12 LENS RELAY SYSTEMS

12.1 INTRODUCTION

12.1.1 Lens relay systems of the type mentioned in Section 7 (Paragraph 7.5 and Figures 7.5 and 7.6) are described in more detail here.

12.1.2 Relay systems are used for two purposes: to provide the proper orientation of the image, and to transfer the light from one region to another. Sometimes the distance between the object and the final image may be large and, in addition, the diameter of the lenses must not become excessive. These conditions may require a series of relay lenses resulting in a system called a periscope.

12.2 THE BASIC LENS PROBLEM OF A RELAY SYSTEM

12.2.1 Suppose a relay system is needed to transfer light from an object plane to an image plane. The two planes are separated by the distance D, Figure 12.1. The magnification should be -1.

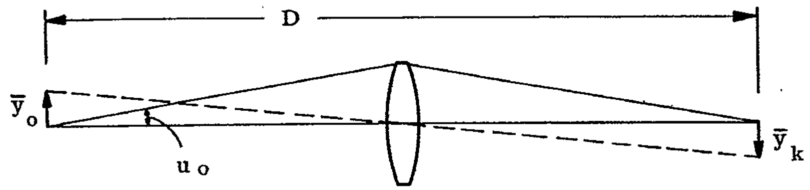

Figure 12.1 - A single-relay-system.

12.2.2 The diameter of the objective will be determined by the angle u_o. The type of lens to use for the objective depends upon the image quality required. For the moment, however, assume that the relay lens will consist of two telescope objectives of the type described in Section 11. The objectives could be placed so that the light would be parallel between them. One could start with any of the lenses shown in Tables 11.3, 11.5, 11.6 or 11.7.

12.3 A VISUAL SYSTEM, NUMERICAL EXAMPLE

12.3.1 To evaluate the visual performance with telescope objectives, first suppose that the system is a visual system, with a 10X eyepiece (see Section 14) used to view the image.

12.3.2 In Section 11.1.3 the effect of the Petzval curvature was described. Equation 8-(28) gives the value of P for a thin lens as $-\phi/n$. The value of P for the relay lens shown in Figure 12.1 would then be $-\frac{4}{Dn_e}$, where n_e is the effective index of the doublets used for the relay lens. The 10X eyepiece could be any of those described in Section 14. For this example, the very common Erfle eyepiece shown in Figure 14.19 will be used. According to Table 14.7, the lens will have a value of P = -0.2125 in reciprocal cm.

12.3.3 From the data shown in Section 11 on doublets, it is evident that the doublets could be used at $f/3.5$. If they could be 10 cm in diameter, each doublet could have a focal length of 35 cm, and the distance D would be approximately 70 cm. The value of P for the relay lens would then be -0.038.

12.3.4 This is only 18% of the Petzval contribution introduced by the eyepiece. It is, therefore, probably negligible as long as the field of view is maintained within the field of the eyepiece. The data in Figure 14.21 show that this eyepiece can cover about 28° half field with a negative distortion of approximately 8%. The maximum image height is therefore 1.32 cm. This means that the maximum object height will be 1.32 cm.

12.3.5 The use of other types of lenses for relay objectives is considered here. If two triplet objectives of the type designed in Section 10 were used, the value of n_e in the equation $P = -\phi/n_e$ could be raised to about 3.0 or 4.0. With a value of $n_e = 4.0$, the P value of the relay lens would be -0.014. This would be completely negligible compared to the value introduced by the eyepiece. On the other hand, the triplet would not be as well corrected on the axis as the doublets. It is usually not a good idea to attempt to correct the Petzval curvature of the relay lenses until they start to introduce a contribution which is one half, or at most equal to, that of the eyepiece.

12.4 SECONDARY COLOR IN A RELAY SYSTEM

12.4.1 In paragraph 6.10.8.3 it was indicated that telescope lenses made out of ordinary glass have an amount of secondary color given by the expression

$$TAch_{F-D} = \frac{y_1}{2200} .$$

12.4.2 The relay lens in the sample problem in Figure 12.1 would have secondary color given by the equation

$$TAch_{F-D} = \frac{2y_1}{2200} = \frac{u_o D}{2200} . \qquad (1)$$

12.4.3 Equation (1) shows that the secondary blur at the focal plane of the eyepiece increases as u_o or D is increased. In the sample problem, if $u_o = 0.14$ and D = 70 cm., the radius of the secondary blur $TAch_{F-D}$ is 0.005 cm. With the 10X eyepiece, this would subtend an angle of 0.002 radians. This is 6.6 minutes, which is definitely noticeable but usually tolerated. Any more than this is objectionable.

12.4.4 Secondary color is usually a serious problem in relay systems. If the distance D must be maintained, then the secondary color can most easily be reduced by making u_o smaller. A value of $u_o = 0.14$ means that the exit pupil diameter will be 7 mm. This is desirable for maximum light transmission, but it could be reduced to 2 mm without impairing the observer's resolution. This would then cut down the secondary color to 2.2 minutes.

12.4.5 The secondary color can be reduced by separating the two doublets. As long as there is parallel light between the two lenses, the space between the lenses can be considered free space. If this distance is d, the secondary color is given by the equation

$$TAch_{F-D} = \frac{u_o (D-d)}{2200} . \qquad (2)$$

12.4.6 As d is made larger, the focal lengths of the lenses are reduced. They therefore introduce more field curvature. It is a fairly general rule that any step taken to reduce secondary color without sacrificing clear aperture will result in more field curvature.

12.5 FURTHER DETAILS ON DESIGN OF DOUBLETS AS RELAY LENSES

For a unit power relay system, there are advantages in using two identical doublets with parallel light between them. Since the doublets are usually air-spaced, this means there are eight glass air surfaces. In principle it is possible to combine the positive elements of the doublets into a single lens surrounded by two negative lenses. One could also combine the two negative lenses and surround the combination with two positive elements. One can see what would be involved in doing this by considering the solutions shown in Table 11.3. In these solutions, the curvature facing the parallel light c_1 is about 0.16. In order to make the positive element in the combined doublets a single lens, it would be necessary to bend these solutions until $c_1 = 0$. Take, for example, Case No. 10 in Table 11.3. If the lens should be bent to make $c_1 = 0$, the remaining curvatures would be $c_2 = -0.4798$, $c_3 = -0.4694$, and $c_4 = -0.2341$. The doublet would no longer be corrected for spherical aberration or coma. The spherical aberration, at least to the third order, could be corrected by adjusting the curvature differences between c_2 and c_3. The coma would, however, be far from corrected. By facing this with an identical doublet, the two plane surfaces could be contacted. This means the positive lens could be made into a single equiconvex lens. The spherical aberration of the doublets would add, but the coma would subtract, to zero. This argument shows that a triplet relay lens is possible, but it also shows that it will probably have considerably more higher order spherical aberration than the two doublets. The use of a triplet of this type could be recommended only when the zonal spherical aberration is tolerable.

12.6 DOUBLE-RELAY SYSTEMS

12.6.1 Often in relay systems one is limited in the diameter of lens which may be used as, for example, when the lens must be confined to a given diameter. In the single-relay example described in 12.3.3, the relay was allowed to have a diameter of 10 cm. Suppose there was a limitation to a 5-cm. diameter for all lenses. In order to maintain $u_o = 0.14$, it would be necessary to use a double-lens relay system as illustrated in Figure 12.2.

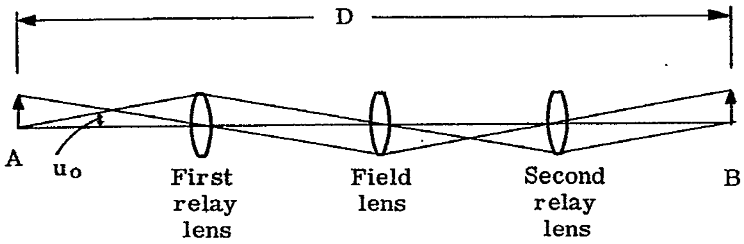

Figure 12.2 - A double-relay system.

12.6.2 In the double-relay lens system, the first and second relay lenses must have just one-half the focal length of the single-relay lens. They will, therefore, each add twice the Petzval contribution.

12.6.3 The two relay lenses will not introduce any more secondary color. Each relay transfers by the distance D/2, so that each has half the secondary color; but they add, so the total comes out the same. Equation (1) still applies for a double relay system.

12.6.4 Note that in Figure 12.2 an extra lens has been added in the intermediate focal plane. This is called a field lens. Its function is to image the chief ray passing through the center of the first relay lens at the center of the second relay lens. This field lens has a focal length equal to D/8. It will also introduce negative field curvature equal to 8/Dn.

12.6.5 One can see then that doubling up the relay system in order to reduce the diameter of the lenses has introduced Petzval field curvature. The relay lenses introduce four times as much field curvature, and the field lens adds as much as one of the relays. The double-relay lens, therefore, introduces six times as much field curvature as the single-relay system of equal length and numerical aperture. The secondary color is not changed.

12.6.6 Reducing the field of view. One can argue that there is little to be gained in reducing the field of view. The eyepiece is designed for, and capable of, viewing an object height of 1.32 cm. It is true that the relay lenses are going to make the image at the end of the field more blurred, but to introduce a stop at the field lens would merely mean a slightly smaller field lens. The savings in cost would be negligible. If one decides to use doublet relay lenses, nothing is lost in using the full field of the eyepiece. It is better to see the wide field, even if it is blurred, than to stop it down. As a rule, the field of a visual instrument should not be reduced if the extra field can be obtained with no increase in cost and size of instrument.

12.6.7 Relay lenses corrected for field curvature. If the problem demands improved quality at the edge of the field, it is necessary to abandon the doublet relay lenses and use a lens with reduced field curvature. Triplets as described in Section 10 may be used if one does not try to increase n_e beyond 4. If this is not enough, a double Gauss lens is recommended.

12.6.8 Field lenses corrected for field curvature. It is possible to introduce compound field lenses, such as triplets, to reduce the Petzval curvature of the field lens. If the field lens is located between the two unit power relay systems, it must be symmetrical. It will have to perform for a finite conjugate, and the region of solution will be quite different from the lenses described in Section 10. One can think of the lens as basically two inverted telephoto objectives with parallel light between them. The lens can be roughed out by first designing one side. One should avoid placing a lens surface directly in the intermediate focal plane, for it will eventually collect dust.

12.7 SUMMARY

12.7.1 Relay systems are inherently limited by problems of field curvature and secondary color. One should always try to use as few relays as possible, until glass weight and cost become a problem.

12.7.2 If it is necessary to use more than one relay, the same rules apply. The relay and field lenses should be kept as large as practicable. The size of the relay or field lenses should never be reduced needlessly.

12.7.3 The secondary color depends on the over-all distance of relay and the value of u_0. If the secondary color becomes serious, it is necessary either to accept it, reduce u_0, or resort to special lens materials.

12.7.4 Whenever attempts are made to correct the field curvature, more secondary color is introduced. See Figure 6.21.

12.7.5 Doublet relay lenses are usually prefered to other more complicated types. They represent a good compromise between simplicity, cost, number of surfaces, and reasonable image quality. It is possible rapidly to reach a point of diminishing returns in trying to reduce field curvature: the result will be a design with increased secondary color and chromatic variation of aberrations.