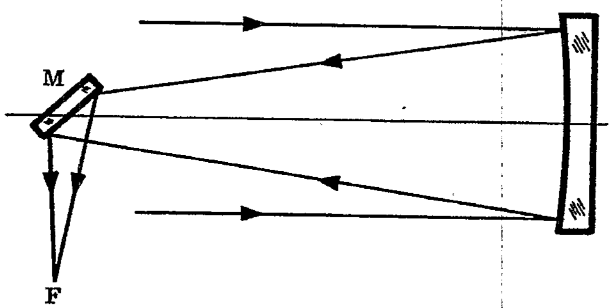


Figure 19.14- Newtonian mirror arrangement.

19.3.2.3 Another serious limitation on the properties of the paraboloid, is the presence of field aberrations, particularly coma. Indeed, the size of the useful field of view of a paraboloid, where the resolution over the field is in accordance with equation (2), can be given by

$$v = \left(\frac{f}{h}\right)^2 \Big/ 25 \qquad (3)$$

where v is the size of the field in inches, f is the focal length of the mirror, and h is the diameter of the mirror. The ratio $\frac{f}{h}$ is the f number of the mirror. Thus an f/5 mirror has a useful field of view of one inch, and an f/10 mirror has a useful field of four inches. The Mt. Palomar 200 inch diameter mirror works at f/3.3, so that its useful field of view is only 0.4 inch. However, over this 0.4 inch field, the Mt. Palomar paraboloid can theoretically resolve better than 0.025 seconds, which for a focal length of 660 inches, would amount to 0.00008 inch.

19.3.3 The Cassegrain system.

19.3.3.1 Another class of reflecting system is the two mirror Cassegrain arrangement.* This system is extremely popular in missile tracking instruments and is shown in Figure 19.15. Rays from a distant object strike a concave mirror (M_1) and are reflected towards a focus at (F_1). Before the rays are converged, a second mirror (M_2), which is convex, interrupts the beam and reflects it to a second focus at (F_2). The position of (F_2) outside the system puts it in an extremely convenient and favorable position for image recording. The hole in the mirror (M_1), is in the region which is blocked out of the original bundle by the physical presence of the convex mirror (M_2). The convex mirror is supported by a mechanical spider, or is cemented to a flat glass plate. In all cases, the mirror (M_2) magnifies the image considerably, since the distance from (F_1) to (M_2) is considerably less than from (M_2) to (F_2). A favorite value for this ratio is 4x, although this figure may vary considerably. This factor lengthens the focal length over that of the mirror (M_1) by the same amount, and similarly increases the focal ratio or f/#. Thus, the Cassegrain arrangement is very suitable for systems of long focal length and relatively low illumination.

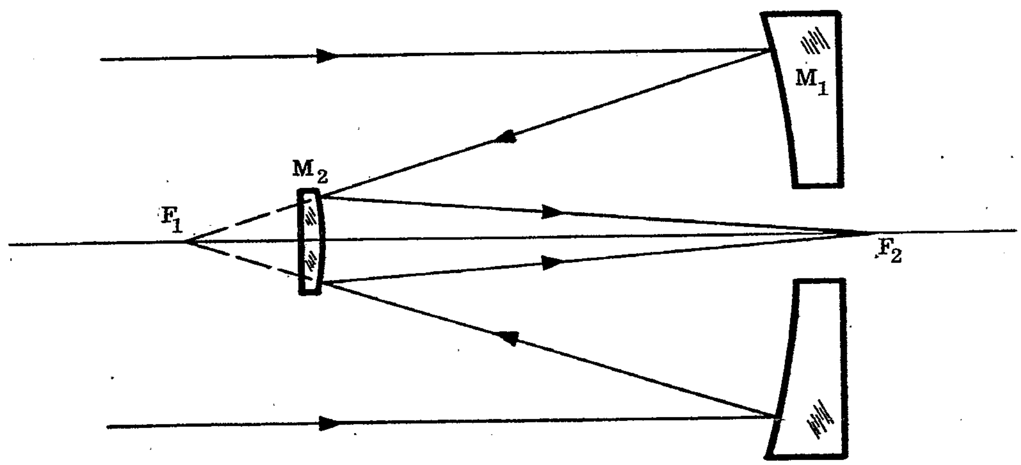


Figure 19.15- Cassegrainian Mirror arrangement.

19.3.3.2 However, there is always the requirement that the image at (F_2) be sharp, that is to say, free from spherical aberration. There are an infinite number of combinations of the two surfaces which will achieve this. For example, the mirror (M_1) can be spherical, in which case (M_2) is a complex, higher order curve. Conversely, the mirror (M_2) can be spherical, in which case the mirror (M_1) will be of a complex nature. Or, in its favorite form, (M_1) is a paraboloid with focus at (F_1), and (M_2) is a hyperboloid with foci at (F_1) and (F_2). Which of these combinations is the best? In most cases, the arrangement which gives freedom from coma will be the most desirable.

* While the system originally proposed by Cassegrain consisted of a paraboloidal primary with an hyperboloidal secondary, accepted usage today has broadened the term "Cassegrain" to apply to any system consisting of a concave primary and a convex secondary.

19.3.3.3 A power equation may be set up for (M_1) with

$$-x = a_1 y^2 + b_1 y^4 + c_1 y^6 + \ldots\ldots, \quad (4)$$

and the equation cut off after the first two terms. For all surfaces

$$a_1 = 1/2 r_1, \quad (5)$$

where r_1 is the instantaneous radius at the vertex of (M_1). Then let b_1 assume a continuous set of values in the equation

$$-x = y^2/2r_1 + b_1 y^4 \quad (6)$$

and for $b_1 = 0$, the surface is a paraboloid. For $b_1 = 1/8 r_1^3$, the surface is spherical, provided the semi-diameter of the mirror is not too large a fraction of the radius r_1.

19.3.3.4 A similar equation can be set up for the mirror (M_2) with

$$-x = y^2/2r_2 + b_2 y^4 \quad (7)$$

where r_2 is determined by the positions of the paraxial arrangement (F_1), (F_2), and (M_2), and b_2 is determined by requiring the marginal ray to pass through (F_2). The form of the mirror (M_2) for any given value of b_1 can then be found. The calculation is difficult and can best be effected on an electronic calculator if available. Then one paraxial and one marginal ray can be traced for each combination (the paraxial ray can be the same for all the combinations) and the departure from the sine condition can be determined. In this procedure, it will be found that the combination most nearly free from coma is not too far from the paraboloid-hyperboloid arrangement, in cases where the magnification of (M_2) is not too different from 4x. If complete freedom from coma is desired by the designer, some departure from this combination may be indicated. However, in all designs known to the author, the paraboloid - hyperboloid form is used, except for extremely low aperture systems, where the spherical aberration is unimportant, and two spheres may be employed.

19.3.3.5 Up to this point, the simple mirror arrangements have been discussed chiefly in the form of the paraboloid and the Cassagrain two mirror system. No color aberrations are involved in purely reflective systems. After spherical aberration is corrected, the paraboloid affords no more degrees of freedom to correct coma. In the Cassagrainian arrangement, the proper choice of form allows both spherical aberration and coma to be controlled. However, no mention has been made of the remaining aberrations, namely astigmatism, field curvature and distortion. These aberrations are handled in precisely the same way as in refractive systems. There is one important difference, relating to the Petzval field curvature.

19.3.3.6 It will be recalled that in refractive systems with a large excess of positive power, the Petzval curvature is concave towards the incident light. The reverse is true in reflective optics. A converging element (concave mirror) has associated with it a heavy Petzval curvature convex towards the incident light. This affords the possibility of combining converging reflective and refractive systems to achieve a flat field, as shall be discussed later. If r is the radius of the mirror, and c ($= 1/r$) its curvature, the contribution to Petzval curvature of any reflecting surface is given by

$$P = 2Nc \quad (8)$$

where N is the index of refraction of the medium in contact with the mirror. For a single concave mirror, the Petzval surface is concentric with the mirror surface as shown in Figure 19.16.

19.4 CATADIOPTRIC SYSTEMS

19.4.1 Introduction. Now consider the second class of reflective systems, which include the Schmidt arrangement and some of its variations and developments. There were two fundamental principles discussed in previous sections of this handbook relating to the change in aberration with a shift in the stop position. In brief, these principles are,

(1) The change in the Seidel coma coefficient, due to a change in the position of the stop, is proportional to the spherical coefficient, multiplied by the shift in the stop position.

(2) The change in the Seidel astigmatic coefficient is proportional to the coma coefficient, multiplied by the shift in the stop position.

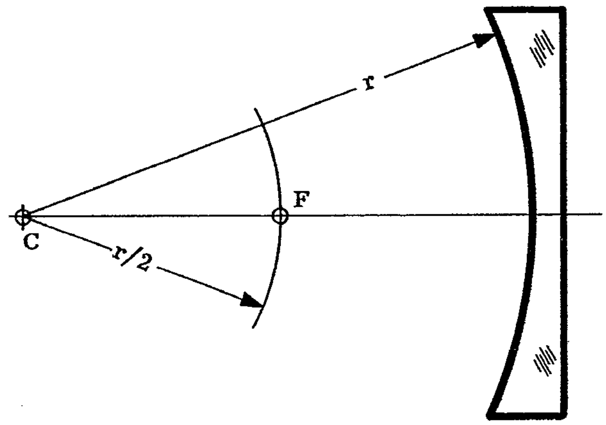


Figure 19.16- Curved focal surface, concave mirror.

A very important principle, relating to aspheric surfaces may then be added,

(3) Any change in form, without a change in the vertex radius, has no effect on the off-axis Seidel coefficients for an optical surface located at the stop position.

The third principle has far reaching consequences in the design of optical systems, since it allows the optical designer to correct spherical aberration by aspherizing the surface at the stop position. At this point, it will only have a mild effect on axial color (no effect at all in the case of reflecting systems), and no effect on coma, astigmatism, field curvature, distortion, and lateral color. It is not to be implied that the optical designer need limit the aspherizing of elements to the stop position, since the effects of such a procedure are subject to calculation, but the use of such surfaces in positions other than the stop brings them into the general juggling procedure characteristic of optical design.

19.4.2 The Schmidt system.

19.4.2.1 With respect to the contributions of Schmidt to optical science, it will be recalled that the single paraboloidal reflector had no spherical aberration but an extremely large amount of coma. In accordance with principle 1, the stop can be anywhere, without changing this coma, since the spherical aberration of the paraboloid is zero. If the stop is considered to be at the mirror, which would be the case if there are no artificial diaphragms in front of it, and the form of the mirror is allowed to change to something else, principle 3 states that the off-axis aberrations, including coma, will be unaffected. Indeed all concave mirrors of equal vertex radius, be they paraboloid, sphere, hyperboloid or off-beat curve will have equal amounts of coma, astigmatism and field curvature. The Seidel coefficient for astigmatism is equal to the refracting power as it is in the case of all thin systems at the stop. For the simple mirror, this is equal to 2c.

19.4.2.2 Now consider the apparently strange consequence of principle 3, and also consider principles 1 and 2. The optical designer might say that perhaps the large amounts of off-axis aberrations can be reduced by shifting the stop position. The designer will be frustrated in the case of the paraboloid, since there is no change in coma due to the absence of spherical aberration. He will probably choose the sphere, since as an accomplished technician he knows there will be less difficulty in making the sphere, than is the case with the hyperboloid or off-beat curve. If the designer allows the stop to recede from the sphere, he will notice a decrease in both coma and astigmatism, until the center of curvature of the mirror is reached. At the center of curvature, these aberrations will vanish and the configuration will be as shown in Figure 19.17.

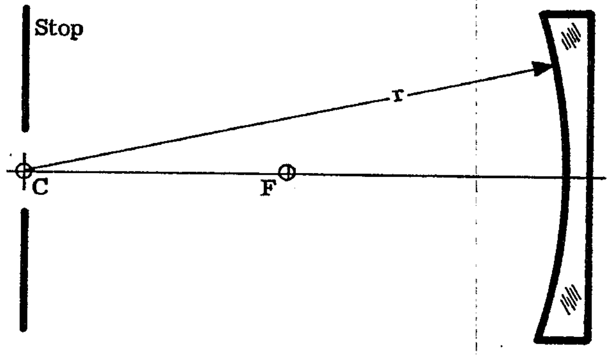


Figure 19.17- Schmidt arrangement, correction plate not shown.

19.4.2.3 At this stage, the design is free of coma and astigmatism, but still contains a large amount of field curvature, which must be tolerated, and a considerable amount of spherical aberration. Principle 3 will allow for the correction of the latter, by placing a plane parallel refracting plate at the stop at c in Figure 19.17, and deforming one surface to correct the spherical aberration. Since the spherical mirror undercorrects the marginal ray, the edge of the plate must have a slight negative power to neutralize it, as compared to the center. In practice, it is common to impart a tiny central positive power to the plate, resulting in a parallel zonal section as shown in Figure 19.18, and a reduced negative marginal region, thus shortening the overall focal length by a very small amount. With the parallel zonal section approximately 0.8 of the distance from the center to the margin, the plate imparts the minimum axial color to the beam.

19.4.2.4 It is fairly certain that Schmidt did not follow this rather involved reasoning to conceive his system. It is quite obvious that for a stop placed at the center of curvature of a spherical mirror, the coma and astigmatism must be zero, since the chief rays strike the mirror normally and can define a new axis just as valid as the central axis. There can be no coma or astigmatic difference on the axis of an optical system. However, the more involved reasoning first given is capable of further extension and application as shown below in the case of the oblate spheroid.

19.4.2.5 The Schmidt telescope has the very great advantage over the paraboloid of enormously extending the field of view over which the image remains sharp. The largest made to date is located on Mt. Palomar, and has a 48 inch diameter aperture with an ability to take sharp pictures over an area 14 inches square. The plates must assume the shape of the field curvature. The focal ratio is F/2.5 with a focal length of 120.9 inches. However, the Schmidt suffers from other defects. Since it has its corrector at the center of the curvature of the mirror, the system must be twice as long as its focal length. Also, to prevent vignetting, the primary mirror must be considerably larger than the aperture. In the Mt. Palomar instrument, the spherical mirror is 72 inches in diameter.

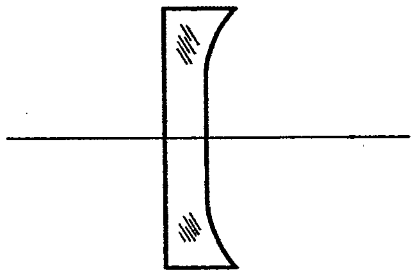
19.4.2.6 If only moderate extension of the field of view is desired, the Schmidt type arrangement, with an oblate spherical primary, can achieve this in a much shorter structure than the Schmidt with a spherical primary. In the following equations,

$$-x = \frac{1}{2r} y^2 , \qquad (9)$$

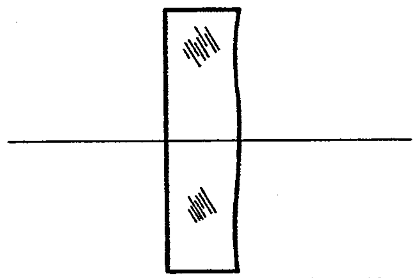
$$-x = \frac{1}{2r} y^2 + \frac{1}{8r^3} y^4 \quad , \qquad (10)$$

$$-x = \frac{1}{2r} y^2 + \frac{2}{8r^3} y^4 \quad . \qquad (11)$$

Equation (9) is the equation of the paraboloid, equation (10) represents a sphere (expansion of its equation to two terms), and equation (11) represents a surface twice as heavily curved away from the paraboloid as the sphere. Any surface in which the coefficient of the y^4 term is in excess of $1/8r^3$ is called an oblate spheroid and equation (11) defines one such surface.



Schmidt with parallel central section.



Schmidt with parallel zonal section.

Figure 19.18- Schmidt plate with parallel central section, (a); and parallel zonal section, (b).

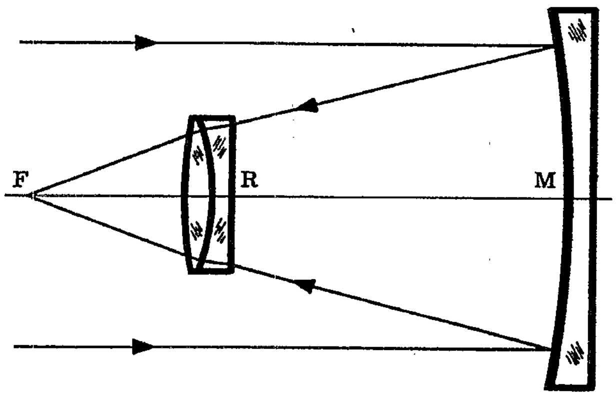
19.4.2.7 The Seidel theory of aberrations as applied to aspheric surfaces states that if equation (9) defines, as it does, a surface with no spherical aberration, and equation (10) defines a surface with a certain amount, then equation (11) is a surface with twice the spherical aberration of that defined in equation (10). Referring back to principle 1, it will be recalled that spherical aberration is needed in order to correct coma by shifting the stop, and that it is possible to correct the coma of the sphere by shifting the stop from the mirror to the center. If now, the designer starts with a surface in accordance with equation (11) having twice the spherical aberration of the sphere, we need to shift the stop only back to the focus to correct the coma, shortening the instrument to half of the Schmidt. The correction plate will now have to be made to correct the doubled spherical aberration.

19.4.2.8 The situation with respect to astigmatism is not so fortunate. Since the stop-at-mirror coma is identical for equation (10) and (11), principle 2 states that shifting the stop back to the focus will correct only half of the astigmatism. For maximum field the stop may be shifted a little further, thus reducing the astigmatism, but allowing the coma of opposite sign to creep back in, until a desirable compromise is reached. The Schmidt principle is capable of a number of variations which will not be discussed further.

19.4.3 The Ross-Baker system.

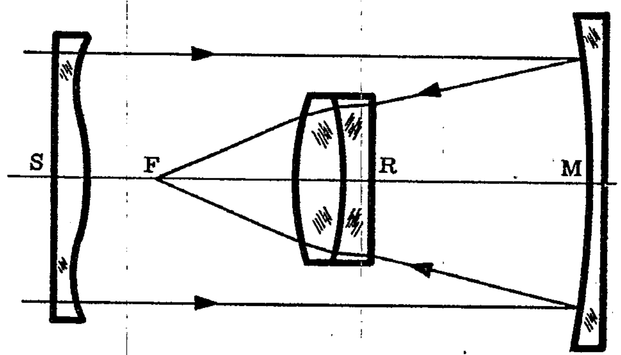
19.4.3.1 Efforts to extend the field of view of the paraboloid have met with some degree of success. Ross constructed some lenses spaced close to the focal plane in accordance with the arrangement in Figure 19.19. A color corrected doublet, placed at R as shown in Figure 19.19, is given sufficient power so that its positive Petzval field curvature contribution will just neutralize that of the mirror M. Its bending and spacing from F offer two degrees of freedom, which are used for the correction of coma and astigmatism. Unfortunately, it proved impossible to prevent R from reintroducing undercorrected spherical aberration into the system so that the images tended to be soft. However the field of view of the paraboloid was greatly extended by this maneuver.

19.4.3.2 James G. Baker has proposed the addition of a Schmidt corrector plate to the Ross system as shown in Figure 19.20. The combination of M and R in Figure 19.20 is designed for correction of coma and astigmatism for a stop position at S. Then insertion of the Schmidt plate takes out the residual spherical aberration. An analysis of this system shows that before the insertion of the Schmidt plate, the residual spherical aberration amounted to a substantial fraction of that of a spherical mirror. However, the amount of depth needed to be hollowed out of the Schmidt was only a few wavelengths of light. This illustrates the tremendous leverage exerted on the rays by systems of this type, and the demanding exactitude required for their construction. Baker proposed this system, which gives definition over a field of view an order of magnitude larger than that of the paraboloid, as a means of correcting simple paraboloids already in existence by the addition of R and S. A similar requirement makes this system desirable for missile recording. The complete system can be used for photographic tracking, while the removal of the refracting elements S and R allows conversion of the system for the detection of targets in the medium infra-red and in the ultraviolet, where the glass would be opaque.



NOT TO SCALE
The size ratio of element R to element M is approximately 5 to 1

Figure 19.19- Ross arrangement.



NOT TO SCALE
The size ratio of element R to element M is approximately 5 to 1

Figure 19.20- Baker arrangement.

19.4.4 Modification of the Ross-Baker arrangement.

19.4.4.1 The author of this section has proposed, in an exchange of letters with Dr. Baker, the elimination of the correcting plate S and transferring its function to the mirror M, thus deforming the paraboloid. The combination of deformed paraboloid, and Ross lens R would have the disadvantage of making the mirror unusable by itself. However, it would have the advantage of eliminating the only large refracting element and enabling the system to be carried to higher physical dimensions.

19.5 APPLIED SYSTEMS

19.5.1 Satellite tracking camera.

19.5.1.1 The principles on which the design of the more complex optical systems used in missile and satellite tracking are based, are illustrated in the Figure 19.21. The design is a classical Schmidt system with just a few variations. For the purpose intended, this camera was designed for high light gathering power and large field, particularly in one direction, preferably the direction of satellite path. The physical aperture of the system is 20 inches, and with a focal length of approximately the same value, the system operates at f/1. To prevent vignetting the primary spherical mirror is 31 inches in diameter.

19.5.1.2 It will be noted that the aperture of the system is very close to the center of curvature of the primary mirror, but the single correcting plate, which normally is located there, is split up into a color corrected triplet for the purpose of eliminating the small amount of residual axial color in the single Schmidt plate. The four inner surfaces of this system are aspheric.

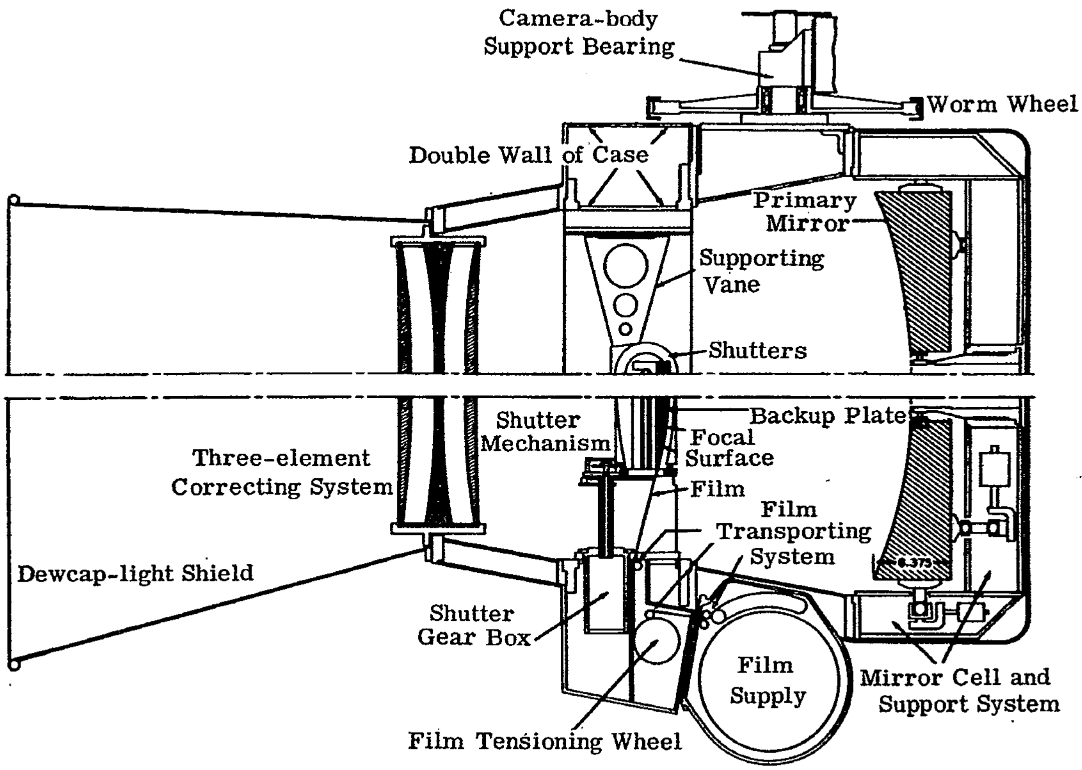


Figure 19.21-Section view of a satellite-tracking camera. (Courtesy of Perkin Elmer Corp.)

19.5.1.3 It is presumed that, because of the high relative aperture of this system (f/1) the curvature of the Schmidt plate required would be more extreme than usual, leading to more axial color than the designer could tolerate. The splitting up of the single plate into three, with the central glass different from the outside, and the distribution of the Schmidt curvature among four surfaces would tend to alleviate this situation.

19.5.1.4 It will be noted from Figure 19.21, that the film is transported over a spherically curved gate, which matches the curved focal plane of the image. The curvature in the plane at right angles must necessarily be zero, because of the mechanical impossibility of bending the moving film into a compound curve. Consequently the field coverage in this direction is limited to only 5 degrees, while in the direction of film travel it reaches the amazing value of 31 degrees. It was found, that at the edges of this extreme field the focal surface departs slightly from a spherical shape so that the film runners are not quite circular. The combination of careful design and excellent execution resulted in a system wherein 80 percent of the point energy anywhere in the field is within a circle 0.001 inch in diameter. This instrument was conceived for the purpose of tracking the U. S. Vanguard satellite, and the first instrument arrived just in time to be used for the original Sputniks.

19.5.2 ROTI Mark II (Recording optical tracking instrument).

19.5.2.1 This instrument and the Igor were made to similar specifications but each has features worth discussing. The original requirements envisioned a versatile instrument capable of a series of fixed focal lengths ranging up to 500 inches in value. Another requirement was that the instrument could be adapted for infrared which necessitates the choice of a paraboloid for the primary mirror.

19.5.2.2 It would be thought that some form of the reflector-corrector system of Baker (Figure 19.20) would be indicated and such is indeed the case. However, the corrector feature is introduced in a rather unique fashion.

19.5.2.3 Referring to Figure 19.22, light enters from the left, passes through the window and strikes the primary mirror, a paraboloid of 100 inch focal length and 24 inch aperture. After reflection by two Newtonian mirrors as shown, the rays are brought to a focus at the reticle. Just before this point a pair of sliding wedges introduce a variable amount of glass into the path. By adjustment of these wedges, the instrument focus can vary between 3000 yards and infinity, the focus being automatically controlled by range data determined from the associated radar equipment.

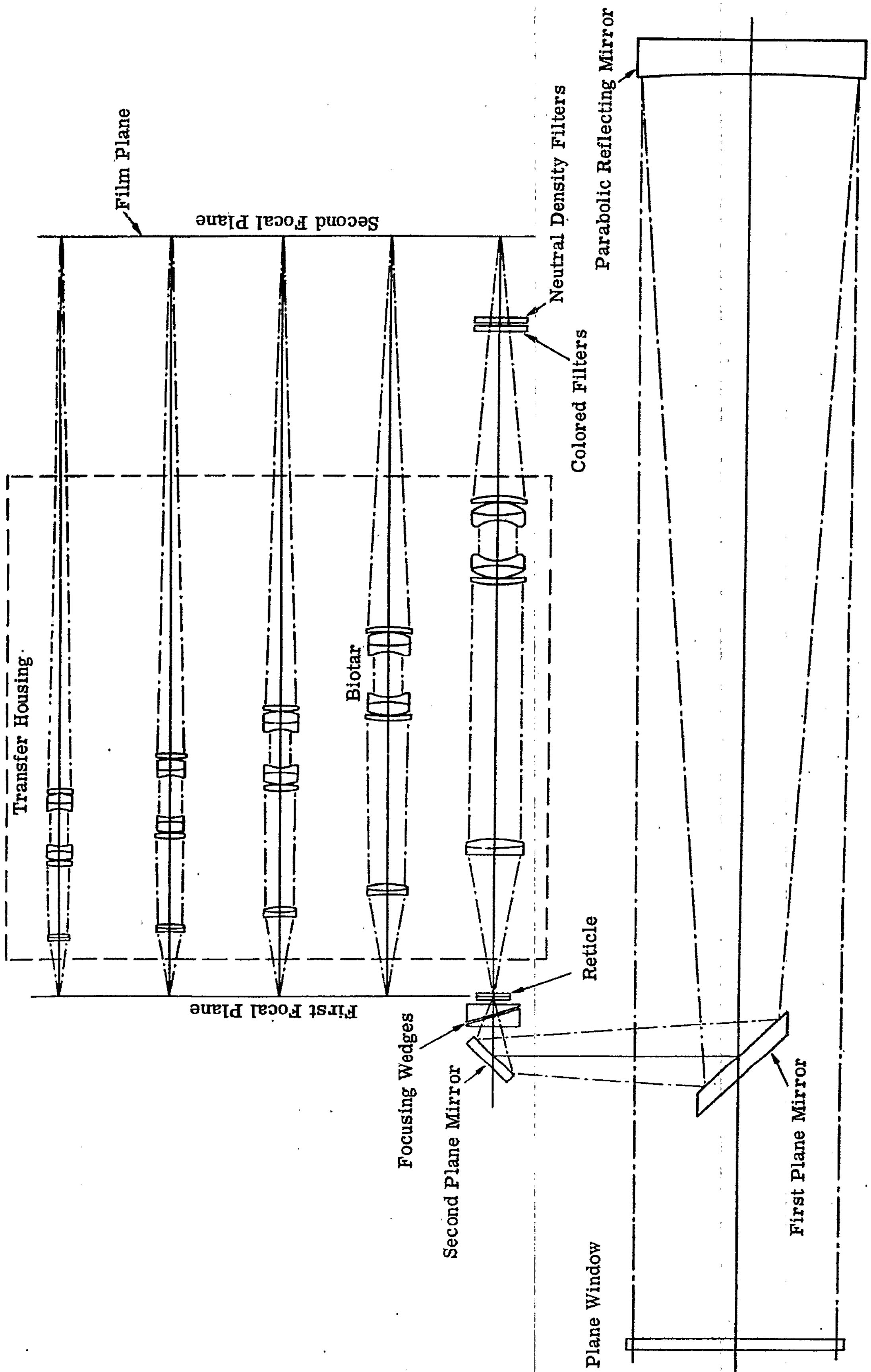


Figure 19.22 – R. O. T. I. Mark II, Optical arrangement. (Courtesy of Perkin Elmer Corp.)

19.5.2.4 As is known from the previous discussion, the primary image is heavily afflicted with coma, and the astigmatism is only partially corrected by placement of the stop at the window. The doublet lens, placed behind the focal plane, serves three functions. First it acts as a Ross lens to neutralize the coma resulting from the paraboloid; second, it is a collector lens to turn the rays into the reimaging Biotar system; finally it imparts an initial magnification (2x) to the image at the prime focus.

19.5.2.5 The five imaging systems can be inserted into the system at will. These have magnifications ranging from 1x to 5x in unit power steps. With the doublet lenses working at 2:1, the Biotar type imaging lenses work at magnifications of 1/2, 1 . . . . 2-1/2x.

19.5.2.6 Where is the Schmidt type correcting surface characteristic of the reflector corrector design? Normally, it would be at approximately the location of the window. However a conjugate stop position exists in this system, namely the midpoint of the Biotar imaging lens. The aspheric correction required is put on the two innermost surfaces of this lens, those adjacent to the aforementioned stop position. This aspheric correction is for the purpose of eliminating the zonal spherical aberration only, since the primary spherical is taken care of in the design of the refractive elements. KZF Schott glass is employed in the system to reduce considerably the secondary spectrum.

19.5.2.7 After passing through the imaging lens, the rays traverse filters, colored or neutral as desired. The latter are automatically controlled by photoelectric means. The final focal plane is at the gate of the 70 mm camera, with a 2-1/4 inch square field of view.

19.5.2.8 Recapitulating, the five systems give a range of focal lengths from 100 inches F/4 to 500 inches F/20 (approximate figures, not allowing for the occlusion by the first Newtonian mirror). When enough light is available, the 500 inch focal length reaches into extremely long distances for the target missile, and the writer believes this has proved the most used focal length. Tracking is effected by means of two operators, one for azimuth and one for elevation, with appropriate telescopes. An elaborate electrical control system is very effective in the accuracy of tracking. A photograph of ROTI is shown in Figure 19.1 .

19.5.3 <u>Igor</u>.

19.5.3.1 This instrument is somewhat smaller than ROTI but serves essentially the same purpose, i.e., the observation of missiles in flight to extreme distances. The original specifications envisaged some situations where a 70 mm camera (2-1/4" x 2-1/4") was to take pictures under some circumstances at the prime focus (P) as shown in Figure 19.23. Consequently the image at this point had to have complete correction for all aberrations over its 2-1/4 inch by 2-1/4 inch format. The combination of Schmidt Plate (S), primary mirror (M), Newtonian mirrors (N1) and (N2), and Ross lens (R) as shown in Figure 19.23 accomplished this purpose very well. Removal of (S) and (R) allows for infrared and ultraviolet measurements if desired. Focussing of the system from infinity to 3000 yards is effected by motion of the Ross lens (R) towards the mirror (N1). The total travel to cover this range is 3/16 inch.

19.5.3.2 The system works at F/5 with a clear aperture of 18 inches, so that the focal length of the system at prime focus is 90 inches. The focal length of the paraboloid itself is 118 inches, reduced to the required 90 inches by the Ross lens. A collector lens at (P) and a re-imaging system at (L) enables final imagery on a 70 mm camera at (F).

19.5.3.3 For a change to longer focal lengths and magnified images a system of Barlow lenses is employed. A Barlow lens is illustrated in Figure 19.24. Suppose rays are converging to a prime focus at P. A negative lens B is inserted into the path and diverges the rays to a more distant focus at F. The image is magnified by the ratio BF/BP.

19.5.3.4 Referring to Figure 19.23, the 1x system of collector and re-imaging lenses are removed and one of the Barlows, B_2 (2x), B_4 (4x) or B_5 (5.7x) is inserted, transferring the final image to the camera at (F) under the particular desired magnification. The highest available magnification corresponds to a focal length in excess of 500 inches.

19.5.3.5 The variable density filter arrangement at (D) is comprised of two oppositely rotating continuously varying density filters, slightly inclined to each other and to the axis, to eliminate multiple reflections. Two are needed to keep the field uniform. As in ROTI, the image is quite good over the field of view in all powers.

19.5.4 <u>S.M.T. (Small Missile Telecamera)</u>.

19.5.4.1 The requirements for this instrument are similar to those for the satellite tracking camera except that a relatively small field of view is required, namely that of a 70 mm camera. High light gathering power (low F/#) is indicated for the tracking of small, high velocity missiles. The focal length of the system is 100 inches with a 30 inch aperture, so that the system is working at F/3.3, and the field is 1° 18'. This system

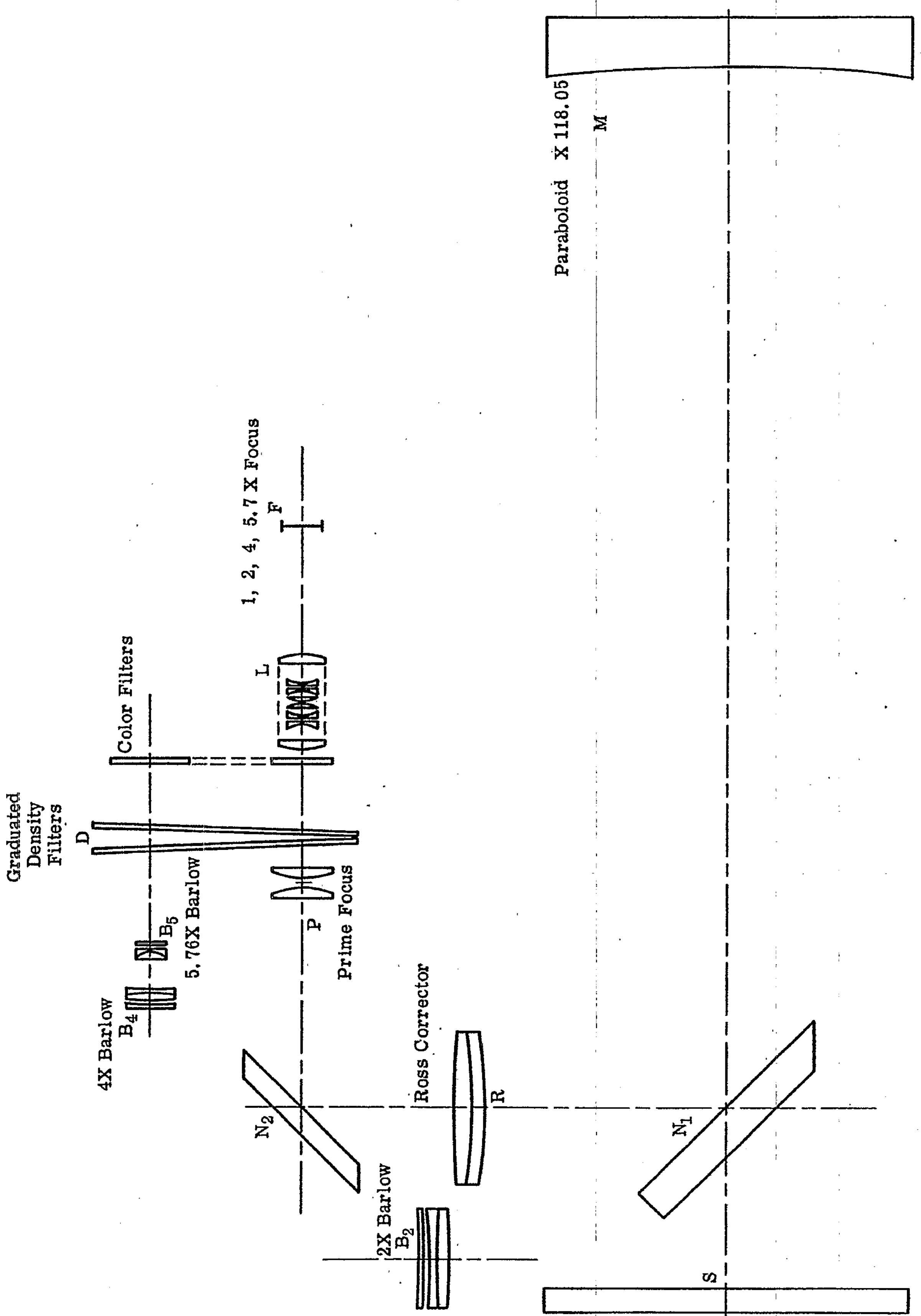


Figure 19.23–I.G.O.R. Optical arrangement. (Courtesy of American Optical Co. Drawing No. 013-0032)

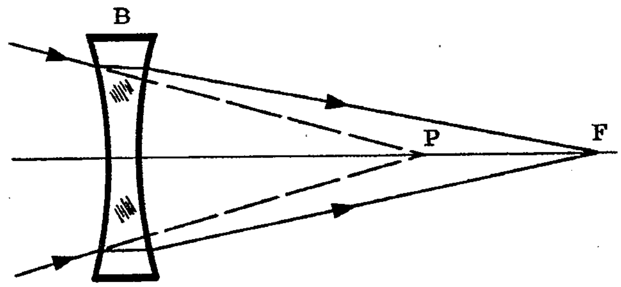


Figure 19.24- A Barlow lens.

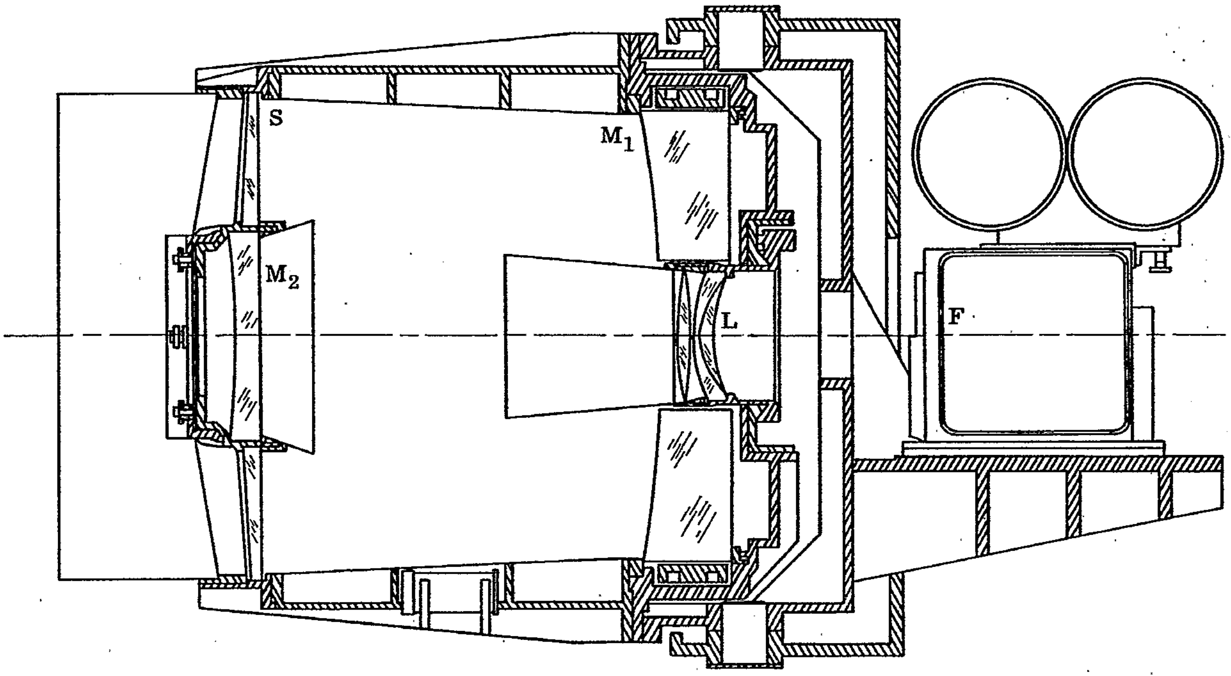


Figure 19.25- S. M. T. Optical arrangement.
(Courtesy of Perkin Elmer Corp.)

is similar to the Cassegrain arrangement in that two mirrors are involved, with the final image behind the hole in the primary as shown in Figure 19.25.

19.5.4.2 Light enters from the left and strikes the spherical mirror (M), after passing through the Schmidt plate (S). The convergent beam is interrupted by the mirror (M_2), which is actually a lens with its rear surface reflecting (often known as a Mangin mirror). The beam thus reflected a second time traverses the lenses placed in the central hole in the primary before reaching the film plane at (F).

19.5.4.3 This arrangement delivers an excellent flat field over its 2-1/4 inch x 2-1/4 inch field. The Mangin mirror and hole lenses afford enough degrees of freedom to correct for the coma and astigmatism of the shortened Schmidt arrangement, as well as to balance out the color aberrations they themselves bring in. The missile trajectory is predicted beforehand and the instrument is aimed from stellar observations made with a calibration camera.

19.5.4.4 It will be noted that heavy emphasis has been given to the optical arrangement of these complex instruments and their illustration of the optical principles discussed earlier. No space is available here for the description of their complex mechanical, electrical and electronic construction which tax all the resources of modern technology.