9 METHOD OF LENS DESIGN

9.1 THE PROCESS OF DESIGNING A LENS SYSTEM

9.1.1 Introduction. The formulae used to design a lens system have now been presented. Ray trace equations were derived in Section 5. Their use in first order design and in aberration analysis were discussed in Sections 6 and 8. In the present section a systematic method for the design of lens systems will be described, and this method will be illustrated with the design of a triplet flat field lens in Section 10.

9.1.2 Approach. The design of a lens system at the present state of the art is an iterative procedure. Certain steps in the procedure are repeated until a satisfactory design is attained. In this sense, lens design involves a trial and error procedure. At present (1962), direct methods of design, proceeding from the desired specifications to the specific lens, do not exist. The following steps are the basic elements of the iterative procedure.

(1) Select a lens type.

(2) Find a first order thin lens solution.

(3) Find a third order thin lens solution.

(4) Find a thick lens solution, and calculate first order and third order aberrations.

(5) Trace a few selected meridional and skew fans.

(6) Adjust third order coefficients to balance higher order aberrations, and repeat steps 5 and 6 until the balance between third and higher order aberrations agrees with desired specifications, or at least is reasonable.

(7) Trace additional fans of skew rays; make spot diagrams and calculate the energy distribution.

(8) Evaluate the image.

(9) Return to a previous step and repeat the process until evaluation indicates desired performance. Which step to return to depends on the problem. The most usual procedure is to return to step (4), but often the designer must return to step (1).

9.2 DESCRIPTION AND ANALYSIS OF THE BASIC PROCEDURE

9.2.1 Step 1 - Selection of a lens type.

9.2.1.1 In order to select the type of lens to be designed, the designer must first survey the complete lens problem. He attempts to equate it to one of the simple basic optical systems. He asks if this is a magnifier problem, a microscope, a telescope or a camera lens. After deciding upon the basic system, he then proceeds to make a layout using simple theory as illustrated in Section 7. This analysis thus generates a possible arrangement picture of how the axial and oblique rays will pass through the system.

9.2.1.2 Suppose, for example, that the system to be designed is a telescope. Given the magnifying power, field of view and over-all instrument length, a designer may conclude that the telescope should consist of an objective, a prism erecting system and an eyepiece. From the preliminary analysis he concludes that the objective must work at $f/3.5$ and the eyepiece must cover a half field of 30°. Looking over objective designs (for example, see Section 11) he may then compute the field curvature for the system and conclude that he will use an objective like the one illustrated in Figure 11.7, and, since the eyepiece must cover a half field of 30°, an Erfle type appears to be a logical choice. Inspection of the eyepieces shown in Section 14 discloses that the Erfle is the simplest design. It represents a good starting point.

9.2.1.3 Other factors may influence the designer's choice. Compatability with other systems, existing hardware, economics or delivery schedule are all valid considerations. Thus, unfortunately for the beginner, this step in the procedure is difficult and requires the most experience. As the process proceeds, the steps become more automatic and less dependent on experience. This means that the beginner finds it difficult to get started and it means that the designer instructing must say in effect at the beginning, "Let us start with a lens of such and such a type. Later I may be able to show you why I picked this particular type of solution." This approach to a problem does not appeal to the analytic mind but at present there is no other way to approach the problem. It would be nice if one could work from the specifications of the image, back to the design required, but there are only very limited procedures which will enable one to establish what lens type is needed for a particular problem. In Sections 10, 11, 12, 13, and 14 the performance and limitations of several types of lenses will be described which it is hoped will help a beginner select the type of lens.

9.2.1.4 The prime accomplishment of this step is the designer's decision to choose a certain lens type to perform a specified function in the system. Thus a starting point is established from which computation and evaluation can proceed. This step, baffling as it is to the beginner, is really the most creative part of the design, and, as experience is gained, this is the part of the design that intrigues the designer and gives him a chance to exercise judgement, which is what humans usually enjoy.

9.2.2 Step 2 - The first order thin lens solution. Once the lens type has been decided on, the next step is to solve the algebraic equations to determine the individual focal lengths and spacings of the elements. It greatly simplifies the procedure to assume that the lenses are thin. At this stage of the problem, there are usually conditions that must be satisfied in the passage of the axial paraxial ray, and the oblique paraxial ray. The entrance and exit pupils may have to be located at special positions, and their sizes may be given. The focal length and back focal length may be specified. It is also necessary to adjust the axial and lateral color, and Petzval sum to appropriate values. The passage of the oblique chief ray has an effect on the distortion. For simple systems it often is possible to write down algebraic equations relating the parameters of the system (ϕ, t, n) and the required conditions to be satisfied, but very often the algebra becomes so complex that graphical or linear approximations are required to find the solutions. The problem basically amounts to trying to solve a set of non-linear equations. Sometimes there are more equations than variables, in other instances the reverse may be true. One can spend a great deal of time on the algebra at this stage of the design. Often, the most sensible procedure is to resort to a systematic trial and error solution. This method will be illustrated in Section 10.2

9.2.3 Step 3 - The third order thin lens solution. By making the thin lens aberration coefficient calculations illustrated in Table 8.4, it is possible to obtain sets of second degree algebraic equations relating the first curvatures of the lenses and the aberrations. Again, in simple systems these can sometimes be solved algebraically or graphically. As a matter of fact, if these equations cannot be solved algebraically there is little justification for using the thin lens approximations, for one can as readily apply the trial and error methods to thick lenses using the surface contribution calculations shown in Table 8.2. By properly choosing the position of the aperture stop it is possible to greatly simplify the equations. The following reasoning is used in the preliminary design. In the preliminary third order design the aberrations are usually all made equal to zero. Equations 8-(18) through 8-(21) show us, that if B, F, C, E and P are all set to zero, then B*, F*, C*, and E* will all be zero. This tells us that the location of the stop position has no effect on the aberrations. Then it is advisable to choose the chief ray to pass through the center of one of the lenses. By so doing, the aberrations for this lens are given by Equations 8-(24) through 8-(28). This eliminates the calculation of E, the C is constant, and F varies linearly with c_1. In practice, it helps to use this procedure even if small residual aberrations are to be left in the system.

9.2.4 Step 4 - First and third order aberrations of a thick lens.

9.2.4.1 During this step in the design, calculations of the type shown in Table 8.2 are made to determine the first and third order aberrations of the lens with actual thicknesses. If the thin lens theory has been worked out completely, then values for the curvatures and the desired angles of the axial and oblique rays are known. Now, the procedure of introducing thicknesses changes all the first order and third order aberrations. The next problem is to modify the thick lens solution to achieve the desired aberrations.

9.2.4.2 Some designers have procedures for computing the positions of the principal planes of each individual element. Then the thick lens system is set up so that the first curvatures of each of the lenses are the same as for each of the thin lenses, and the angles the axial ray makes with the axis is the same as for the thin lenses. Finally, the spaces between the lenses are adjusted to make the spacings between the image principal plane (P_{2a}) and the next object principal plane (P_{1b}) of the thick lenses equal to the spacing between the thin lenses.

9.2.4.3 The designer should not spend too much time trying to adjust the spacings in this way since there is no direct and easy way to set up a thick lens equivalent of the thin lens. The procedure just described always fails to keep all the aberrations the same as for the thin lens; some changes in the power distribution are necessary.

9.2.4.4 If the designer is setting up for the first time a thick lens from thin lens data, there is really very little point in trying to make the thick lens aberrations exactly equal to the thin lens aberrations. The reason for this is that until one has ray traced a design, and determined the magnitude of the higher order aberrations, it is not possible to tell just what third order aberrations are needed to balance out those of a higher order. Usually, a perfectly satisfactory way to set up a thick lens from thin lens data is to assume the positions of the principal planes, from a simple sketch of the lens, using curvatures from the thin lens solution and thicknesses from 10.4.

9.2.4.5 A major problem in lens design is the problem of adjusting a thick lens to arrive at some definite third order aberrations. This can be done by a trial and error method if some information is known about how the aberrations vary with parameter changes. Sometimes the information in the form of curves for the thin lenses provides indications to the designer which help him decide how to adjust the thick lens to find a solution.

9.2.4.6 The problem of adjusting a thick lens system resolves itself into the problem of solving a set of simultaneous equations. One can systematically change one parameter at a time and recalculate all the total aberrations of the new system. By finding the differences in the total aberrations due to the parameter change, it is possible to compute the parameter differential for all the third and first order aberrations. This method will now be discussed in detail.

9.2.4.7 Since B, F, C, E, P, a, and b are functions of all the system parameters, it is possible to write

$$\Delta\Sigma B = \sum_{j=1}^{j=k-1} \left[\left(\frac{\partial \Sigma B}{\partial c}\right)_j \Delta c_j + \left(\frac{\partial \Sigma B}{\partial t}\right)_j \Delta t_j + \left(\frac{\partial \Sigma B}{\partial n}\right)_j \Delta n_j \right], \qquad (1)$$

$$\Delta\Sigma F = \sum_{j=1}^{j=k-1} \left[\left(\frac{\partial \Sigma F}{\partial c}\right)_j \Delta c_j + \left(\frac{\partial \Sigma F}{\partial t}\right)_j \Delta t_j + \left(\frac{\partial \Sigma F}{\partial n}\right)_j \Delta n_j \right], \qquad (2)$$

$$\Delta\Sigma C = \sum_{j=1}^{j=k-1} \left[\left(\frac{\partial \Sigma C}{\partial c}\right)_j \Delta c_j + \left(\frac{\partial \Sigma C}{\partial t}\right)_j \Delta t_j + \left(\frac{\partial \Sigma C}{\partial n}\right)_j \Delta n_j \right], \qquad (3)$$

$$\Delta\Sigma E = \sum_{j=1}^{j=k-1} \left[\left(\frac{\partial \Sigma E}{\partial c}\right)_j \Delta c_j + \left(\frac{\partial \Sigma E}{\partial t}\right)_j \Delta t_j + \left(\frac{\partial \Sigma E}{\partial n}\right)_j \Delta n_j \right], \qquad (4)$$

$$\Delta\Sigma P = \sum_{j=1}^{j=k-1} \left[\left(\frac{\partial \Sigma P}{\partial c}\right)_j \Delta c_j + \left(\frac{\partial \Sigma P}{\partial t}\right)_j \Delta t_j + \left(\frac{\partial \Sigma P}{\partial n}\right)_j \Delta n_j \right], \qquad (5)$$

$$\Delta\Sigma a = \sum_{j=1}^{j=k-1} \left[\left(\frac{\partial \Sigma a}{\partial c}\right)_j \Delta c_j + \left(\frac{\partial \Sigma a}{\partial t}\right)_j \Delta t_j + \left(\frac{\partial \Sigma a}{\partial n}\right)_j \Delta n_j + \left(\frac{\partial \Sigma a}{\partial \nu}\right)_j \Delta \nu_j \right], \qquad (6)$$

$$\Delta\Sigma b = \sum_{j=1}^{j=k-1} \left[\left(\frac{\partial \Sigma b}{\partial c}\right)_j \Delta c_j + \left(\frac{\partial \Sigma b}{\partial t}\right)_j \Delta t_j + \left(\frac{\partial \Sigma b}{\partial n}\right)_j \Delta n_j + \left(\frac{\partial \Sigma b}{\partial \nu}\right)_j \Delta \nu_j \right]. \qquad (7)$$

9.2.4.8 In order to correct a finite thickness lens system to any desired third and first order aberrations, a designer must, in effect, solve this set of simultaneous equations. Now since B, F, C, E, P, a, and b do not change linearly with parameter changes, these equations will not, in general, provide the correct changes, so the process must be repeated for a series of iterations. Without a large computer it was a hopelessly long procedure to systematically correct a system to a given set of third order values. Therefore, designers had to resort to other techniques. They did this by separating the problem into two parts. First, a solution was found which corrected a, b, and P, with some consideration given to E. Second, this solution was corrected for B, F, and C.

9.2.4.9 The first step, correction of a, b, P, and E, was done by adjusting the focal lengths of the lenses and the spacings between the lenses. Sometimes different coefficients were found for the changes of a, b, and P, and simultaneous equations solved, but the index and dispersion of glass were usually not included because glasses are manufactured in finite steps. Usually designers resorted to a simple trial and error method of adjusting focal lengths, spaces, and glasses. It is surprising how rapidly an experienced designer can adjust variables and arrive at a solution without actually solving the above equations.

9.2.4.10 The second step, correction of B, F, and C, was done by the technique called bending. Lens bending means changing the shape of a lens without affecting its focal length. Equation 6-(22)

gives the expression for the power ϕ of a lens as $(c_1 - c_2)(n-1)$. As long as $c_1 - c_2$ remains constant, c_1 may take on any value without affecting ϕ. If the lens is thin, then bending does not change the angles of the axial and oblique paraxial rays after passage through the lens. If the lens is thick, keeping $(c_1 - c_2)$ constant is not quite the same thing as keeping the focal length fixed because f' depends on t as well as $(c_1 - c_2)$. Usually in bending thick lenses, it is advisable to solve for the second curvature so that the axial ray remains at a constant angle with the optical axis. Bending of a lens has no effect on a, b and P for a thin lens and a very small effect in a thick lens. The bending affects primarily B, F, and C.

9.2.4.11 Therefore, before the widespread use of computers, designers found solutions for given values of B, F, and C by setting up three simultaneous equations. Usually many of the possible degrees of freedom were not used. Experienced lens designers seldom actually solved the equations, but they would keep adjusting the lens by a trial and error method. In the lens designers' slang, the method for finding a solution is jiggle in or poke at it. It is amazing how successfully an experienced designer could jiggle in a design. This method appears to be an art. With experience a designer apparently develops a procedure analogous to solving these equations in his head, by developing a feel for the system.

9.2.4.12 With the modern computer it is now feasible to find automatically a solution of Equations (1) through (7). In Section 10 several examples will be shown illustrating how this is done. Up to the present, the equations solved automatically by the computer have not included the terms with the glass type as a variable. Many problems have been solved using curvatures and thicknesses as variables. The automatic program does essentially the following:

(1) All the first and third order calculations are computed for an initial system. Call this system No. 1.

(2) Each system parameter (c or t) is varied one at a time, and all the first and third order aberrations are calculated for each altered system. The designer may specify which curvatures and thicknesses to change. Each parameter is changed by 0.01% of its initial value.

(3) Differential coefficients are then computed for each variable and aberration. For example,

$$(c_{new} - c_{old})_j = \Delta c_j ,$$

and

$$(\Sigma B_{new} - \Sigma B_{old})_j = \Delta \Sigma B_j .$$

Then

$$\left(\frac{\partial \Sigma B}{\partial c}\right)_j \approx \frac{\Delta \Sigma B_j}{\Delta c_j}$$

(4) When all the differential coefficients are known, the data for the seven equations (1) through (7) are known. The numbers on the left hand side of the equation are found by taking the difference between the aberrations in system No. 1 and the final target (desired) values for the aberrations. For example,

$$\Delta \Sigma B = (\Sigma B_{target} - \Sigma B_1) \text{ etc.}$$

(5) If there are seven variables in the optical system then there will be seven equations with seven unknowns. If there are more variables than equations then the set of equations cannot be uniquely solved. One technique is to impose the condition, that the sum of the squares of the changes in the parameters shall be a minimum. If there are fewer variables than equations then it is not possible to obtain an exact solution. In this case it is customary to solve for a least squares solution. This means a solution is found when the sum of the squares of the differences between the final aberrations and their target values is a minimum.

(6) If the aberrations changed linearly with parameter changes, the target values for the aberrations would be found in one step. However the changes are not usually linear, so the process has to be repeated several times. If the target values for the aberrations are far removed from the initial values, there is the real possibility that this inherently simple procedure will not converge to a solution. Knowledge of the regions of solution is an invaluable aid in helping to select the initial values for system No. 1.

9.2.5 Step 5 - Tracing a few selected meridional and skew fans.

9.2.5.1 After the third order solution is found, the next step is to trace a few selected rays to evaluate the effects of higher order aberrations. The number of rays to trace depends on the stage of the design. On the first ray trace of a new system, only a small number of rays need be traced, but as the design proceeds, additional rays may be necessary for added refinement.

9.2.5.2 One suggested plan for the ray tracing of a design is as follows:

(1) The 0° image. In D light trace three rays at $Y_1 = (Y_1)_{max.}$, $Y_1 = 0.7\ (Y_1)_{max.}$, $Y_1 = 0.5\ (Y_1)_{max.}$, where $(Y_1)_{max.}$ is the radius of the entrance pupil. Trace the same rays in F and C light.

(2) If the object is at infinity, trace three meridional fans of rays at angles corresponding to $L_o = (L_o)_{max.}$, $L_o = 0.7\ (L_o)_{max.}$ and $L_o = 0.5\ (L_o)_{max.}$. If the object is at a finite distance, trace the rays from three object points $\overline{Y}_o = (\overline{Y}_o)_{max.}$, $\overline{Y}_o = 0.7\ (\overline{Y}_o)_{max.}$ and $\overline{Y}_o = 0.5\ (\overline{Y}_o)_{max.}$. For each obliquity, trace at least five meridional rays to enter the entrance pupil at uniform intervals ranging from $Y_1 = (Y_1)_{max.}$ to $Y_1 = -(Y_1)_{max.}$.

(3) For each obliquity, trace three skew rays with coordinates in the entrance pupil as follows:

$(X_1)_{max.}$, $Y_1 = 0$

$0.7\ (X_1)_{max.}$, $Y_1 = 0$

$0.5\ (X_1)_{max.}$, $Y_1 = 0$

$(X_1)_{max.} = (Y_1)_{max.}$ since the entrance pupil is assumed to be a circle.

(4) Repeat steps 2 and 3 for F and C light.

9.2.5.3 The data from the ray tracing may be plotted as illustrated in Figure 8.5, and Figures 8.7 through 8.10. In practice, this data is plotted on a single diagram usually leaving out the plots shown in Figures 8.8b and 8.9. A plot of this type is shown in Figure 9.1. In making these plots, it is advisable to use the same scale for all the plots of Y_k and X_k. At first it might appear that lenses of different focal lengths should be plotted using different scales. Actually, for most applications, the scale shown in Figure 9.1 represents the size of images used most frequently. Therefore, it simplifies plotting and helps one to assess rapidly a lens if these plots are made on this standard scale. Notice that 0.01 division on the vertical scale corresponds to 1 cm. (But this has been reduced to 0.86 cm in reproduction.) If the lens is calculated in centimeters, then 1 cm on the vertical scale of the graph corresponds to 100 microns. If the lens is calculated in inches, the 0.01 division should be replaced by 0.004, so that again 1 cm indicates a 100 micron image. If it turns out that the aberrations are so large they cannot be plotted on this scale, they are so large that they probably are not worth plotting.

9.2.6 Step 6 - Adjusting third order aberrations. Usually one attempts to make the curves in Figure 9.1 as flat as possible. In a perfect lens the curves would be horizontal straight lines. In most cases this can not be achieved, even to practical limits. The usual curves look more like the ones shown in Figure 9-1. Take for example the curves shown for the image point at 1.76. The meridional rays are focused within a strip 0.012 wide. The skew rays are confined within a strip 0.016 wide. One can say with fair assurance that the complete image is confined to an area 0.012 by 0.016. Since the meridional ray plot

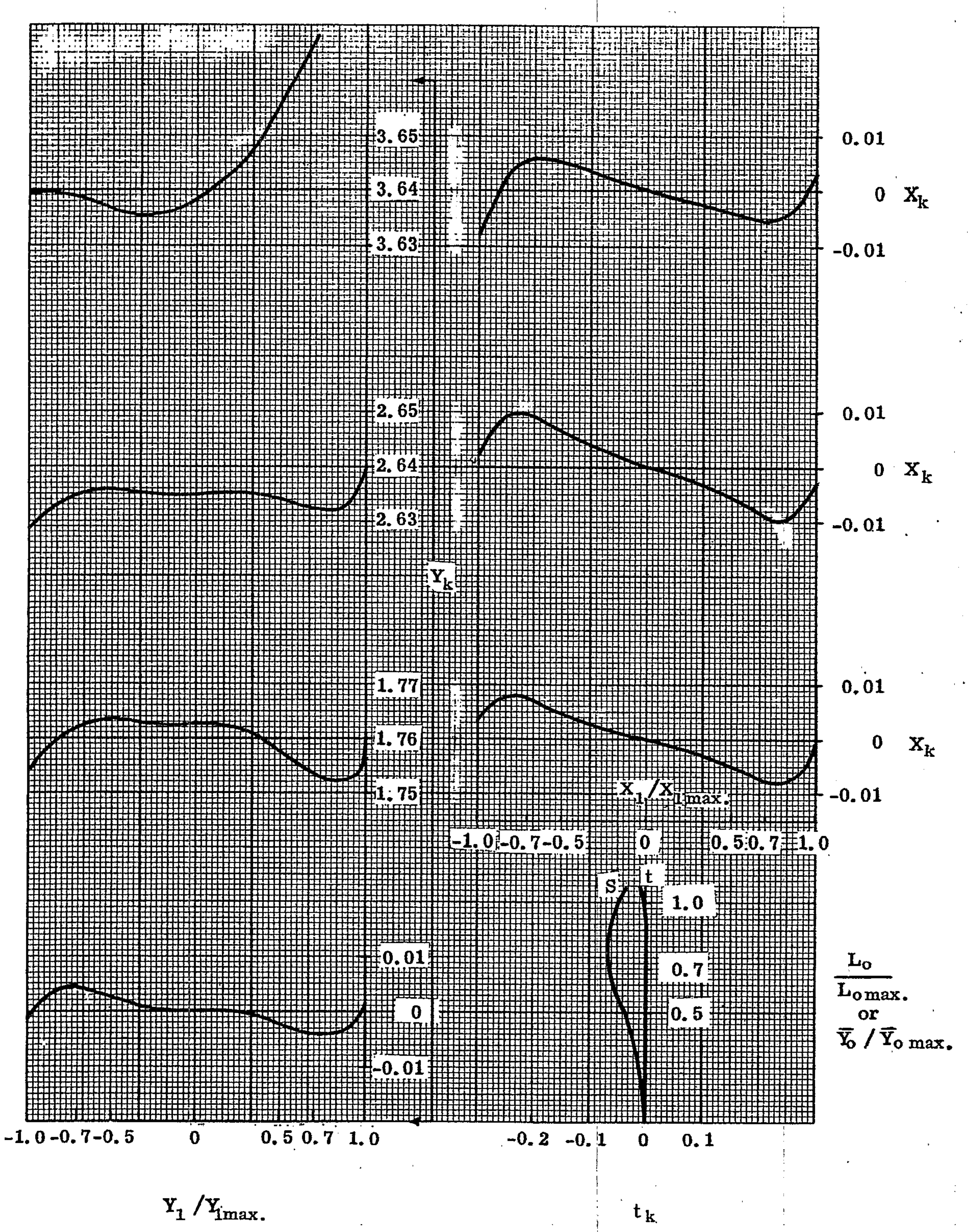

Figure 9.1- Sample plot of selected ray trace data.

shows a region where the curve is flat and horizontal, one would expect to get some concentration of energy towards the center of the spot. When one begins adjusting a design it is usually possible to tell from these curves what is needed to improve the energy concentration. For example, the basic difficulty with the design represented by Figure 9.1 is that the Petzval sum is too negative. This is the reason the skew curves are so far from the horizontal. One can also see that the image at a height of 3.64 will be poor because of the over-corrected spherical aberration in the upper meridional rays. These defects might suggest to the designer that he should try to find a solution with less negative Petzval curvature and introduce more negative third order spherical aberration. If so he would then return to Step 4 in paragraph 9.2.4 and solve for new third order aberrations, and repeat Step 5. Several alternate solutions may therefore evolve, but eventually it will be necessary to evaluate the energy concentration by proceeding to Step 7.

9.2.7 Step 7 - Calculation of spot diagrams and energy distributions. The energy distribution curves should be computed as described on page 8.1. Usually it is advisable to compute the energy distribution curves for a field point on the axis, for one half-way out in the field and for one at the edge of the field. Strictly speaking, one should also compute curves for two or three wavelengths, but this takes a great deal of computing and usually is not necessary for the average problem.

9.2.8 Step 8 - Image evaluation.

9.2.8.1 Once the designer has computed the energy distributions in several images in the field he is able to compare these with the design requirements. Seldom can one achieve the required results in the first system analyzed. The designer must then decide whether to continue with this design or to shift over to another type of lens. If he shifts over to another lens type he may then return to Step 1. If he decides to stick with the present lens type, he must decide whether to continue trying to meet the original specifications or whether to seek to modify the specifications and provide an alternative compromise solution. Usually the modern design problems end up with a give and take solution. The designer must therefore completely understand how the lens will perform, and be able to show what can be achieved by making variations in the original specifications. This means he may have to carry several designs up to the energy distribution curves in Step 7 in order to make a wise decision. It is imperative therefore that he devise ways to quickly evaluate the design.

9.2.8.2 The energy distribution curves of Step 7 may be used to check the image quality. This is a satisfactory method for many optical systems, but if the image quality is high one must consider the calculation of diffraction effects. As a general rule, one does not need to worry about diffraction effects if the wavefront departs from a perfect sphere by more than two to five wavelengths. (A method of computing this departure from ray trace data is described by H. H. Hopkins.*) There are several criteria one can apply to gauge the influence of diffraction, but this is a subject in itself. (See Sections 16, 25, 26.) However, a designer should be familiar with the wavefront tolerances suggested by Conrady.**

9.2.8.3 One must remember that it is impossible to concentrate the energy in an image into a smaller spot size than predicted by diffraction. In Figure 9.2 a plot of energy distribution is shown for a perfect lens. The abscissa Z is the following:

$$Z = \frac{\pi \, Y_e d}{\lambda \, \ell'} ,$$

where

Y_e is the radius of the exit pupil

d is the diameter of the image spot

λ is the wave length of light

ℓ' is the distance from the exit pupil to the image plane which is located at the perfect focus.

The first dark ring occurs at a value of Z equal to 3.83. This has a spot diameter

$$d = \left(\frac{3.83\,\lambda}{\pi}\right)\left(\frac{\ell'}{Y_e}\right) = 1.22 \frac{\lambda \ell'}{Y_e}$$

* H. H. Hopkins, Wave Theory of Aberrations (Oxford University Press, London, 1950) pp. 21-23.

**A. E. Conrady, Applied Optics and Optical Design, Part 1 (Oxford University Press, New York, 1943) pp. 126-141. See also Part I, 2nd ed. (Dover, New York, 1957) pp. 126-141, and Part II (Dover, 1960) pp. 626-639.

It is always a good idea to plot this curve on the same graph with the energy distribution curves computed for the actual lens. If the geometrical energy distribution curves lie to the left of the diffraction curve one knows that the light will not concentrate as well as the geometrical distribution curves indicate. The actual distribution curve will be inclined to follow the diffraction curve. Quite often the geometrical energy distribution curve will cross the diffraction image curve as shown in Figure 9.3 One can then estimate the energy concentration by using the formula

$$Z_{G+D} = \sqrt{Z_{G}^{2} + Z_{D}^{2}}\ .$$

Where Z_G ~ geometrical spot diameter

Z_D ~ diffraction spot diameter of a perfect aperture

Z_{G+D} ~ estimated spot diameter.

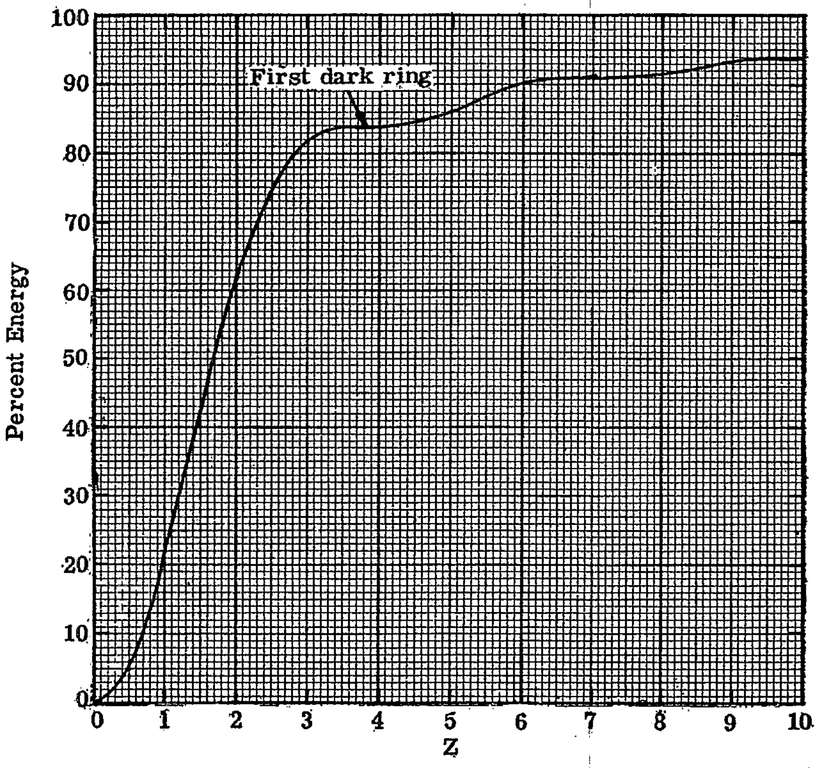

Figure 9.2 - Energy distribution for a perfect lens.

9.2.8.4 Some designers object to the energy distribution method for image evaluation because it does not take into account the orientation of the energy distribution. For example, if there is astigmatism the energy will be concentrated in a line image. The fact that the image of a point is a line might actually be favorable in some types of optical systems. For example, if the image is scanned by a slit one could certainly use this to advantage. For most optical systems however the circular energy distribution curves are adequate.

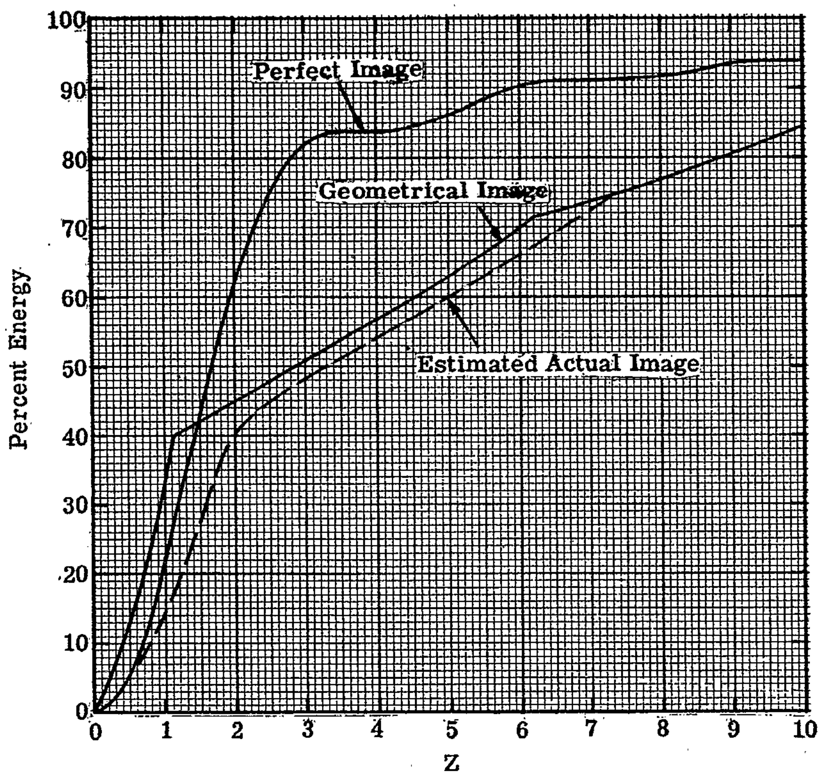

Figure 9.3 - Energy distribution curves.

9.2.8.5 The most modern method for evaluating images is to compute the optical transfer function (often called the sine wave frequency response) for the image. This can be done by performing a Fourier transform of the energy distribution in the image of a point source or a line source. Figure 9.4 shows a series of energy distribution curves. Figure 9.5 shows the corresponding modulation transfer curves. The modulation transfer function is the modulus of the complex optical transfer function. In Figure 9.4 all the curves are normalized to a maximum spot diameter of 10 mm. In Figure 9.5 the frequency is given in lines/mm. These spot diameters may of course be scaled to any other size. For example, suppose the maximum spot diameter is 100μ. Then the frequency scale should be multiplied by 100. One can multiply the modulation transfer function of a lens by the function for a detector to obtain the overall function for the entire optical system. Finally one can estimate the cutoff frequency at some particular response. A review of this approved image evaluation method may be found in Sections 26.2, 26.3 and 26.4, and in the article by Perrin ("Methods of Appraising Photographic Systems," J. Soc. Motion Picture and Television Eng., 69, 151-156, 239-249 (1960).

9.2.8.6 The problem of image evaluation is so involved that actually a designer is always forced to refer to some system which is known. Before attempting to improve a new system, a designer should try to do the following:

(1) Find out what systems have already been designed for conditions as nearly identical as possible with those specifying the new system.
(2) Evaluate the energy distribution of the nearest equivalent system.
(3) Compare the energy distribution in the new design with that of the closest equivalent to determine if improvement has been made.

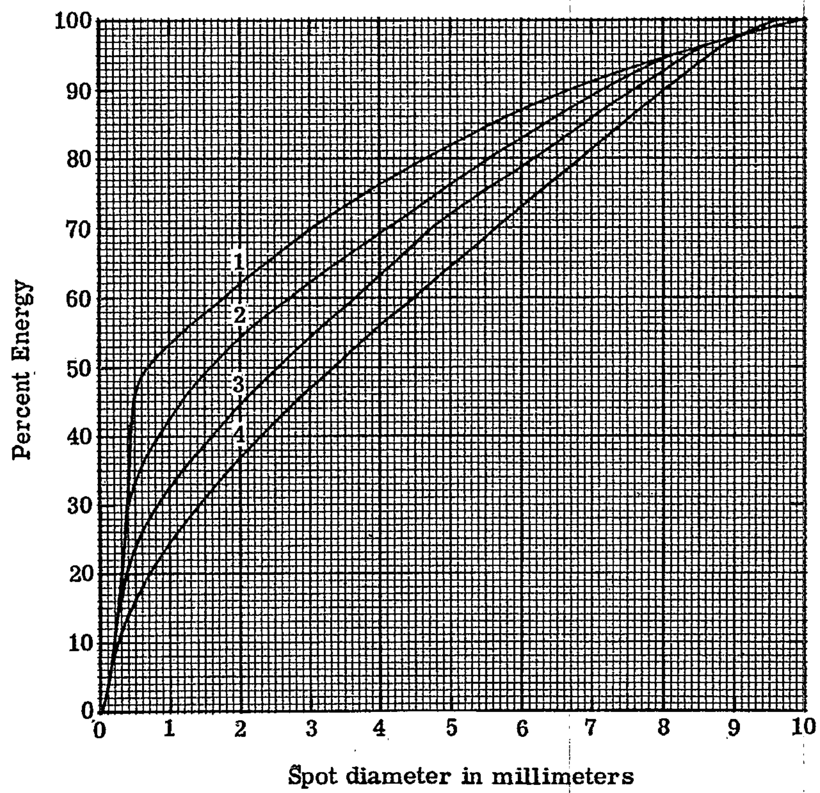

Figure 9.4 - A series of energy distribution curves.

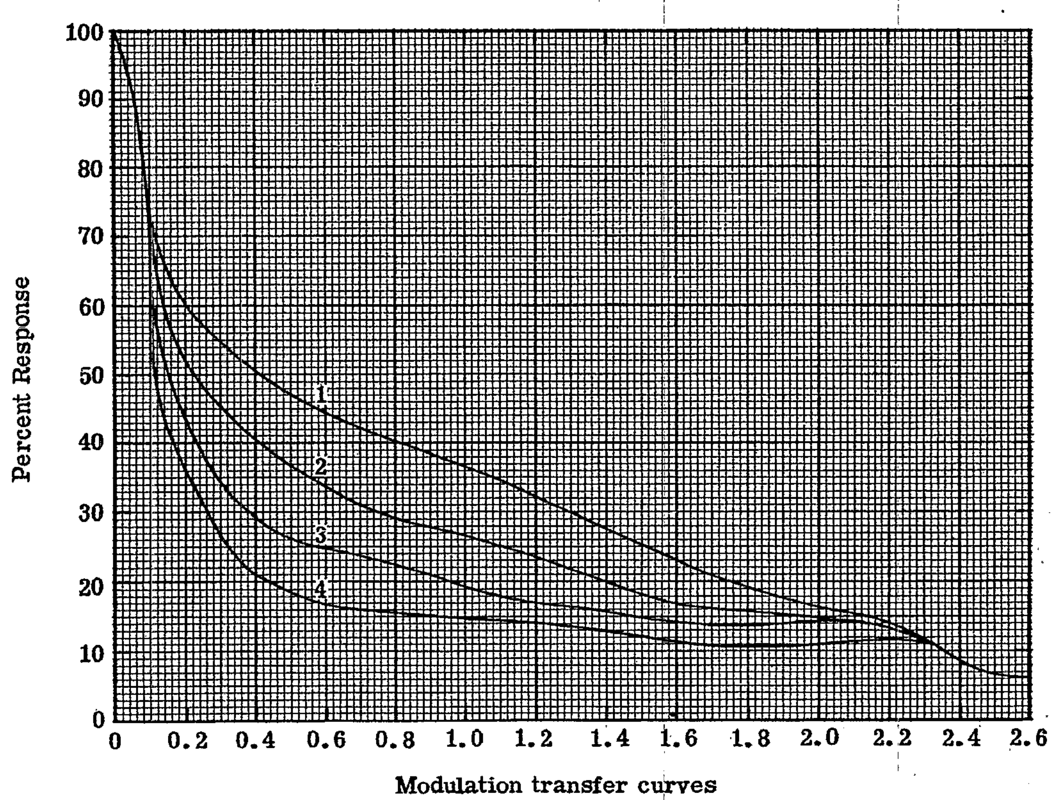

Figure 9.5 -

9.3 SUMMARY OF EQUATIONS USED IN THE CALCULATION OF THIRD ORDER ABERRATIONS

9.3.1 Paraxial ray trace equations.

$$y = y_{-1} + (t_{-1}/n_{-1})\ (n_{-1}\ u_{-1}) \qquad 5\text{-}(56)$$

$$nu = n_{-1}\ u_{-1} + y\ (n_{-1} - n)\ c \qquad 5\text{-}(57)$$

Alternate equations,

$$y = y_{-1} + t_{-1}\ u_{-1} \qquad 5\text{-}(56)$$

$$u = u_{-1} + i\left(\frac{n_{-1}}{n} - 1\right) \qquad 6\text{-}(3)$$

$$i = yc + u_{-1} \qquad 6\text{-}(4)$$

$$\Phi = \bar{y}\ (nu) - y\ (n\bar{u}) = \bar{y}\ (n_{-1}\ u_{-1}) - y\ (n_{-1}\ \bar{u}_{-1}) \qquad 6\text{-}(6)$$

9.3.2 Chromatic contribution formulae.

$$a = -yn_{-1}\ i\left(\frac{dn}{n} - \frac{dn_{-1}}{n_{-1}}\right) \qquad 6\text{-}(34)$$

$$b = -yn_{-1}\ \bar{i}\left(\frac{dn}{n} - \frac{dn_{-1}}{n_{-1}}\right) \qquad 6\text{-}(35)$$

9.3.3 Third order surface contributions.

$$S = yn_{-1}\left(\frac{n_{-1}}{n} - 1\right)(u + i) \qquad 8\text{-}(5)$$

$$\bar{S} = \bar{y}n_{-1}\left(\frac{n_{-1}}{n} - 1\right)(\bar{u} + \bar{i}) \qquad 8\text{-}(13)\ \text{footnote}$$

$$B = Si^2 \qquad 8\text{-}(4)$$

$$F = Si\ \bar{i} \qquad 8\text{-}(11)$$

$$C = S\bar{i}^2 \qquad 8\text{-}(12)$$

$$E = \bar{S}i\ \bar{i} + \Phi\ (\bar{u}_{-1}^2 - \bar{u}^2) \qquad 8\text{-}(13)$$

$$P = \frac{c\ (n_{-1} - n)}{n_{-1}\ n} \qquad 8\text{-}(14)$$

For an aspheric surface with a fourth order coefficient of e,

$$B = 8\ (n_{-1} - n)\ ey^4 \qquad 8\text{-}(4a)$$

$$F = B\bar{y}/y \qquad 8\text{-}(11a)$$

$$C = B\ (\bar{y}/y)^2 \qquad 8\text{-}(12a)$$

$$E = B\ (\bar{y}/y)^3 \qquad 8\text{-}(13a)$$

9.3.4 Stop shift equations.

$$\Sigma B^* = \Sigma B \qquad 8\text{-}(18)$$

$$\Sigma F^* = Q\ \Sigma B + \Sigma F \qquad 8\text{-}(19)$$

$$\Sigma C^* = Q^2\ \Sigma B + 2Q\Sigma F + \Sigma C \qquad 8\text{-}(20)$$

$$\Sigma E^* = Q^3\ \Sigma B + 3Q^2 \Sigma F + Q\Sigma(3C + P\Phi^2) + \Sigma E \qquad 8\text{-}(21)$$

$$\Sigma P^* = \Sigma P \qquad 8\text{-}(21a)$$

$$a^* = a \qquad 8\text{-}(22)$$

$$b^* = Qa + b \qquad 8\text{-}(23)$$