11 TELESCOPE OBJECTIVES

11.1 INTRODUCTION

11.1.1 Scope. Sections 11 through 15 will be devoted to the demonstration of design principles using the simple telescope as a working example. Section 11 will discuss the detailed design of objectives; Section 12, lens erecting systems; Section 13, mirror and prism systems; Section 14, eyepieces; Section 15, complete telescope. It is assumed at this point that the reader has studied all the foregoing material and now has considerable knowledge of optical design. The inexperienced reader should not be concerned if he does not fully grasp the following material on first reading. As he gains experience it will become more and more useful.

11.1.2 The complete telescope. In describing the telescope, it will be assumed that the object is at infinity and that the eyepiece will be focused to place the virtual image at infinity. This means that the distant object will be brought to focus or imaged at the second focal point of the objective and that the first focal point of the eyepiece will coincide with this image. The telescope is then said to be in afocal adjustment.

11.1.3 The Petzval curvature of the system.

11.1.3.1 To start the design of an optical system, one of the first calculations to be made is an estimate of the amount of field curvature in the system, i.e., an estimate of ΣP. This is done by calculating P for each surface using Equation 8-(14), and summing. From Equations 8-(15) and 8-(16), if ΣB, ΣF, and ΣC are all zero, then

$$\alpha_y = \left(\frac{L_{k-1}}{M_{k-1}} - \frac{\bar{L}_{k-1}}{\bar{M}_{k-1}}\right) = \frac{\Sigma P \Phi^2}{2n_{k-1}\; y_{k-1}} \left(\frac{Y_1}{y_1}\right)\left(\frac{\bar{Y}_o}{\bar{y}_o}\right)^2 * \qquad (1)$$

$$\alpha_x = \frac{K_{k-1}}{M_{k-1}} = \frac{\Sigma P\, \Phi^2}{2n_{k-1}\; y_{k-1}} \left(\frac{X_1}{y_1}\right)\left(\frac{\bar{Y}_o}{\bar{y}_o}\right)^2 . \qquad (2)$$

These equations show that P introduces angular aberrations, α_y and α_x, which are linearly proportional to Y_1/y_1 and to X_1/y_1. This means that for oblique object points the fan of rays entering the objective from a distant object point, do not emerge from the eyepiece as a parallel bundle. Instead the rays come to a finite focus. When $\bar{Y}_o$ is zero, Equations 8-(15) and 8-(16) indicate that the rays from the object point do emerge parallel to the axis because the telescope is in afocal adjustment and $\Sigma B = 0$. For the case described by Equations (1) and (2), the amount of the angular aberration varies as the square of the object height, $\bar{Y}_o$.

11.1.3.2 If the telescope objective is assumed to be thin, then from Equation 8-(28), $P = -\phi/n$. It is possible to change the value of P by using a photographic type of objective, but a simple doublet objective is by far the most common type of lens used in telescopes. In order to derive a general formula for the aberration, it is possible to write $P_o = -\phi_o/\gamma_o$, where γ_o is a constant ranging in value from 1.5 to ∞. For a doublet γ_o is approximately 1.5.

11.1.3.3 The value of P for the eyepiece also depends on how it is designed, but here again it is possible to write $P_e = -\phi_e/\gamma_e$, where ϕ_e is the power of the eyepiece and γ_e is a constant depending on the type of eyepiece. γ_e will range in value from 2.6 to 0.7 for most eyepieces.

11.1.3.4 Using these values of P for the objective and the eyepiece, it is possible to compute, from Equation (1), the angular aberration,

$$\alpha_y = -\frac{n_o^2 \; u_o^2}{2\,n_{k-1}\; y_{k-1}} \left(\bar{Y}_o^2\right)\left(\frac{Y_1}{y_1}\right)\left(\frac{\phi_o}{\gamma_o} + \frac{\phi_e}{\gamma_e}\right) .$$

Since a telescope is usually used in air, n_o and n_{k-1} are 1. Then,

$$\alpha_y = -\frac{1}{2}\left(\frac{\bar{Y}_o}{t_o}\right)^2 \left(\frac{Y_1}{y_1}\right) MP^2 \; y_{k-1} \; \phi_e \left(\frac{-1}{\gamma_o\, MP} + \frac{1}{\gamma_e}\right) . \qquad (3)$$

*For a telescope with an object point at infinity one should strictly speaking use $(\tan U_o)/u_o$ instead of $\bar{Y}_o/\bar{y}_o$ but it is satisfactory to use the above equations and assume that the object distance is very large but finite.

This equation shows that the angular aberration due to Petzval curvature only varies

(1) as the square of the object height,

(2) approximately as the square of the MP,

(3) linearly with y_{k-1} .

11.1.3.5 For a given $(\overline{Y}_o / t_o)$ MP, the angular aberration in a telescope can be made small by making ϕ_e or y_{k-1} small. It is customary to specify the angular aberration of telescopes in diopters using the definition $d = 100\alpha / y_{k-1}$, where d is in diopters when y_{k-1} is given in centimeters and α is expressed in radians.

11.1.3.6 As a numerical example consider Table 11.1 which shows the values of α in minutes of arc and in diopters for telescopes of three different magnifications. To make these calculations the following assumptions were made:

$$y_{k-1} = -0.35 \text{ cm}$$

$$f_e = 2.00 \text{ cm}$$

$$(\overline{Y}_o / t_o)\,\text{MP} = -0.6$$

$$Y_1 / y_1 = 1.0$$

$$\gamma_o = \gamma_e = 1.5$$

The angular aberrations in Table 11.1 are all positive which means that the oblique bundles are focused behind the observer's eye making it impossible for him to focus on the image. It also means that the eyepiece must be focused towards the objective in order to remove the angular aberration for the off-axis object point. If it is moved towards the objective then the telescope will have angular aberration of the opposite sign for the central image. This indicates that the observer can accommodate and completely focus-out the angular error. The large angular aberration due to field curvature in telescopes is therefore not as serious as it may appear for it is possible for the eye to change focus as the observer views different parts of the field.

$\frac{\overline{Y}_o}{t_o}$. MP \ MP	-2	-5	-10	
-0.6	-9.0	-7.2	-6.6	Diopters
-0.6	107'	87'	79'	Minutes

Table 11.1- Angular aberration for telescopes of three magnifying powers.

11.1.3.7 In military telescopes it is often necessary to insert a reticle in the focal plane of the objective. Since a reticle is used to measure distances in the object space, it is important to design the objective with a flat field on the reticle, which usually means that the lens has to be more complex than the usual telescope doublet objective. Because the Petzval curvature of the eyepiece cannot be made zero, the eyepiece cannot focus the entire reticle with a single setting. Hence the reticle may not appear perfectly sharp, but if the objective is well corrected there is no parallax between the object and the reticle.

11.1.3.8 If a reticle is not needed in the design there is usually very little need to attempt to reduce the Petzval curvature of the objective by using a compound photographic lens type of objective. Equation (3) shows that the objective contribution, γ_o , is multiplied by the magnifying power of the telescope. For high power telescopes therefore, the objective adds a negligible amount of field curvature. This is why the majority of telescopes use simple doublets for the objective. If the power of the telescope is low then one must consider using some type of lens other than a doublet objective.

11.2 DESIGN PROCEDURE FOR A THIN LENS TELESCOPE OBJECTIVE

11.2.1 First order, thin lens.

11.2.1.1 The doublet, of course, consists of two lenses, and one can immediately start to fill out a table as was done with the triplet objective in Section 10.2. This has been done in Table 11.2.

		Lens a		Lens b		Image Plane
$-\phi$		$-\phi_a$		$-\phi_b$		0
t			0		$\frac{1}{\phi_a + \phi_b}$	
y		1		1		0
u	0		$-\phi_a$		$-\phi_a - \phi_b$	
$\bar{y}$		0		0		$\frac{1}{\phi_a + \phi_b}$
$\bar{u}$	1		1		1	
ν		ν_a		ν_b		0
$-\phi y^2/\nu$		$-(\phi/\nu)_a$		$-(\phi/\nu)_b$		$\Sigma a = -\frac{\phi_a}{\nu_a} - \frac{\phi_b}{\nu_b}$
$\frac{-\phi y \bar{y}}{\nu}$		0		0		$\Sigma b = 0$
$-\phi/n$		$-\phi_a/n_a$		$-\phi_b/n_b$		$\Sigma P = -\frac{\phi_a}{n_a} - \frac{\phi_b}{n_b}$

Table 11.2 - Computing table for the thin lens telescope doublet.

11.2.1.2 In order to solve for ϕ and have the axial color zero, assuming the two elements are close together, the following two equations must be satisfied:

$$\phi_a + \phi_b = \phi \ , \tag{4}$$

and

$$\frac{\phi_a}{\nu_a} + \frac{\phi_b}{\nu_b} = 0 \ . \tag{5}$$

The solution of these equations is

$$\phi_a = \phi \frac{\nu_a}{\nu_a - \nu_b} \ , \tag{6}$$

and

$$\phi_b = \phi \frac{\nu_b}{\nu_b - \nu_a} \ . \tag{7}$$

By using these equations, the value of ΣP from Equation 8-(28) is

$$\Sigma P = -\phi \left(\frac{\nu_a / n_a - \nu_b / n_b}{\nu_a - \nu_b} \right) . \qquad (8)$$

11.2.1.3 These equations show that any two glasses with a difference in ν may be used to design a doublet. As will be seen later however, $\Delta \nu$ should be large in order to reduce the monochromatic aberrations. In principle the ΣP may be made equal to zero by the proper choice of glass. Actually, the ratio

$$\frac{\nu_a / n_a - \nu_b / n_b}{\nu_a - \nu_b}$$

is nearly constant for any glasses chosen with a reasonable value of $(\nu_a - \nu_b)$. It is therefore not practical to attempt to reduce P in a doublet by choosing the proper glasses. Once the two glasses for the doublet are chosen the following is known about the lens:

(1) The focal lengths of the (a) and (b) lenses, using Equations (6) and (7).

(2) The axial color, which was set equal to zero using Equation 6-(42).

(3) The transverse color, which is zero, using Equation 6-(41), because the objective is the entrance pupil.

(4) The Petzval curvature, using Equation (8).

(5) The third order astigmatism, using Equation 8-(26).

(6) The third order distortion, using 8-(27).

Only two aberrations of the third order, B and F, remain uncorrected.

11.2.2 Third order, thin lens.

11.2.2.1 As explained in Section 8.10.1, it is possible to compute the coefficients (α and β) for the following thin lens equations:

$$B_a = \alpha_{1a} + \alpha_{2a} c_1 + \alpha_{3a} c_1^2 \qquad (9)$$

$$B_b = \alpha_{1b} + \alpha_{2b} c_3 + \alpha_{3b} c_3^2 \qquad (10)$$

$$F_a = \beta_{1a} + \beta_{2a} c_1 \qquad (11)$$

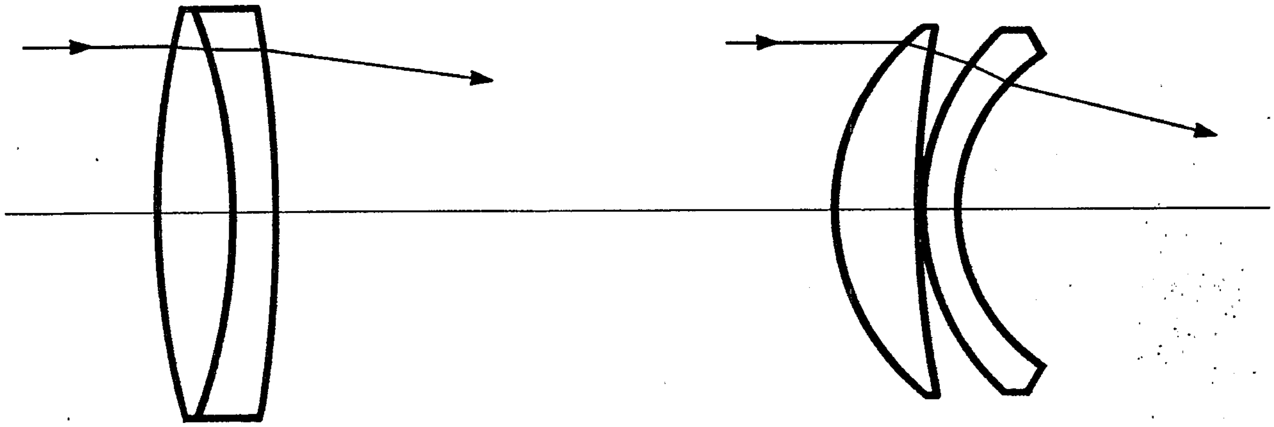
$$F_b = \beta_{1b} + \beta_{2b} c_3 \qquad (12)$$

11.2.2.2 By setting $B_a + B_b = 0$ and $F_a + F_b = 0$, the above equations may be reduced to a second degree equation in c_1. There are then two real solutions called the left and right hand solutions. Examples of the two solutions are shown in Figure 11.1. The doublet used in the example was computed for the following glasses:

Lens (a) $n_D = 1.511$ $\nu = 63.5$

(b) $n_D = 1.649$ $\nu = 30.6$

11.2.2.3 The doublets shown in Figure 11.1 have the low dispersion glass in the front element facing the infinite conjugate side of the lens. Doublet solutions can equally well be found with the high dispersion glass in the front. Figure 11.2 shows the left hand solution for the same glasses with the positions reversed. The left hand solutions with the positive lens in front have the most favorable shape for the passage of the axial rays. Therefore most telescope objectives are solutions of this type, and they are referred to as Fraunhofer objectives.



Left hand solution

Right hand solution

Figure 11.1 - Two types of doublets. For both types, the positive lens is in front and is of low dispersion glass.

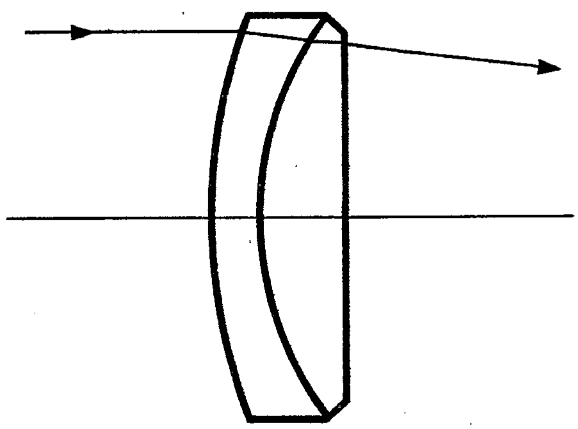


Figure 11.2 - Doublet with negative lens in front. The glasses are the same as the left hand solution in Figure 11.1 with the positions reversed.

11.2.3 Thin lens solutions.

11.2.3.1 Table 11.3 contains a selection of left hand solutions for 38 thin lens systems of the Fraunhofer type. Table 11.4 includes a list of the left hand solution for lenses with the negative lens in front. Systems with the negative lens in front appear to offer no advantages over the Fraunhofer type and so will not be discussed further.

11.2.3.2 There are of course numerous other combinations of glass one can pick, but those shown in Table 11.3 provide a sufficient variety of choices to build up an understanding of the solutions. The following points should be noted about the solutions:

(1) The series of solutions 1 to 8 have the same glass for the positive lens. The negative lenses however are all selected from the ordinary glass line shown in Figure 6.18. Notice that the first curvature for each solution is approximately 0.165 and the last curvature is about -0.08. The outside appearance of these lenses is therefore very similar. The difference between the lenses is in the curvatures of the internal surfaces.

(2) When combinations of glasses with small ν differences are selected, the curvature of the second surface is stronger than that of the third surface. This means that the surfaces do not edge contact and a spacer is needed. As the ν difference is increased, the difference in curvature of these two surfaces decreases, and at large ν differences the third surface becomes stronger than the second. The lenses then contact at the edge.

11.3 DESIGN PROCEDURE FOR A THICK LENS TELESCOPE OBJECTIVE

11.3.1 The thick lens doublet.

11.3.1.1 Very little choice can be made, from the thin lens data alone, between the numerous telescope objectives. In order to make a selection from the possible glass choices, it is necessary to trace rays through the designs.

11.3.1.2 As was pointed out in Section 10.4.1.7, when a lens is studied by ray tracing it is necessary to decide on a definite f - number. Changing the f - number requirement changes completely the conclusions one draws from the ray tracing. In order to illustrate the design procedure, a study will be made for a particular lens of f - number 3.57 with a focal length of 10.

11.3.2 Automatic correction of the third order aberrations.

11.3.2.1 After thicknesses are added to the thin lens solutions, it is necessary to readjust the curvatures to correct the spherical aberration, coma, and axial color to the required residuals. One cannot set the third order aberrations to zero if the total aberrations are to be zero, for the actual ray tracing will reveal the presence of higher order aberration.

11.3.2.2 In order to compare various glass choices in doublets it is necessary to hold constant many variables. In the study to be described the following parameters were held constant.

(a) The focal length of each lens was 10.0.

(b) The marginal ray entered the lens at $Y_1 = 1.4$.

(c) The third order spherical aberration was adjusted until the marginal ray at $Y_1 = 1.4$ in D light focused at the paraxial D focus.

(d) The axial first order color was adjusted until the rays traced at a value of $Y_1 = 1.0$ for F and C light united in the paraxial focal plane.

(e) The thickness of the positive and negative element was made 0.5 and 0.3, respectively

(f) The space between the lenses was maintained automatically. There were two alternatives. Where $r_2 > r_3$, the elements were spaced by 0.01 at the aperture height of $y \doteq 1.4$; where $r_2 < r_3$, a space of 0.01 was set at the vertex.

(g) The third order coma was corrected to zero in all cases.

Glass values, Element a: n_{Da} / ν_a	Glass values, Element b: n_{Db} / ν_b	Total curvatures c_a	Total curvatures c_b	Individual surface curvatures c_1	c_2	c_3	c_4	Case No.
1.511 / 63.5	1.5795 / 41.0	0.5523	−0.3145	0.1652	−0.3871	−0.3802	−0.0658	1
1.511 / 63.5	1.621 / 36.2	0.4552	−0.2135	0.1658	−0.2893	−0.2848	−0.0713	2
1.511 / 63.5	1.649 / 33.8	0.4184	−0.1754	0.1660	−0.2525	−0.2499	−0.0746	3
1.511 / 63.5	1.689 / 30.9	0.3812	−0.1376	0.1659	−0.2152	−0.2163	−0.0787	4
1.511 / 63.5	1.720 / 29.3	0.3634	−0.1190	0.1659	−0.1975	−0.2009	−0.0819	5
1.511 / 63.5	1.8052 / 25.5	0.3270	−0.0833	0.1656	−0.1614	−0.1730	−0.0896	6
1.511 / 63.5	1.5704 / 48.1	0.8070	−0.5476	0.1563	−0.6507	−0.6351	−0.0875	7
1.511 / 63.5	1.5838 / 46.0	0.7101	−0.4503	0.1595	−0.5506	−0.5354	−0.0851	8
1.511 / 63.5	1.605 / 43.6	0.6245	−0.3622	0.1620	−0.4625	−0.4476	−0.0854	9
1.511 / 63.5	1.617 / 36.6	0.4798	−0.2353	0.1657	−0.3141	−0.3077	−0.0723	10
1.511 / 63.5	1.717 / 48.2	0.8123	−0.4394	0.1411	−0.6711	−0.6259	−0.1865	11
1.511 / 63.5	1.720 / 50.3	0.9415	−0.5293	0.1228	−0.8186	−0.7630	−0.2337	12
1.511 / 63.5	1.8037 / 41.8	0.5727	−0.2397	0.1631	−0.4095	−0.3779	−0.1382	13
1.517 / 64.5	1.689 / 30.9	0.3713	−0.1335	0.1648	−0.2065	−0.2091	−0.0756	14
1.611 / 58.8	1.5795 / 41.0	0.5516	−0.4090	0.1594	−0.3922	−0.4009	0.0080	15
1.611 / 58.8	1.617 / 36.6	0.4397	−0.2734	0.1538	−0.2859	−0.2903	−0.0170	16
1.611 / 58.8	1.621 / 36.2	0.4318	−0.2638	0.1535	−0.2782	−0.2824	−0.0186	17
1.611 / 58.8	1.649 / 33.8	0.3895	−0.2126	0.1525	−0.2369	−0.2403	−0.0277	18
1.611 / 58.8	1.689 / 30.9	0.3482	−0.1637	0.1519	−0.1963	−0.2002	−0.0366	19
1.611 / 58.8	1.720 / 29.3	0.3290	−0.1403	0.1518	−0.1773	−0.1819	−0.0416	20
1.611 / 58.8	1.689 / 30.9	0.3448	−0.1607	0.1519	−0.1930	−0.1973	−0.0366	21
1.620 / 60.3	1.617 / 36.6	0.3910	−0.2308	0.1521	−0.2389	−0.2459	−0.0151	22
1.620 / 60.3	1.649 / 33.8	0.3526	−0.1827	0.1510	−0.2016	−0.2080	−0.0253	23
1.620 / 60.3	1.7506 / 27.8	0.2904	−0.1067	0.1500	−0.1404	−0.1503	−0.0437	24
1.620 / 60.3	1.8052 / 25.5	0.2731	−0.0861	0.1498	−0.1232	−0.1362	−0.0502	25
1.620 / 60.3	1.8037 / 41.8	0.4902	−0.2537	0.1490	−0.3412	−0.3253	−0.0716	26
1.638 / 55.5	1.617 / 36.6	0.4603	−0.3139	0.1527	−0.3075	−0.3144	−0.0005	27
1.638 / 55.5	1.649 / 33.8	0.4009	−0.2400	0.1503	−0.2506	−0.2551	−0.0151	28
1.638 / 55.5	1.689 / 30.9	0.3536	−0.1823	0.1493	−0.2044	−0.2083	−0.0260	29
1.638 / 55.5	1.720 / 29.3	0.3308	−0.1542	0.1490	−0.1818	−0.1861	−0.0319	30
1.5286 / 51.6	1.5497 / 45.8	1.6831	−1.437	0.1206	−1.563	−1.548	−0.1113	31
1.5286 / 51.6	1.5795 / 41.0	0.9209	−0.6675	0.1481	−0.7729	−0.7568	−0.0893	32
1.5286 / 51.6	1.621 / 36.2	0.6339	−0.3785	0.1584	−0.4755	−0.4604	−0.0819	33
1.5286 / 51.6	1.649 / 33.8	0.5484	−0.2926	0.1610	−0.3874	−0.3736	−0.0810	34
1.5286 / 51.6	1.689 / 30.9	0.4716	−0.2167	0.1629	−0.3087	−0.2975	−0.0808	35
1.5286 / 51.6	1.72 / 29.3	0.4378	−0.1825	0.1635	−0.2742	−0.2646	−0.0822	36
1.5286 / 51.6	1.72 / 42.0	0.4981	−0.2268	0.1623	−0.3357	−0.3193	−0.0926	37
1.5286 / 51.6	1.76 / 29.3	0.4378	−0.1729	0.1639	−0.2738	−0.2620	−0.08913	38

Table 11.3 – Thin lens aplanatic doublets of focal length 10.

Glass values Element a n_{Da} ν_a	b n_{Db} ν_b	Total curvatures c_a	c_b	Individual surfaces curvatures c_1	c_2	c_3	c_4	Case No.
1.5795 41.	1.511 63.5	-0.3145	0.5523	0.2270	0.5415	0.5473	-0.0050	1
1.621 36.2	1.511 63.5	-0.2135	0.4552	0.2332	0.4467	0.4496	-0.0056	2
1.649 33.8	1.511 63.5	-0.1754	0.4184	0.2369	0.4123	0.4127	-0.0057	3
1.689 30.9	1.511 63.5	-0.1376	0.3812	0.2417	0.3793	0.3755	-0.0057	4
1.720 29.3	1.511 63.5	-0.1190	0.3634	0.2453	0.3643	0.3577	-0.0057	5
1.8052 25.5	1.511 63.5	-0.0833	0.3270	0.2543	0.3376	0.3217	-0.0053	6
1.5704 48.1	1.511 63.5	-0.5476	0.8070	0.2487	0.7962	0.8109	0.0039	7
1.5838 46.0	1.511 63.5	-0.4503	0.7101	0.2465	0.6968	0.7108	0.0007	8
1.605 43.6	1.511 63.5	-0.3622	0.6245	0.2471	0.6092	0.6227	-0.0018	9
1.617 36.6	1.511 63.5	-0.2353	0.4798	0.2342	0.4695	0.4744	-0.0055	10
1.717 48.2	1.511 63.5	-0.4394	0.8123	0.3500	0.7895	0.8315	0.0193	11
1.720 50.3	1.511 63.5	-0.5293	0.9415	0.3974	0.9267	0.9791	0.0376	12
1.8037 41.8	1.511 63.5	-0.2397	0.5727	0.3029	0.5426	0.5699	-0.0027	13
1.689 30.9	1.517 64.5	-0.1335	0.3713	0.2385	0.3718	0.3666	-0.0045	14
1.5795 41.0	1.611 58.8	-0.4090	0.5516	0.1532	0.5622	0.5540	0.0023	15
1.617 36.6	1.611 58.8	-0.2734	0.4397	0.1788	0.4521	0.4476	0.0079	16
1.621 36.2	1.611 58.8	-0.2638	0.4318	0.1805	0.4443	0.4399	0.0082	17
1.649 33.8	1.611 58.8	-0.2126	0.3895	0.1900	0.4026	0.3986	0.0092	18
1.689 30.9	1.611 58.8	-0.1637	0.3482	0.1994	0.3631	0.3580	0.0098	19
1.720 29.3	1.611 58.8	-0.1403	0.3290	0.2049	0.3452	0.3390	0.0100	20
1.689 30.9	1.611 58.8	-0.1607	0.3448	0.1995	0.3602	0.3548	0.0098	21
1.617 36.6	1.620 60.3	-0.2308	0.3910	0.1769	0.4077	0.4007	0.0097	22
1.649 33.8	1.620 60.3	-0.1827	0.3526	0.1876	0.3703	0.3634	0.0108	23
1.7506 27.8	1.620 60.3	-0.1067	0.2904	0.2074	0.3141	0.3023	0.0118	24
1.8502 25.5	1.620 60.3	-0.0861	0.2731	0.2147	0.3007	0.2851	0.0121	25
1.8037 41.8	1.620 60.3	-0.2537	0.4902	0.2361	0.4898	0.5031	0.0130	26
1.617 36.6	1.638 55.5	-0.3139	0.4603	0.1623	0.4762	0.4696	0.0094	27
1.649 33.8	1.638 55.5	-0.2400	0.4009	0.1774	0.4174	0.4127	0.0118	28
1.689 30.9	1.638 55.5	-0.1823	0.3536	0.1889	0.3712	0.3665	0.0128	29
1.720 29.3	1.638 55.5	-0.1542	0.3308	0.1951	0.3494	0.3439	0.0131	30
1.5497 45.8	1.5286 51.6	-1.437	1.6831	0.2721	1.7087	1.7231	0.0399	31
1.5795 41.0	1.5286 51.6	-0.6675	0.9209	0.2505	0.9180	0.9333	0.0124	32
1.621 36.2	1.5286 51.6	-0.3785	0.6339	0.2438	0.6223	0.6360	0.0021	33
1.649 33.8	1.5286 51.6	-0.2926	0.5484	0.2433	0.5359	0.54795	-0.00047	34
1.689 30.9	1.5286 51.6	-0.2167	0.4716	0.2438	0.4604	0.4692	-0.0024	35
1.72 29.3	1.5286 51.6	-0.1825	0.4378	0.2456	0.4281	0.4347	-0.0030	36
1.72 42.0	1.5286 51.6	-0.2268	0.4981	0.2560	0.4827	0.4962	-0.0018	37
1.76 29.3	1.5286 51.6	-0.1729	0.4378	0.2531	0.4260	0.4344	-0.0034	38

Table 11.4 - Telescope objectives with flint in front.

11.3.2.3 In designing a doublet of a particular glass choice, it is necessary to estimate the values at which to set the third order spherical aberration, B, and axial color, a. Then surfaces 1, 2, and 3 are varied to provide derivatives for B, F, and a. The following three equations are then solved for the required values of B, F, and a.

$$\Delta \Sigma B = \frac{\partial \Sigma B}{\partial c_1} \Delta c_1 + \frac{\partial \Sigma B}{c_2} \Delta c_2 + \frac{\partial \Sigma B}{\partial c_3} \Delta c_3 ,$$

$$\Delta \Sigma F = \frac{\partial \Sigma F}{\partial c_1} \Delta c_1 + \frac{\partial \Sigma F}{\partial c_2} \Delta c_2 + \frac{\partial \Sigma B}{\partial c_3} \Delta c_3 ,$$

$$\Delta \Sigma a = \frac{\partial \Sigma a}{\partial c_1} \Delta c_1 + \frac{\partial \Sigma a}{\partial c_2} \Delta c_2 + \frac{\partial \Sigma a}{\partial c_3} \Delta c_3 .$$

This is the method described in Section 9.2.4.12. Since the changes are not linear, it is necessary to repeat the procedure for several iterations. When the desired third order values are found, the solution is then ray traced in D, F, and C light at 0° with values of Y_1 = 1.4, 1.2, 1.0, and 0.8. From this ray tracing data it is possible to determine if condition c and d are fulfilled. If they are not it is necessary to assign a new value to B and a and repeat the process.

11.3.2.4 Tables 11.5, 11.6, and 11.7 show the data for three doublets designed in this manner. The design shown in Table 11.7 illustrates a careful balance of high order aberrations for D light. In order to arrive at this design it was necessary to choose just the right ν number for the negative lens. If a flint with a larger ν number had been chosen, the rays at an aperture of Y_1 = 1.4 would have crossed the paraxial focal plane at a larger negative value and this would have been impossible to correct without introducing a large positive zonal aberration. Notice how the F light starts out to be under-corrected (negative Y_k), but as Y_1 is increased it becomes over-corrected (positive Y_k). The C light starts out positive and then turns toward the negative side. This is evidence of chromatic variation of spherical aberration. Note also how the aberration for the lenses in Tables 11.5 and 11.6 are larger than in Table 11.7 even though all the curvatures are smaller. If a different f - number is needed one would choose a different flint element for optimum correction.

11.3.2.5 The aberration curves in Figure 11.3 are for the ray data given in Table 11.6. The D light curve is typical for a telescope objective. The third order spherical aberration is undercorrected. The curve, for small values of Y_1, starts out below the reference axis following the third order aberration, but it then starts to depart and swings towards the positive side. This is due to higher order aberrations which, in this case, are positive. By evaluating the constants in Equation 8-(1) for this aberration curve we find that

$$b_o = 0, \quad b_3 = -0.0014, \text{ and } b_5 = 0.000619$$

Lens specifications				
c	t	n	ν	
0.1672				
	0.5	1.511	63.5	f'=9.995478
-0.1594				
	0.0218	1.0		l'=9.624807
-0.1709				
	0.3	1.80489	25.4	
-0.0886				
Ray-trace data				
Y_1	Y_D	Y_F	Y_C	
1.4	0.000279	0.002047	0.000618	Y is the height of the ray in the D light paraxial focal plane.
1.2	-0.000689	0.000362	-0.000233	
1.0	-0.000777	-0.000202	-0.000291	
0.8	-0.000542	-0.000267	-0.000089	

Table 11.5 - Lens specification and ray trace data for an achromatic doublet with large ν difference.

Lens specifications				
c	t	n	ν	
		1		
0.167876				
	0.5	1.511	63.5	f'= 10.000000
-0.244936				
	0.01	1.0		l'= 9.597161
-0.243789				
	0.3	1.649	33.8	
-0.073862				
Ray-trace data				
Y_1	Y_D	Y_F	Y_C	
1.4	0.000201	0.002359	0.000334	
1.2	-0.000423	0.000786	-0.000079	
1.0	-0.000526	0.000060	-0.000089	
0.8	-0.000387	-0.000176	0.000055	

Table 11.6 - Lens specification and ray trace data for an achromatic doublet with moderate ν difference.

Lens specifications				
c	t	n	ν	
0.168413				
	0.5	1.511	63.5	f'= 10.0000
-0.290972				
	0.01	1.0		l'= 9.578138
-0.289068				
	0.3	1.605	38.0	
-0.067287				
Ray-trace data				
Y_1	Y_D	Y_F	Y_C	
1.4	-0.000366	0.001065	0.000038	
1.2	+0.000083	0.000618	0.000673	
1.0	-0.000034	-0.000064	0.000628	
0.8	-0.000092	-0.000411	0.000542	

Table 11.7 - Lens specification and ray trace data for an achromatic doublet with small ν difference.

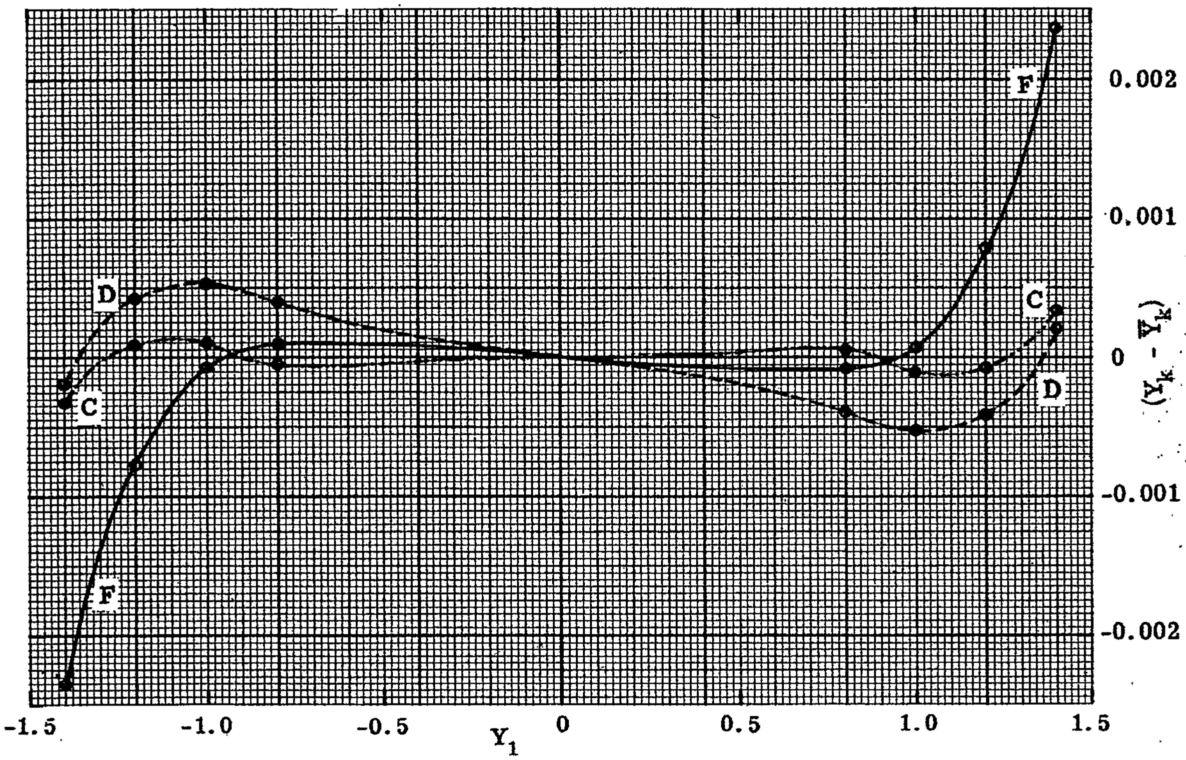


Figure 11.3 - Meridional ray plot at 0° for a doublet lens.

11.3.2.6 The third order calculations for this lens are included in Table 11.8. Inspection of these data gives a clue to why b_5 is positive. Surfaces 1, 2 and 4 have negative values of B. The only source of positive B is the 3rd surface. A surface always adds higher order aberration of the same sign as the third order. Since the 3rd surface introduces such a large positive third order contribution it over-balances the negative higher order contributions from the other surfaces. The result is a positive fifth order term. In designing the doublet it is considered advisable to adjust the third order aberration so that the higher order correction brings the curve back to $Y_k = 0$ for the rays at $Y_1 = Y_{1\,max}$. This leaves a residual aberration called zonal aberration. In figure 11.3 the zonal aberration for D light amounts to -0.000526.

c	0.16788	-0.24494	-0.24379	-0.07386	
t	0.50	0.01	0.30		
n	1.511	1.0	1.649		
y	1	0.97161	0.96954	0.95972	
u	-0.05677	-0.20739	-0.03274	-0.1	
B	-0.00106	-0.03273	0.03580	-0.00225	$\Sigma B = -0.00023$ $_3Y_k = -0.00115$

Table 11.8 - Third order spherical contribution on each surface of the doublet shown in Table 11.6

11.3.3 Tolerance on zonal aberration.

11.3.3.1 The question of the tolerance on the zonal aberration cannot be explained fully here, but a few guide lines can be given.

(1) If the axial image must be corrected to be physically perfect it is necessary to reduce the zonal aberration to the following tolerance.

$$(Y_k)_{zone} = \frac{4.6\lambda}{L_{k-1}}$$

L_{k-1} = optical direction cosine of the emerging ray.

This tolerance assumes that the ray traced at a height of $Y_{1\,max.}$ is adjusted so that $Y_k = 0$. Tolerance is calculated as a positive number.

(2) If the objective is used in a telescope, one can compute the angular aberration presented to the eye, and set the tolerance by using the principle that the angular resolution of the eye is limited to one part in 3000. However, there is no need to attempt to reduce the zonal aberration below the value given above for the diffraction image case (1). In this region the size of the image is actually determined by the physical nature of the light and not by the geometrical aberrations.

11.3.3.2 The $Y_{1\,max.}$ for the lens shown in Table 11.6 is 1.4. The focal length is 10. Therefore, L_{k-1} is approximately -0.14 The zonal tolerance $(Y_k)_{zone}$ is then calculated as follows.

$$(Y_k)_{zone} = \frac{4.6 \times 0.5893 \text{ microns}}{0.14}$$

$$= 19.4 \text{ microns} = 0.0194 \text{ mm.}$$

The zonal aberration for the lens in Table 11.6 is -0.000526. This means that the lens could have a focal length 10 x 0.0194/0.000526 = 369mm and remain corrected within the tolerance. This assumes, of course, that the light is monochromatic. One can see that F and C light are not corrected as well as this. More will be said about this in a later section (Section 11.4) on secondary spectrum.

11.3.4 Methods for reducing the zonal aberration.

11.3.4.1 If the zonal aberration is too large in a lens it may be reduced by four methods. These methods will now be described for they illustrate a powerful technique of design. The methods are:

(1) Choosing the proper glasses.

(2) Using an air space.

(3) Introducing an aspheric surface.

(4) Adding an extra positive lens.

11.3.4.2 Tables 11.5, 11.6, and 11.7 illustrate the influence of glass choice.

11.3.4.3 If the air space is made larger the marginal rays have a chance to drop more before they strike the negative lens. The higher order negative aberration on the positive lens then causes the rays to actually strike the over-correcting surface at a lower aperture than predicted from first and third order theory. This cuts down on the higher order over-correcting tendency of this surface. Therefore as the space is increased the positive fifth order term is reduced. The third order value can then be made less negative, resulting in a reduced zone. Figures 11.4, 11.5, and 11.6 show some of the aberration curves for doublets where the air space has been adjusted to minimize the spherical aberration in D light. These lenses were also corrected so that the $Y_{1\,max.}$ was 1.4. The zonal aberration has been reduced to a remarkable degree. Table 11.9 contains the curvatures and thicknesses for many optimum solutions of this type. The last two columns are headed OSC', which stands for offense against the sine condition. This quantity, OSC', is proportional to coma, for a given image height, Y_k. These last two columns, therefore, are a measure of third order coma, and total coma for the marginal ray, respectively.

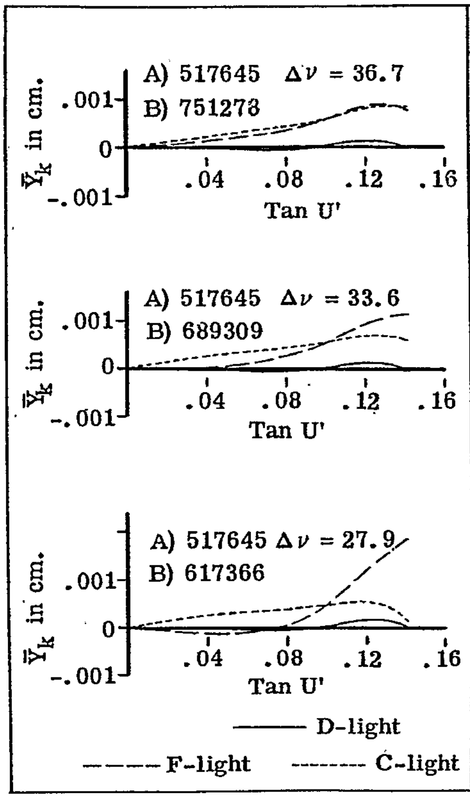


Figure 11.4- Effects of ν difference on spherochromatism (glass A used in positive lens, glass B used in negative lens).

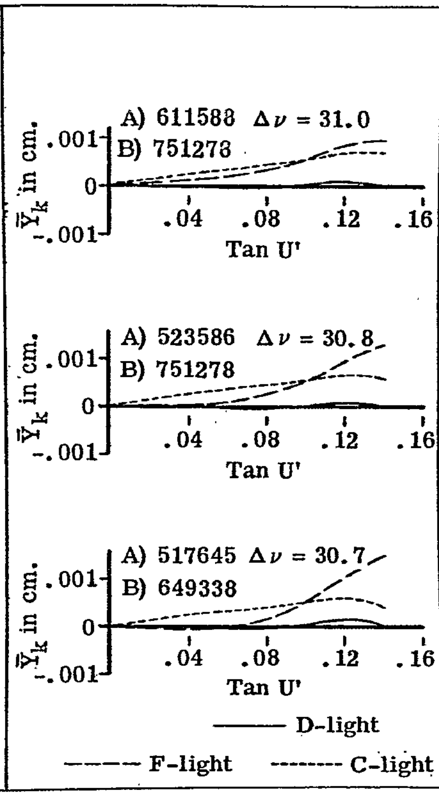


Figure 11.5- Effects of index of positive (A) lens on spherochromatism.

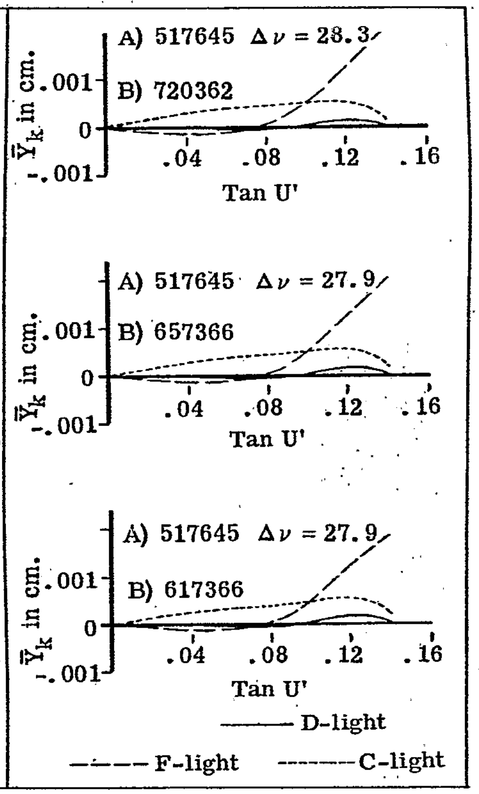


Figure 11.6- Effects of index of negative lens (B) on spherochromatism.

Glasses								OSC'	
+ lens	- lens	Δν	c_1	c_2	c_3	c_4	t_2	Third order	Marginal
517 645	751 278	36.7	+0.2582	−0.1450	−0.2222	−0.0489	+0.7364	−0.0030	−0.0017
517 645	720 293	35.2	+0.2379	−0.1630	−0.2159	−0.0470	+0.5353	−0.0026	−0.0014
517 645	689 309	33.6	+0.2205	−0.1825	−0.2173	−0.0468	+0.3596	−0.0022	−0.0012
517 645	649 338	30.7	+0.1955	−0.2215	−0.2348	−0.0516	+0.1412	−0.0015	−0.0009
517 645	617 366	27.9	+0.1780	−0.2638	−0.2663	−0.0575	+0.0400	−0.0007	−0.0006
517 645	657 366	27.9	+0.1634	−0.2716	−0.2671	−0.0783	+0.0022	+0.0003	−0.0000
517 645	720 362	28.3	+0.1526	−0.2733	−0.2632	−0.0986	−0.0261	+0.0010	+0.0004
523 586	751 278	30.8	+0.2091	−0.1801	−0.2067	−0.0593	+0.3141	−0.0016	−0.0009
523 586	720 293	29.3	+0.1955	−0.2016	−0.2161	−0.0612	+0.1826	−0.0013	−0.0008
523 586	689 309	27.7	+0.1829	−0.2260	−0.2317	−0.0641	+0.0883	−0.0008	−0.0006
523 586	649 338	24.8	+0.1611	−0.2776	−0.2730	−0.0751	−0.0014	+0.0004	+0.0000
523 586	617 366	22.0	+0.1373	−0.3417	−0.3334	−0.0926	−0.0221	+0.0028	+0.0015
523 586	657 366	22.0	+0.1149	−0.3787	−0.3653	−0.1282	−0.0311	+0.0048	+0.0026
523 586	720 362	22.4	+0.1009	−0.3636	−0.3478	−0.1523	−0.0444	+0.0064	+0.0039
611 588	751 278	31.0	+0.2149	−0.1381	−0.1858	−0.0111	+0.4904	−0.0020	−0.0011
611 588	720 293	29.5	+0.2052	−0.1551	−0.1913	−0.0086	+0.3540	−0.0019	−0.0011
611 588	689 309	27.9	+0.2006	−0.1731	−0.2027	−0.0030	+0.2606	−0.0020	−0.0011
611 588	649 338	25.0	+0.1971	−0.2091	−0.2308	+0.0079	+0.1481	−0.0025	−0.0015
611 588	617 366	22.2	+0.2161	−0.2426	−0.2662	+0.0389	+0.1128	−0.0050	−0.0032
611 588	657 366	22.2	+0.1713	−0.2598	−0.2652	−0.0142	+0.0347	−0.0013	−0.0009
611 588	720 362	22.6	+0.1439	−0.2676	−0.2623	−0.0538	−0.0128	+0.0008	+0.0003

Table 11.9- Final solution lens data resulting from least-squares correction program. For all lenses, $f' = 10.0$ cm, $t_1 = 0.5$ cm, $t_3 = 0.3$ cm.

11.3.4.4 Probably the simplest (from a theoretical point of view) method to reduce the zonal aberration is to introduce an aspheric surface on the last surface. Thus one can introduce high order terms of deformation and geometrically correct any amount of zonal aberration in the lens. The method used to compute the necessary coefficients is usually quite straight forward; briefly, it is as follows:

(1) Add a fourth order deformation term to reduce the third order aberration to zero. See Equation 8-(4a).

(2) Make an arbitrary guess at a sixth order coefficient (f). See Section 5.5.2.

(3) Trace through a ray at a finite aperture and determine how much of a deflection ΔY_k this aspheric term causes.

(4) It may then be assumed that this deflection

$$\Delta Y_k = \lambda \left[6fS^5 + 8gS^7 + 10hS^9 + \ldots\ldots \right]$$

(5) It is then possible to write a set of equations to bring as many rays to the axis as there are aspheric constants. It is possible to write as many equations as rays traced through the system, but if there are more equations than terms in the aspheric, one has to resort to a method of least squares rather than expect an exact solution.

(6) Since the equation in step 4 is not exact, it may be necessary to repeat the process a few times.

This method is usually satisfactory, but if either the aperture of the lens or the zonal aberration is large, it may not be possible to fit a power series deformation which will reduce the aberration for all rays. This is because not enough terms are used in the expansion. In practice if one cannot fit a curve with a 10th degree polynomial then it helps very little to add a few more terms in the series; it takes a large number of terms to reduce the aberration for many rays. Sometimes it becomes necessary to abandon the use of the polynomial expression. The aspheric must then be expressed as a series of Y and Z coordinates. This is computed by actually calculating the optical path along the ray, and adding glass thickness to produce a spherical wavefront. This procedure is almost never necessary, but if it is, then one seriously questions whether it would be possible to make an aspheric of this type.

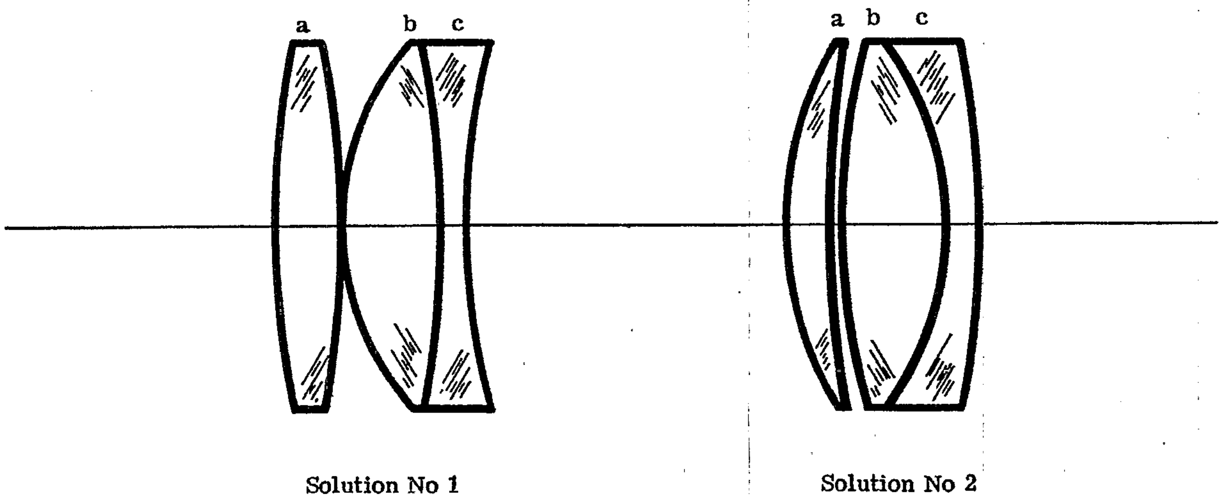


Figure 11.7 - Two types of triplet solution.

11.3.4.5 The zonal spherical aberration can be reduced by splitting off some of the power in the positive lens in the doublet. This would make a triplet similar to the types illustrated in Figure 11.7. By so doing, two extra degrees of freedom are created, namely the power ϕ of the extra positive lens, and its first curvature, c_1. One degree of freedom will be eliminated by cementing the last two elements. The problem then becomes, what is ϕ_a? In other words, how should ϕ_a and ϕ_b be distributed? The manner one uses to find an answer to this problem is typical of one of the techniques used by a lens designer. The reasoning goes as follows:

(1) If the lens is assumed to be thin, the power $\phi_a + \phi_b$ will equal the same power as for the positive lens in a doublet. Therefore one may start by using the powers for the lenses as given in Table 11.3.

(2) By cementing the b and c lenses, there are only two degrees of freedom left. The first curvatures, c_1 and c_3, are the variables. If ϕ_a is decided upon, then the thin lens formulae, described in Section 8.9, enable one to compute the coefficients of the equations

$$B_a = \alpha_{1a} + \alpha_{2a} c_1 + \alpha_{3a} c_1^2 ,$$

$$B_{b+c} = \alpha_{1(b+c)} + \alpha_{2(b+c)} c_3 + \alpha_{3(b+c)} c_3^2 ,$$

$$F_a = \beta_{1a} + \beta_{2a} c_1 ,$$

$$F_b = \beta_{1b} + \beta_{2b} c_3 .$$

In exactly the same way as for the doublet, two types of solutions may be found. They are illustrated in Figure 11.7.

(3) Next plot B_c for these two solutions on a plot as shown in Figure 11.8. B_c is the total spherical aberration due to the negative lens. By finding the two solutions for several values of ϕ_a it is possible to plot the curves shown in Figure 11.8. These curves show that if $\phi_a = 0$ we then have a simple doublet and the two solutions require different amounts of positive spherical aberration. For the doublet, of course, the (b) and (c) lenses must be considered to be separated. As more and more power is put into lens (a), less and less positive B is required of lens (c). Now we know that the higher order aberrations will be minimized when the lens is corrected with B_c having a minimum positive value because the higher order spherical aberration has the same sign as the third order. From this reasoning one would predict that the type 1 solution with a value of $\phi_a = 0.066$ would provide an optimum solution. Solution 1, shown in Figure 11.7, is a lens of this type. The type 2 lens was also chosen with a value of $\phi_a = 0.066$.* Table 11.10 shows the results of ray tracing these solutions after adjustments were made to the residual third order aberration so that the marginal ray comes to focus at $Y_k = 0$. Table 11.11 contains the data for these two solutions.

(4) It is interesting to see that the type 1 lens is remarkably well corrected. The zonal aberration is 10 times less than the type 2. The type 2 lens may be thought of as a derivative of a separated doublet of the left hand branch. With the choice of glass used, the doublet would be an air spaced lens. By cementing it, it would be under-corrected for spherical aberration. Now by splitting off a small part of the positive lens and by bending slightly the lens can be re-corrected for spherical aberration, thus leading to the lens type 2. The type 1 lens is actually a derivative from the right hand branch of the doublet. Note that the better solution comes from the poorer doublet type. This is mentioned because, in designing this type lens, if one started by trying to modify a left hand doublet lens he might easily converge on a type 2 solution and find no advantage in using the split positive lens. Notice that the zonal aberration in the type 2 lens is

* It is true that the type 2 solution would probably be better at $\phi_a = 0.082$ or at $\phi_a = 0.138$, but the value of $\phi_a = 0.066$ was selected to illustrate that for a given value of ϕ_a there are two solutions quite close together.

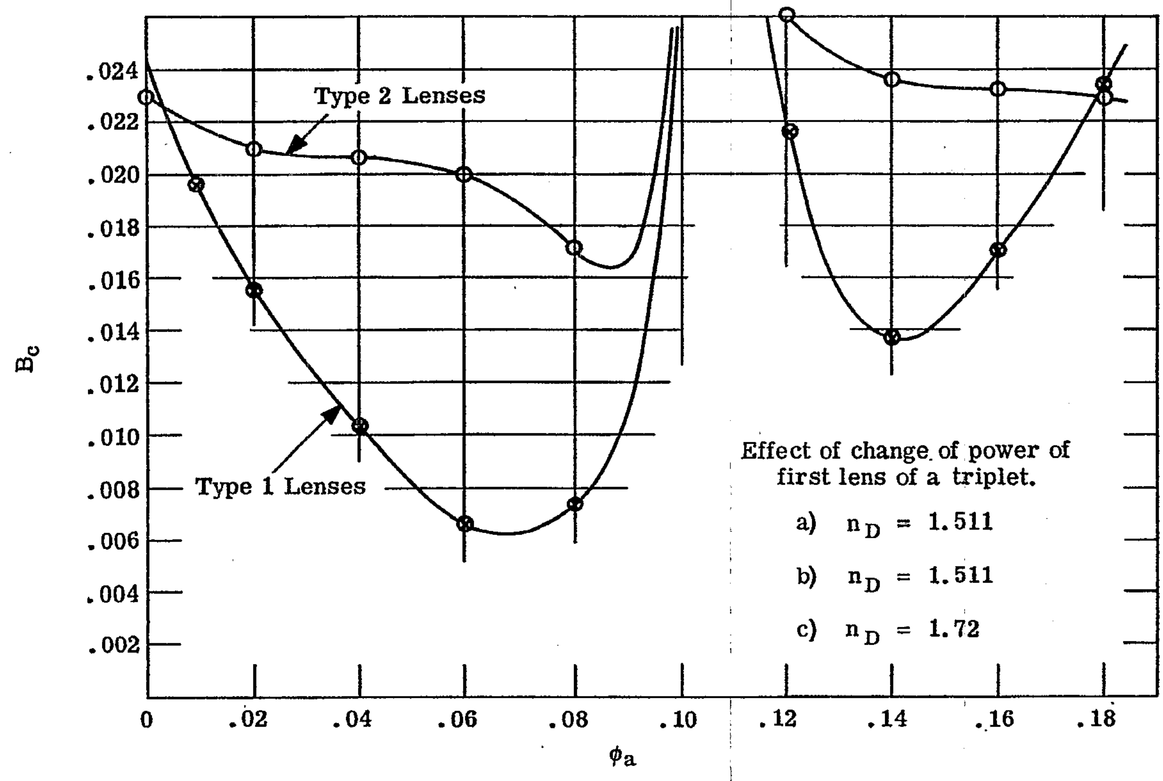


Figure 11.8 - A plot of B_c the total spherical aberration of the negative lens as a function of ϕ_a.

Y_1	TYPE 1 Y_k	TYPE 2 Y_k
1.4	0.000210	-0.000101
1.2	0.000070	-0.000706
1.0	0.000010	-0.000696
0.8	-0.000007	-0.0004681

Table 11.10 - Ray trace data in D light showing a comparison between type 1 and type 2 solutions in triplet telescope objectives

Type 1 c	t	n_D	ν	Type 2 c	t
0.07121				0.1641	
	0	1.511	63.5		0
-0.05525				0.0349	
	0				0
0.20132				0.0472	
	0	1.511	63.5		0
-0.03572				-0.1871	
	0	1.72	29.3		0
0.08337				-0.0680	
		f' = 10			

Table 11.11 - Lens data on type 1 and type 2 lenses

almost identical with the doublets shown in Table 11.6. If we had happened to choose a glass combination which would have resulted in a cemented doublet, then a type 2 solution would offer no advantage. One would have to go to type 1. The curves shown in Figure 11.8 change as the glass is varied. In Figure 11.9 the type 1 branch is shown for another pair of glasses. One can see that it is quite different, for most of the positive power should be placed in the (a) lens.

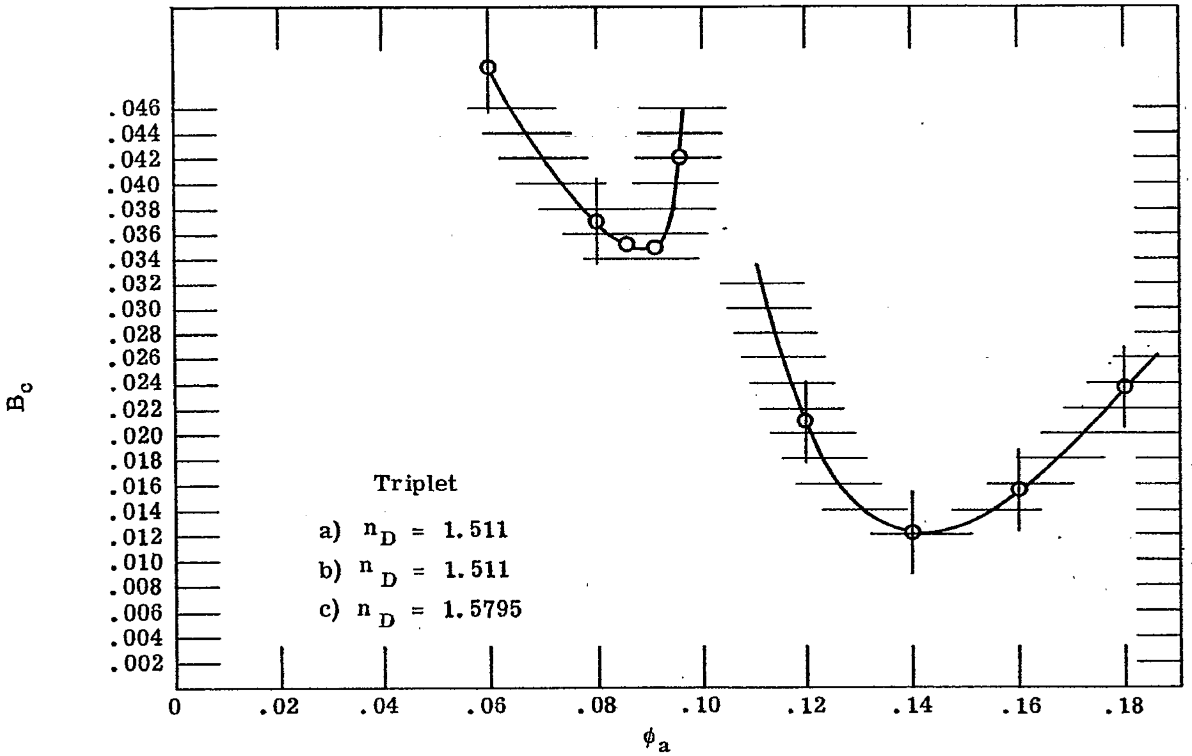


Figure 11.9 - A plot of B_c as a function of ϕ_a for a triplet.

11.3.5 Discussion of zonal correction methods.

11.3.5.1 This study illustrates that there are many possible solutions, and the designer must investigate them all in order to be sure he has exhausted the possibilities. This study also illustrates that there are regions where no solutions exist even though there appear to be a sufficient number of degrees of freedom.

11.3.5.2 If possible, a doublet design should be used. However if the zonal spherical aberration exceeds the tolerance, there are the four methods for reducing the zonal aberration described in the above section. Of these methods the fourth method of adding the extra element is strongly recommended, for this method does not involve the balance of large aberrations. The other three methods depend upon the balancing of large aberrations. The extra element method provides a solution with no large surface contributions. One will find in practice that it also will provide a lens much easier to manufacture, for it will be much less sensitive to decentering or spacing errors.

11.3.6 Coma correction. So far practically nothing has been said about coma correction. All the doublet solutions were corrected to have the third order coma exactly zero. This provides optimum correction for most of the designs. However, if the zonal spherical aberration is corrected by a large air space, one will encounter high order coma aberration, and it may be necessary to introduce residual coma in the third order. This however is very unsatisfactory, so everything possible should be done to find a solution with minimized high order coma. For this reason also, the split doublet lens offers a far better way to reduce the zonal aberrations than does the unsplit doublet, as the former has excellent coma correction.

11.4 SECONDARY SPECTRUM OF TELESCOPE OBJECTIVES

11.4.1 The difference in focus for F, C, and D light.

11.4.1.1 In the preceding paragraphs, it was shown (11.3.4.3) how the zonal spherical aberration could be reduced by a large factor. However, the aberration curves of Figures 11.4 through 11.6 clearly show that since the lens must be designed to image F and C light, as well as D light, the high degree of zonal correction in D light is of small practical significance. The F and C light focus does not coincide with the D focus. This defect in focus for F, C, and D light is a paraxial ray defect and was briefly discussed in Section 6.10.8, where the transverse aberration, $TAch_{F-D}$, was defined as the secondary spectrum.

11.4.1.2 It was also stated (6.10.8.4) that the three principal methods for reduction of this aberration are:

(1) Use special materials with equal partial dispersions.

(2) Use more than two types of glass.

(3) Use proper combination of lenses.

Paragraphs 11.4.2 and 11.4.3 will describe methods (1) and (2) respectively.

11.4.2 Reduction of secondary spectrum in a doublet. ($\tilde{P}_a - \tilde{P}_b = 0$ Method).

11.4.2.1 Equation 6-(49) tells us that when the partial dispersion ratios of both lenses of a doublet are equal for F and D light, then $\tilde{P}_a - \tilde{P}_b = 0$ and the F, C, and D light will unite in a common focus. Now, depending on the shapes of the dispersion curves of the glasses used in the doublet, there is still the question of where other wavelengths will focus.

11.4.2.2 Equation 6-(49) can also be used to calculate the $TAch_{\lambda-D}$ for any other wavelength. If the partial dispersion ratios for other wavelengths are not equal, i.e., $\tilde{P}_a - \tilde{P}_b \neq 0$, then there is still residual chromatic aberration. In choosing glass types, then, a designer must consider the following compromises:

(1) Should he settle for a small $(\nu_a - \nu_b)$ by setting $(\tilde{P}_a - \tilde{P}_b)$ exactly equal to zero, or should a larger $(\nu_a - \nu_b)$ be chosen and some secondary color be allowed? The decision, of course, will depend on the focal length and the numerical aperture required of the objective.

(2) Does the need for correction for a large range of wavelengths require that $(\tilde{P}_a - \tilde{P}_b)$ be set at a value other than zero?

11.4.2.3 It is clear that these considerations, combined with the task of correcting the spherical aberration, and the variation of spherical aberration with wavelength, pose a formidable array of problems.

11.4.2.4 The designer's difficulties are further increased by the need for extremely accurate measurements of the index of refraction. One can, by differentiating Equation 6-(49), determine that the following relation holds for an achromatic doublet.

$$dn_\lambda = \frac{d\left(TAch_{\lambda-D}\right)}{TAch_{\lambda-D}} \left(\frac{n_D - 1}{2200}\right) \qquad (13)$$

Therefore, if it is desired that the secondary spectrum of a doublet be held to 1/10 of its normal value, then it should be sufficient to know the index of refraction at each wavelength with an accuracy of 2 in the fifth decimal place. With an index error of this magnitude in each of the wavelengths used to calculate $\tilde{P}$, and ν, it is possible for the errors to combine so as to cause a doubling of the total error. Thus, it is necessary that the index be accurate to at least half of this value, or 1.0 in the fifth decimal place.

11.4.2.5 It is not only difficult to make measurements of the index of refraction with this accuracy; it is even more difficult to manufacture glass to specification with this degree of precision. The reputable optical glass manufacturers claim the required accuracy of their measurements but do not claim to be able to furnish samples to catalog values with this exactness. If, in the manufacture of precision lenses, it is necessary to have glass whose index of refraction is accurate to this degree, then it is necessary to have measurements of index made on a sample of the actual glass to be used in the lens.

11.4.3 Correction of secondary spectrum in a triplet lens. (Multiple glass-type method).

11.4.3.1 By using three glass types in the telescope objective it is possible in principle to bring at least three wavelengths to a common focus. If it is assumed the lenses are all thin and closely spaced then it is possible to write the following equations.

$$\phi_a + \phi_b + \phi_c = \phi \qquad \text{(Focal length)} \qquad (14)$$

$$\frac{\phi_a}{\nu_a} + \frac{\phi_b}{\nu_b} + \frac{\phi_c}{\nu_c} = 0 \qquad \text{(F and C light brought to same axial focus)} \qquad (15)$$

$$\frac{\phi_a P^*_a}{\nu_a} + \frac{\phi_b P^*_b}{\nu_b} + \frac{\phi_c P^*_c}{\nu_c} = 0 \qquad \text{(D light brought to the F-C axial focus)} \qquad (16)$$

11.4.3.2 By defining $\tilde{P}^* = \dfrac{n_F - n_D}{n_F - n_C}$, the third equation must be fulfilled if D light is to be focused at the F and C focus. The above equations may be solved to give the following values of ϕ_a, ϕ_b, and ϕ_c.

$$\phi_a = \phi \frac{\nu_a \left[\tilde{P}^*_c - \tilde{P}^*_b\right]}{\Delta}, \qquad (17)$$

$$\phi_b = \phi \frac{\nu_b \left[\tilde{P}^*_a - \tilde{P}^*_c\right]}{\Delta}, \qquad (18)$$

$$\phi_c = \phi \frac{\nu_c \left[\tilde{P}^*_b - \tilde{P}^*_a\right]}{\Delta}, \qquad (19)$$

where

$$\Delta = \begin{vmatrix} \tilde{P}^*_a & \nu_a & 1 \\ \tilde{P}^*_b & \nu_b & 1 \\ \tilde{P}^*_c & \nu_c & 1 \end{vmatrix} . \qquad (20)$$

One can recognize that Δ is a determinant, the value of which is the area of a triangle connecting points plotted with $\tilde{P}^*$ as the ordinate and ν as the abscissa.

11.4.3.3 Figure 11.10 is such a plot for several glasses. From the above equations one can see that points for three glasses must be found so as to form a triangle of finite area on the $\tilde{P}^*$ versus ν plot. It is important to pick glasses that will have the smallest possible values of ϕ_a, ϕ_b, and ϕ_c. If three glasses are picked, as shown in Figure 11.10 marked as a, b, c, then the (b) lens becomes negative, for $(\tilde{P}^*_a - \tilde{P}^*_c)$ is negative and Δ is positive. In order to minimize ϕ_b, the ratio of $(\tilde{P}^*_a - \tilde{P}^*_c)/\Delta$ must be made a minimum. If one draws a line from a to b it is clear that any glasses located on this line will have the same ratio of $(\tilde{P}^*_a - \tilde{P}^*_c)/\Delta$. If $\tilde{P}^*_a - \tilde{P}^*_c$ is made smaller, the area Δ of the triangle is made smaller by the same ratio. This can be seen to be true by remembering that ac is the base of the triangle and a perpendicular from b to this base line is the altitude of the triangle. Therefore,

$$\frac{(\tilde{P}^*_a - \tilde{P}^*_c)}{\cos\theta} h = \Delta,$$

and

$$\left[\frac{\tilde{P}^*_a - \tilde{P}^*_c}{\Delta}\right] = \frac{\cos\theta}{h} .$$

$\cos\theta$ is the angle between the line connecting a and c and the vertical axis. As long as $\cos\theta$ and h remain constant then $(\tilde{P}^*_a - \tilde{P}^*_c)/\Delta$ is constant. The procedure to pick glasses, then, is to try to find a triangle with as large an h as possible. It is also logical to suggest that the two positive lenses [the (a) and the (c) lenses] should have approximately the same power.

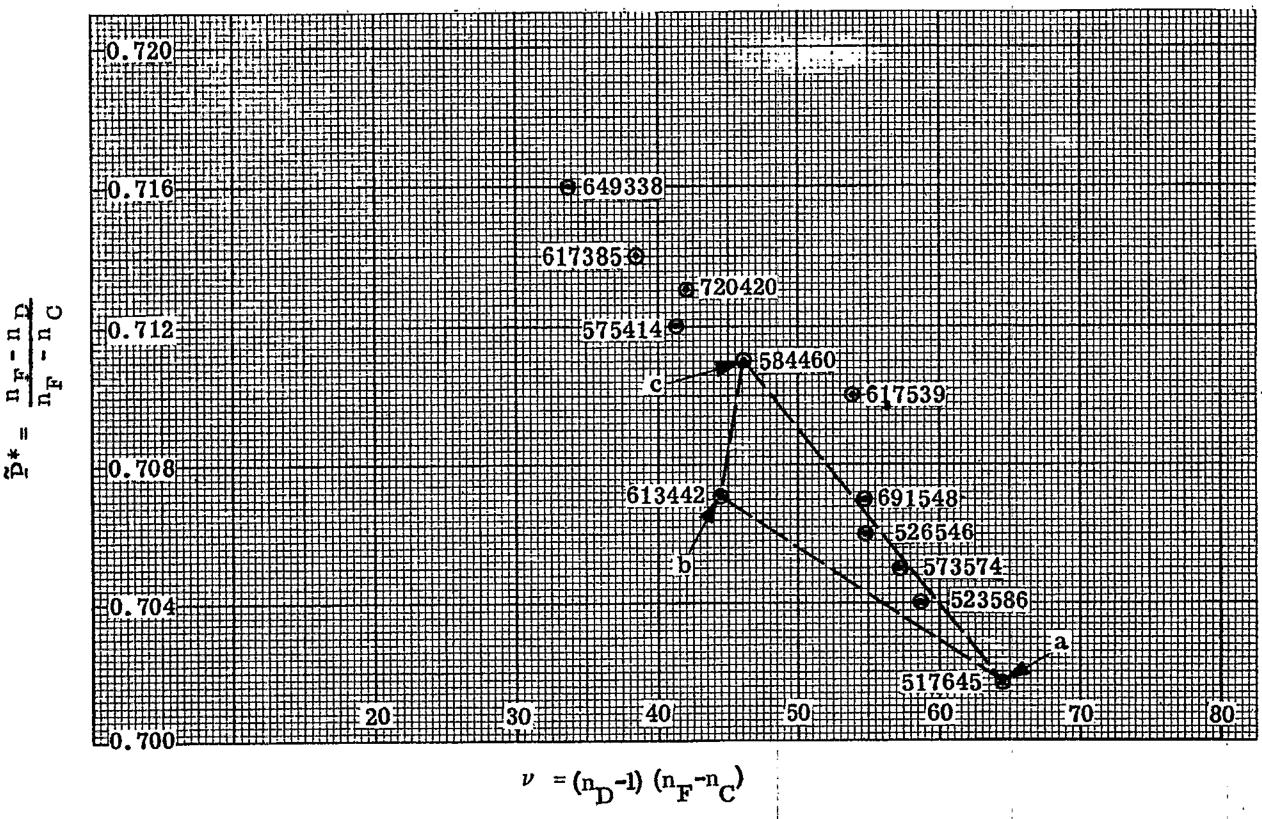


Figure 11.10 - A plot of $\tilde{P}^*$ vs ν for different glasses.

11.4.3.4 Dividing equation (17) and (19) provides the ratio ϕ_a / ϕ_c .

$$\frac{\phi_a}{\phi_c} = \frac{\nu_a}{\nu_c}\left(\frac{\tilde{P}^*_c - \tilde{P}^*_b}{\tilde{P}^*_b - \tilde{P}^*_a}\right)$$

Since ν_a / ν_c is greater than 1, it then follows that $(\tilde{P}^*_c - \tilde{P}^*_a)$ should be less than $(\tilde{P}^*_b - \tilde{P}^*_a)$. A glass combination selected with these considerations is shown in Figure 11.10. There are, however, other factors one must consider in selecting the glasses for the reduction of secondary color, namely, tertiary color as described below.

11.4.3.5* By satisfying the conditions in Equations (14), (15) and (16), the three wavelengths F, D, and C focus at a common axial point. One may now calculate the residual transverse aberration for any other wavelength λ , from the equation

$$\left(\frac{\phi\tilde{P}_{\lambda-D}}{\nu}\right)_a + \left(\frac{\phi\tilde{P}_{\lambda-D}}{\nu}\right)_b + \left(\frac{\phi\tilde{P}_{\lambda-D}}{\nu}\right)_c = \text{TAch}_{F-\lambda}\left(\frac{n_{k-1}\,u_{k-1}}{y^2}\right) \qquad (21)$$

$\tilde{P}_{\lambda-D}$ will be hereafter referred to as $\tilde{P}^{**}$. By inserting the expressions for (ϕ/ν) given in Equations (17), (18), (19), and (20), Equation (21) becomes

$$\frac{\tilde{P}^{**}_a\,(\tilde{P}^*_c - \tilde{P}^*_b) + \tilde{P}^{**}_b\,(\tilde{P}^*_a - \tilde{P}^*_c) + \tilde{P}^{**}_c\,(\tilde{P}^*_b - \tilde{P}^*_a)}{\Delta} = \text{TAch}_{F-\lambda}\,\frac{n_{k-1}\,u_{k-1}}{y^2\,\phi} \quad . \qquad (22)$$

* The notation P* and P** and the ideas suggested in this section have been described by Herzberger, Optica Acta, 6, 197 (1959).

The left hand side of the equations is equal to

$$\frac{\begin{vmatrix} \tilde{P}^*_a & \tilde{P}^{**}_a & 1 \\ \tilde{P}^*_b & \tilde{P}^{**}_b & 1 \\ \tilde{P}^*_c & \tilde{P}^{**}_c & 1 \end{vmatrix}}{\Delta}$$

The value of the determinant in the numerator is again the area of a triangle in a coordinate system with P^* plotted as abscissa and P^{**} plotted as ordinate. This tells us then, that if we wish to have small residual aberration for other wavelengths it is necessary to pick three glasses that lie on a straight line when plotted on the P^* versus P^{**} diagram. Plots of this type are shown in Figure 11.11. There are three sets of wavelength data plotted on this graph. The values of P^{**} are $P_{A'-D}$, P_{e-D}, and P_{g-D}. The glasses used in a sample calculation are shown connected by dotted lines. These plots show that A', g and e light will have residual aberration because the three glasses do not lie on a straight line. As the data is plotted the triangles show that A' will have a positive TAch. The e light will be slightly negative, and g will be slightly positive. An actual curve for these three glasses is shown in Figure 11.12. It is plotted on the same coordinates as the data in Figure 6.20. The corresponding curve for a doublet is shown in the same figure. The powers of the lenses in the triplet are shown compared with a doublet in Table 11.12. The strong curves in the triplet indicate clearly the reason why lenses corrected for secondary color must have small relative apertures.

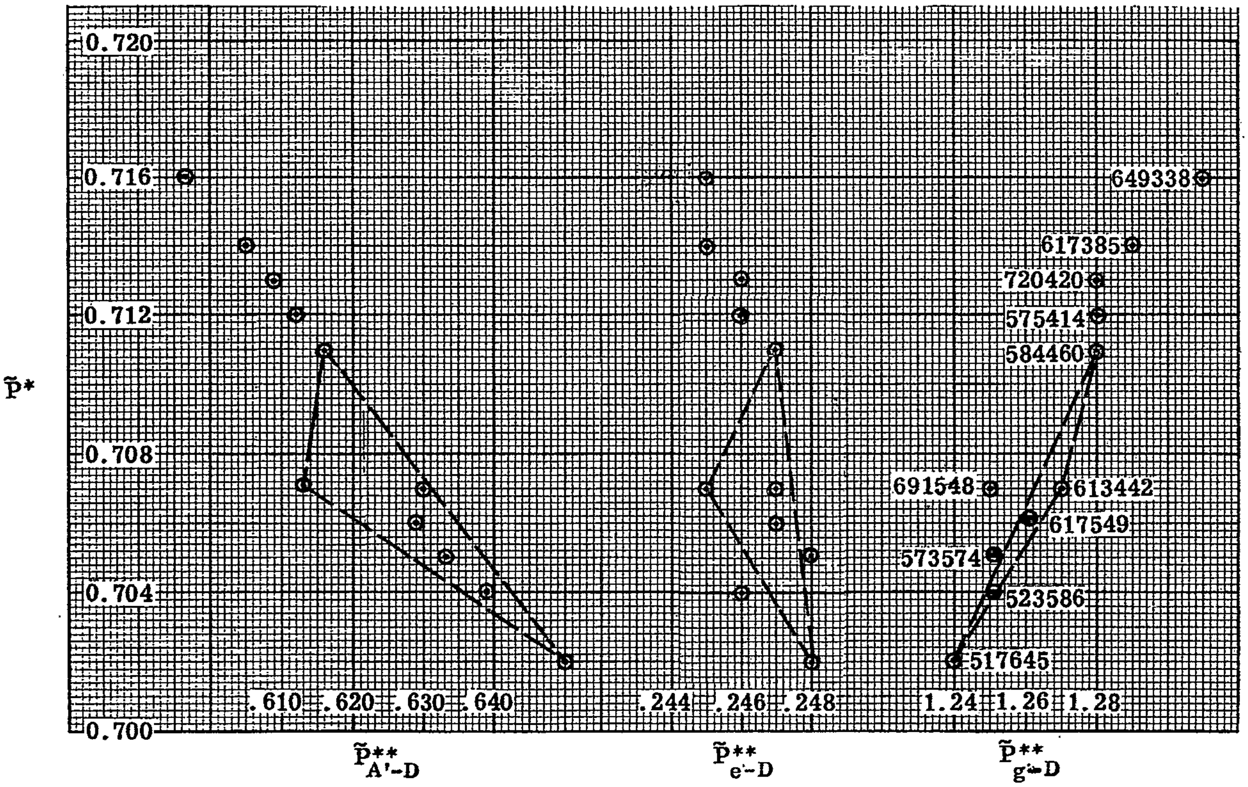


Figure 11.11 - A plot showing tertiary spectrum.

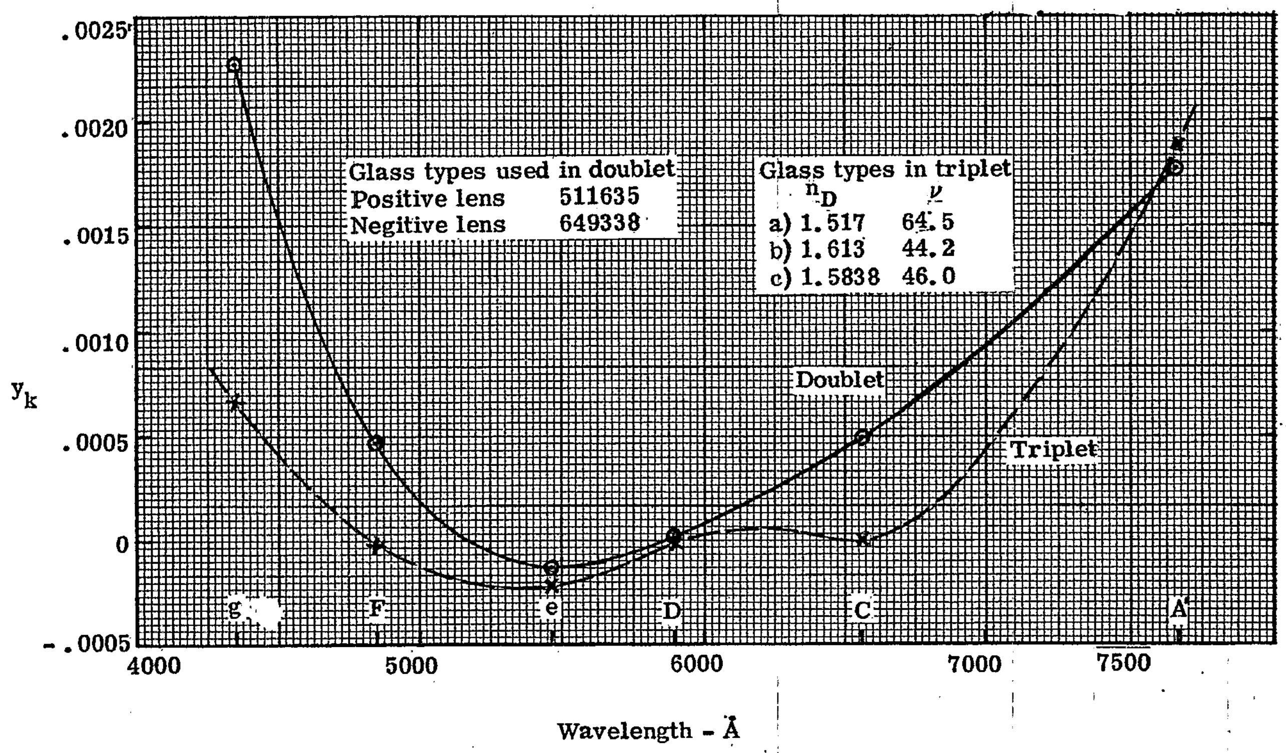


Figure 11.12 - Plot of y_k vs λ for a triplet corrected for secondary color.

Doublet	Triplet
ϕ_a = 0.21380 ϕ_b = -0.1138 ϕ = 0.1	ϕ_a = 0.2812 ϕ_b = -0.4747 ϕ_c = 0.2935 ϕ = 0.1

Table 11.12 - Comparison between powers in a triplet corrected for secondary color and an ordinary doublet.

11.4.4 Additional readings on secondary color. For further reading on secondary color, refer to the following articles:

(a) Three color achromats, R. E. Stephens, J. Opt. Soc. Am. 49, 398 (1959)

(b) Four-color achromats and superchromats, R. E. Stephens, J. Opt. Soc. Am. 50, 1016 (1960)

11.4.5 Sample design of a triplet corrected for secondary color.

11.4.5.1 A sample lens has been fully corrected for secondary color and ray traced. The glasses used in the design were all Schott glasses. The thin lens solution was given by R. E. Stephens in the second of the above papers. The thin lens glasses and powers were given as follows:

Glass	Power
F-1	0.338
KzFS-4	-0.721
PKS-1	0.483

These powers add up to 0.1. The focal length is therefore 10. The lens was corrected for an f-number of 12. The final lens specifications are shown in Table 11.13.

LENS SPECIFICATIONS

c	t	Glass
0.1989	0.1683	F-1
-0.0791	0.0175	Air
-0.1502	0.1400	KzFS-4
0.6180	0.0100	Air
0.6387	0.2964	PKS-1
-0.1272	9.2728	Air

RAY TRACE DATA

Y_1	$(Y_k)_D$	$(Y_k)_F$	$(Y_k)_C$
0.25	0.00000369	0.000092	0.000023
0.375	-0.000155	0.000074	-0.000164
0.500	-0.001056	-0.000574	-0.001137

Table 11.13 - Three lens system corrected for secondary color.

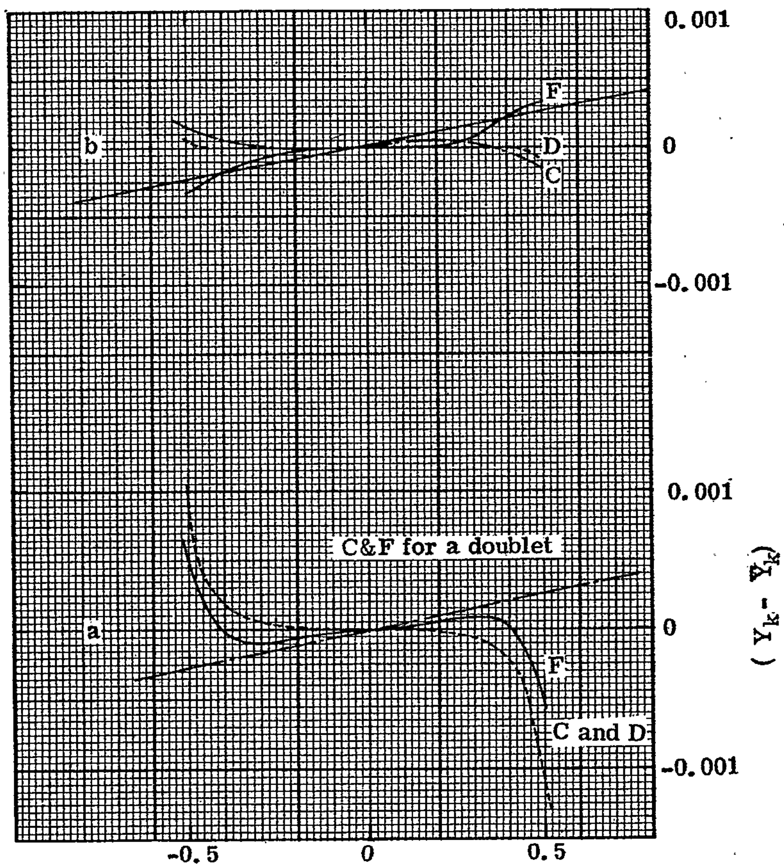


Figure 11.13 - Meridional ray plot at 0°, for triplet.

The meridional ray plot for this lens is shown in Figure 11.13a. It is plotted on the same scale as the doublet shown in Figure 11.3. These data show that the triplet corrected for secondary color has to be made to a much larger f-number than the doublet. One can see that the curves are stronger than the doublet, and the higher order spherical aberration is large. The straight dashed line in Figure 11.13a shows the best possible aberration curves for F and C light in a simple doublet. The curves for the triplet out to a value of $Y_1 = 0.4$ show that some advantage is gained by using the triplet. Beyond $Y_1 = 0.4$ there is no gain at all, for the high order spherical aberration is so heavily under-corrected. If the lens had been stopped down to a value of $Y_1 = 0.3$ it could have been corrected with a smaller residual aberration. The lens would then be corrected with approximately one third the aberration in a simple doublet. This would give an f-number of 16.7.

11.4.5.2 Other solutions. The triplet described in Table 11.13 is not a completely optimized solution. The higher order aberrations might have been further reduced by adjusting the air spaces. One must also consider other orientations of the lenses. To illustrate this effect a solution was corrected with the PKS-1 as the first element, and F-1 as the final element. This solution is shown in Table 11.14. This shows some improvement over the first solution shown. This lens could probably be used at $f/11$. This lens is better corrected than the one with F-1 in front. It is not, however, certain that this is a characteristic of the lens. The second system was designed much more carefully than the first one. Many, many solutions were found for the second design. The solutions were found automatically for varying amounts of primary and secondary color, and finally, the zonal spherical aberration was reduced by adjusting the air space between the second and third lens. The second solution is, we believe, nearly as good a solution as it is possible to design with this combination of glass; but we are not sure, for there are many things that should be studied. For example, the axial color should probably be made slightly more negative. This would lower the F light curve slightly and raise the C light curve. The two curves would therefore cross further out in the aperture.

LENS SPECIFICATIONS

c	t	Glass
0.5334	0.3180	PKS-1
-0.3322	0.0100	Air
-0.2792	0.1400	KzFS-4
0.7079	0.1000	Air
0.5469	0.1969	F-1
-0.1723	9.018	Air

RAY TRACE DATA

Y_1	$(Y_k)_D$	$(Y_k)_F$	$(Y_k)_C$
0.25	0.0000021	0.0000127	0.0000517
0.375	0.0000028	0.0001495	0.0000265
0.500	-0.000085	0.0003700	-0.0001531

Table 11.14- Three lens system corrected for secondary color.
The meridional ray plot is shown in Figure 11.13b.

11.4.5.3 The amount of computing that went into the above designs is beyond the comprehension of anyone not familiar with the problem. We found 16 automatic third order solutions. Usually it took a minimum of four iterations. For each solution a fifth order and 9 rays were traced. Only one out of five possible thicknesses was used as a variable. Before one could say he really had an optimum solution it would be necessary to check the effectiveness of varying the thickness, trying the negative lens out front and use other glasses. Thanks to the modern computer it is beginning to become practical to do this at a reasonable cost. When we realize how limited our present approaches are to the problem, we can look forward to promising solutions in the future.

11.4.6 Evaluation of lens from optical path. The gain in image quality is however, misleading. This is an example of why one must always remember to consider the physical optics of the problem. Figure 11.13 shows that if the perfect doublet is stopped down to $f/16.7$ the C and F light would have a transverse aberration of 0.00015, with respect to the D light focus. If we assume the focal length is 10 cm then this corresponds to a transverse aberration of 0.00015 cm. There is a relation between the transverse aberration and optical path difference for shift of focus. The equation is

$$OPD = Y_k \cdot \frac{Y_1}{2 f'}$$

where Y_k is the transverse aberration for the ray entering the lens at Y_1. Inserting the above aberration into this equation shows that the OPD in the simple doublet is 0.038 wavelengths. Now if the Rayleigh tolerance of $\lambda/4$ is assumed, this means an $f/16.7$ doublet with a focal length of 10 cm has so little secondary spectrum it will never be noticed. Its focal length could be scaled up 0.25/0.038 times the 10 cm focal length. This amounts to a focal length of 65.8 cm. Therefore, there is no point whatsoever in designing a lens to reduce the secondary spectrum, when the focal length is less than 65.8 cm, if it has to be stopped down to $f/16.7$. The above triplet therefore would not show any advantage until scaled to focal lengths longer than 65.8 cm. To realize a two to one gain over a doublet, it will have to be scaled to 131.6 cm focal length. At $f/16.7$ this would be a lens 7.9 cm in diameter. This is getting to be a fairly expensive size lens in which to use the unusual glass KzFS-4.

11.5 SUMMARY

One can now see the relation between a doublet and a triplet corrected for secondary color. One can design a doublet with two types of glasses and split the positive lens into two and make the system a triplet. Then it is necessary to find two glasses with the same values of P* and P**. Since there are only very few glasses removed from the $\tilde{P}$ versus $\check{\nu}$ line this means that relatively few glasses are available for the positive lens. By using a third glass type it is possible to use a much larger selection of glasses. In order to find an optimum solution it is necessary to study many combinations of glass. One must completely correct the lens and ray trace the solution before deciding what glass choices are most suitable. There will be variation of spherical aberration with wavelength. This aberration may become so large that all the advantage of using the special glasses to correct the first order effects may be completely lost.