7 SIMPLE THIN LENS OPTICAL SYSTEMS

7.1 INTRODUCTION

7.1.1 Thin lens solution. The first two steps in designing optical systems, which will be discussed further in Section 9, are (1) selecting lens types for the various elements, and (2) finding a first order solution assuming thin lenses. The methods and procedures used in step (2) were developed in Section 6. In Section 7 the optics of several thin lens optical systems will be described to illustrate the usefulness of the paraxial equations and to indicate the reasoning a designer uses in following step (1). With information obtainable from only paraxial ray data, a designer can conclude many of the important features needed for a final design. There are numerous discussions in text books of simple optical systems such as the microscope and the telescope. The following discussion, assuming that the reader has read some explanation of these systems, will concentrate on the numerical analysis.

7.1.2 Optical systems used with the eye. The optical systems considered in this section are all used with the human eye. Because of this the eye is an integral part of the system and must be considered in the design. There are four basic types of lenses: (1) microscope objectives, (2) telescope objectives, (3) eyepieces and (4) photographic objectives. The first three are used with the human eye and systems employing these types are discussed in Section 7.

7.2 THE SIMPLE MAGNIFIER

7.2.1 A single lens. One of the simplest of optical devices is the simple magnifier. A single positive lens works as a magnifier because it makes an object appear to subtend a larger angle at an observer's eye than is possible with the unaided eye. Without a magnifier, an observer can make an object appear larger only by bringing it close to his eye. As an object is moved closer and closer to an observer's eye it is necessary for the eye to increase its refractive power in order to continue to focus the image on the retina. There is a minimum distance V at which the eye has increased its refractive power to its maximum capability. For object to eye distances less than V the image will no longer be sharply focussed on the retina. For the standard observer this distance V is approximately 10 inches or 250 mm (V is always considered positive). Therefore, in order to make the object appear still larger, it is necessary to add refractive power to the eye, so that the object may in effect be brought closer. The magnifier provides the extra refractive power required.

7.2.2 Magnifying power. The magnifying power of a visual instrument may be defined as

$$MP = \frac{\text{size of retinal image obtained with instrument}}{\text{size of retinal image obtained with the unaided eye}} .$$

In the region of the paraxial approximation this is equivalent to the definition, $MP = \beta/\alpha$ where β equals one half the angle subtended by the object as seen through the instrument, and α equals one half the angle subtended by the object as seen by the naked eye. (These particular definitions of α and β assume that the object and image are centered with respect to the optical axis. This is usually the case with visual instruments. According to this assumption, when reference is made to "an object $\bar{y}_o$," the object height is rigorously $2\bar{y}_o$.) β is called the half image field angle, and α is called the half object field angle. Magnifying power, then, is the ratio of the field angles.

7.2.3 Diagram of a single lens magnifier. In Figure 7.1 (a) an object $\bar{y}_o$ is shown viewed with the unaided eye. Figure 7.1 (b) shows the same object viewed by a single lens magnifier. The eye is placed at a distance d from the lens. The object is placed in relation to the lens so that the visual image $\bar{y}_k$ lies to the left of the eye at a distance, A. (A is negative). For the eye to focus on the image, A must be numerically equal to or larger than V. The numerical formulation of this problem may be handled with simplicity by the usual methods adequately covered in most text books. In the following analysis, it will be handled formally using the ray trace format in order to illustrate a method of analysis which can be used for any system, regardless of its complication.

7.2.4 Ray trace format.

7.2.4.1 The system consists of an object plane, an entrance pupil plane, a thin lens, an aperture stop and exit pupil, and a final image plane. Table 7.1 contains a layout of a computation sheet for this simple magnifier system. The data may be filled in as follows. First, all the ϕ values are zero except that of the lens. We also know that $y_o = 0$ and we may choose to trace a paraxial ray at any angle from the point $y_o = 0$.

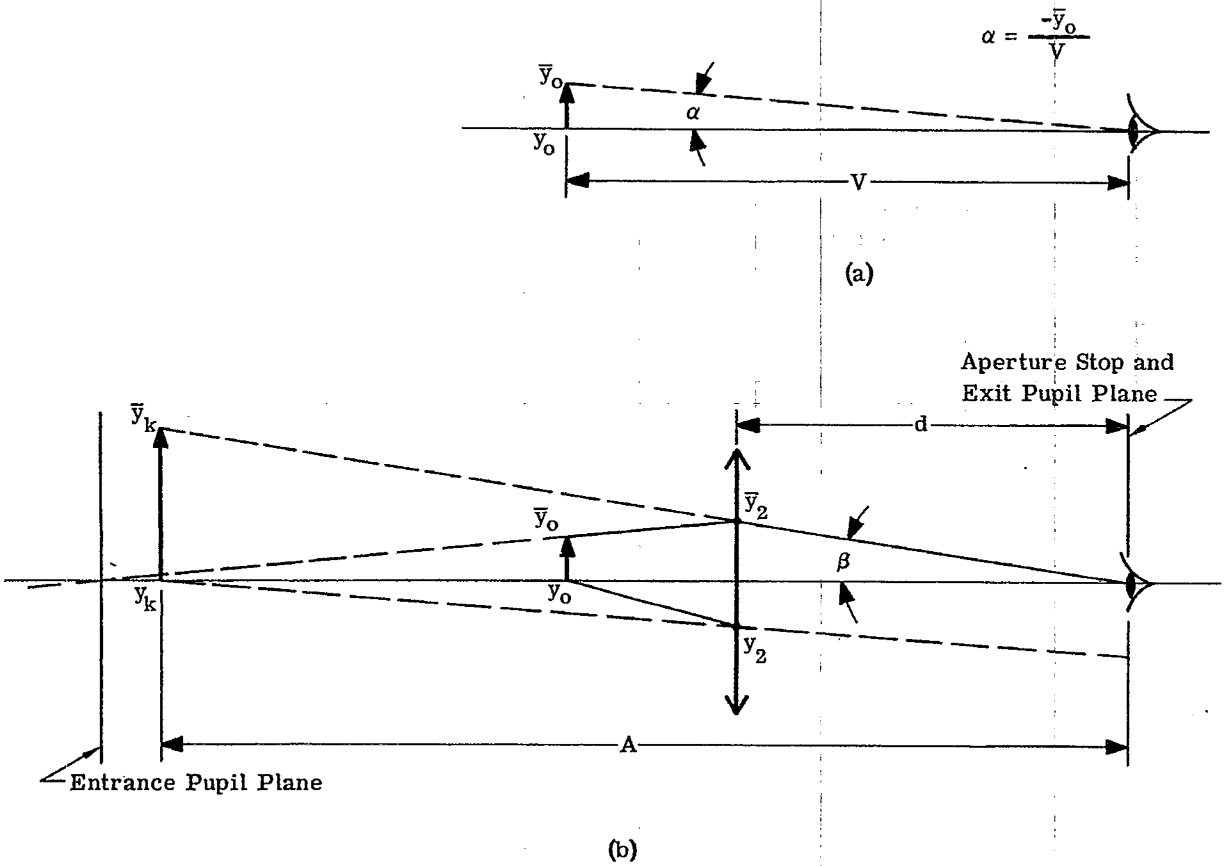

Figure 7.1 - Diagrammatic illustration of a single lens magnifier.

Surface	Object 0	Entrance Pupil Plane 1	Lens 2	Exit Pupil Plane 3	Image Plane 4
$-\phi$	0	0	$-\phi$	0	0
t	$-\bar{y}_o/\beta(1-d\phi)$	$-d/(1-d\phi)$	d	A	
y	0	$A/(1-d\phi)(d+A)$	1	$A/(d+A)$	0
u	$\left[\phi - \frac{1}{d+A}\right]$	$\left[\phi - \frac{1}{d+A}\right]$	$-1/(d+A)$	$-1/(d+A)$	
$\bar{y}$	$\bar{y}_o$	0	$-d\beta$	0	$\bar{y}_k$
$\bar{u}$	$\beta(1-d\phi)$	$\beta(1-d\phi)$	β	β	

Table 7.1 - Computation sheet for a simple magnifier

Therefore, we elect to make $y_2 = 1$. (Figure 7.1 (b) shows y_2 negative; this has been so done for pictorial clarity). The system has only two physical stops, the lens and the eye pupil. One of these is the aperture stop. To find which one, we can image each stop in the system preceding it, and see which image subtends the smallest angle at the base of the object, $y_o = 0$. The image of the lens in the lens is of magnification unity and is at the lens; hence if the size of the lens and its distance from the object is known, the angle subtended can be found. Likewise if the eye pupil size and location is known, its image in the lens can be determined, and the angle subtended by this image compared with the previous angle. We see, therefore, that which of the two stops is the aperture stop depends on the size and location of the two elements, i.e. on the design of the system. In such systems it is usual to assume that the lens is so much larger than the eye pupil that the latter is the aperture stop. Hence it is also the exit pupil. Therefore we can fill in d, the distance from the lens to the aperture stop plane, and A the distance from the aperture stop plane to the final virtual image plane. Since $y_k = 0$ and $u_2 = u_3$ (no refraction occurs at surface 3), it can be concluded that $u_2 = u_3 = -1/(d + A)$. With y_2 and u_2 known it is possible to calculate u_1 from equation 6-(24). Then $u_1 = \phi - 1/(d + A)$. Also $u_o = u_1$.

7.2.4.2 The oblique principal ray may now be traced backward through the system from the center of the exit pupil. Let this go back at the angle β with the optical axis. $\bar{y}_2$ is now determined as $-\beta d$. $\bar{u}_1$ and $\bar{u}_o$ are also determined. Knowing $\bar{y}_o$, $\bar{y}_1$, $\bar{y}_2$, $\bar{u}_o$ and $\bar{u}_1$, t_o and t_1 may be computed. Since all the spaces are now determined y and u are known on every surface for each ray.

From Equation 6-(7),

$$\bar{y}_k = \bar{y}_o \frac{u_o}{u_{k-1}} .$$

Therefore,

$$\bar{y}_k = \bar{y}_o \left[1 - \phi (d + A) \right] .$$

Since

$$\beta = \frac{\bar{y}_k}{A} \quad \text{and} \quad \alpha = -\frac{\bar{y}_o}{V} ,$$

$$MP = -\frac{V}{A} \left[1 - \phi (d + A) \right] . \qquad (1)$$

7.2.5 Analysis of magnifying power equation. There are several cases of special interest which should be noted.

a) If $A = -\infty$

$MP = V\phi$

b) If $d = f' = \frac{1}{\phi}$

$MP = V\phi$

c) If $A = -V$ with $d = 0$

$MP = 1 + V\phi$.

One can see by inspection of these equations, that $MP = V/f'$ is the minimum magnifying power and $MP = (V/f') + 1$ is the maximum. Hence for maximum magnifying power, the final image is at the near point of the eye, a distance V from the eye. Therefore the eye has maximum refractive power. For the relaxed eye, the image is at the far point; this is ∞ for the normal eye and results in a minimum magnifying power. The relative increase in magnifying power, as the eye accommodates for smaller A, is small and offset by the greater likelihood of eye strain. (For a typical magnifier of 1 inch focal length, the maximum magnifying power is only 10% higher than the minimum). Therefore simple magnifiers should be designed and used so that the final image is at infinity, or at the far point of the eye, if these cases differ.

7.3 THE MICROSCOPE

7.3.1 Limitations of a simple magnifier. It is clear from Section 7.2.5, that for large magnifying power, ϕ must be large, hence the focal length, f', must be small. Because the final image is to be at infinity, the object must be at F_1. Therefore for large magnifying power, the object must be placed very close to the lens. By Equation 6-(22) we see that the lens surfaces must have very short radii, and therefore the diameter of the lens will be small. Because it was assumed that the eye pupil was the aperture stop, for the case of the simple magnifier, the only other stop in the system, namely the lens, is the field stop. Whereas the aperture stop limits the bundle of rays traversing the system, the field stop (a physical stop) limits the field of view. Hence a small diameter magnifying lens means a small object field.

7.3.2 The simple microscope. A practical method of overcoming the limitations of the simple magnifier is to use a relay lens as shown in Figure 7.2. While the object is being relayed it may also be magnified. The magnifying power of the microscope is then the product of the lateral magnification of the objective and the magnifying power of the eyepiece. As with the magnifier, it is advisable to adjust the microscope so that the final virtual image is at ∞ ; the microscope is then in afocal adjustment. In this case the eyepiece magnifying power is $MP_e = V/f'_e$, and the magnifying power of the microscope may be written

$$MP = m_o \frac{V}{f'_e} \tag{2}$$

where m_o is the lateral magnification of the objective. The focal length of the objective can, in principle, have any value. If the focal length is made long, the overall length of the system will also be long.

7.3.3 Paraxial ray trace. Table 7.2 contains the paraxial ray trace for a microscope with an objective focal length of 16 mm and an eyepiece focal length of 25 mm. In order to use the objective lens symmetrically, i.e. in order that the chief ray pass through the center of the lens, the entrance pupil is placed in contact with the objective. The axial ray is traced from the object at an angle of 0.25. This corresponds to the sine of the angle of the actual ray to be traced through the system. This paraxial ray then passes through the optical system at very nearly the same heights as an actual true ray. As discussed in Section 23, the resolving power of a microscope depends on the wavelength and a quantity called the numerical aperture, or N.A. The numerical aperture $= n_o \sin U_o$. Since the object space has an index of $n_o = 1$, the system, as laid out, has a numerical aperture of 0.25. The chief ray was traced through the entrance pupil from an object height $\bar{y}_o = 1.0$. From this trace the exit pupil is found to lie 28.55 to the right of the lens (b).

7.3.4 Aperture stop and pupils. It is now possible to gather information about the pupils of this system from these paraxial rays. One can read directly from the table that the radius of the exit pupil is 0.625. Since the calculations are made in millimeters, the exit pupil is therefore 1.25 mm. In order that this exit pupil be the true exit pupil of the system, it is necessary to have the pupil of the observer's eye located fairly centrally in this exit pupil plane. Since the normal eye pupil is approximately 2 mm in diameter, the microscope exit pupil will definitely be the exit pupil of the entire microscope - eye system. The objective is the aperture stop and the entrance pupil of the system.

7.3.5 The f - number. The f - number of a lens (always considered positive) is defined as f/D where D is the diameter of the lens. It is very useful to calculate this quantity because from it one can estimate the difficulty of optically correcting the lens for image errors. Equation 6-(24) gives a relation between f and y for a thin lens. From this equation then, assuming $y = D/2$,

$$f\text{ - number} = 0.5/|\Delta u| .$$

In Table 7.3 the f - numbers for the objective and eyelens are listed for both the axial and oblique rays.

7.3.6 Difficulties in designing the eyepiece. The lenses should have sufficient diameter for the smallest f - number in each case. Therefore the objective should have an f - number of 1.82 and the eyepiece an f - number of 1.09. Figure 7.3 (a) shows a picture of an $f/1$ lens. This is an extremely fat lens and the chief ray would strike the curved surfaces at very large angles of incidence. The large angles of incidence introduce appreciable aberrations and the paraxial assumptions no longer hold. Hence the image at $\bar{y}_k$ would not be near the position predicted by first order theory. This lens is also uncorrected for color. Since correcting for color has the effect of approximately doubling the power of the positive element if the total power is to remain constant - because of the necessary addition of a negative element - one can see that it would be out of the question to color correct this lens. Therefore, it is clear that it will not be practical to use a single lens eyepiece. Either the size of the object will have to be reduced considerably, thereby reducing $\bar{u}_4$ and hence increasing the f - number, or several lenses will have to be used for the eyepiece.

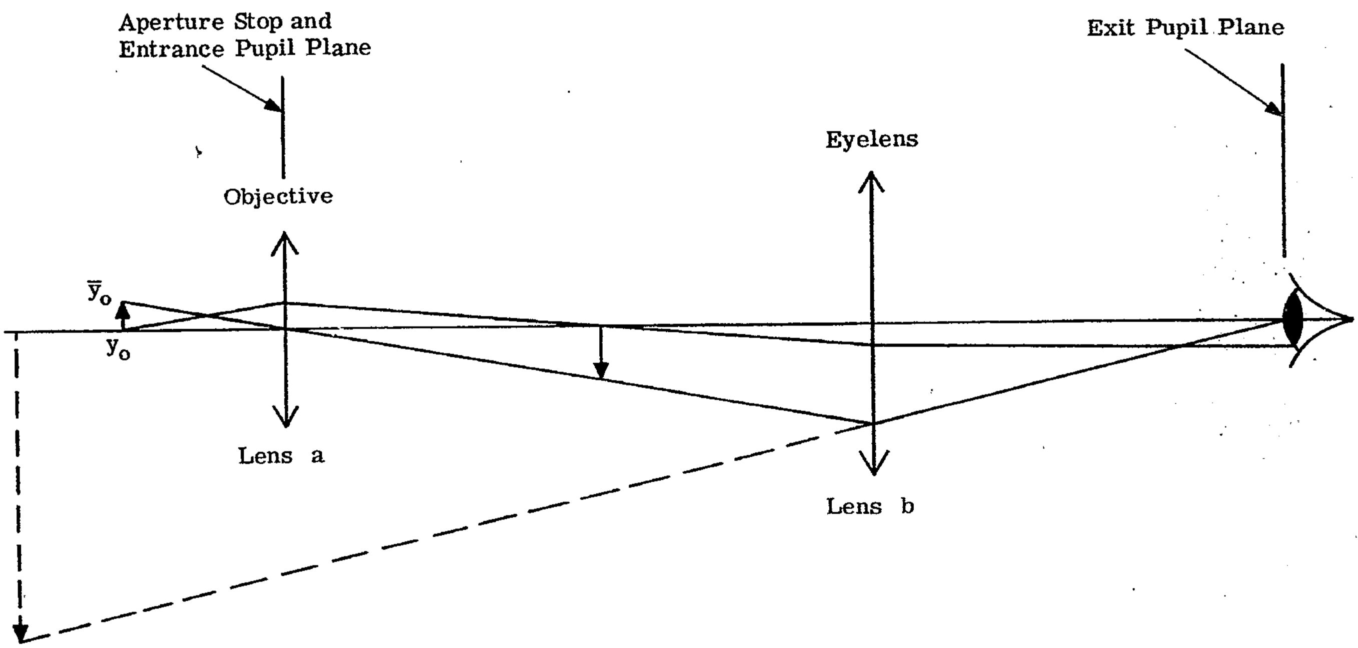

Figure 7.2 - The simple microscope. Lens (a) is the relay lens.

Surface	Object Plane 0	Entrance Pupil 1	Objective (a) 2	First Image Plane 3	Eyelens (b) 4	Exit Pupil Plane 5
$-\phi$	0	0	-0.0625	0	-0.04	
t	17.6	0	176.0	25.0	28.55	
y	0	4.4	4.4	0	-0.625	-0.625
u	0.25	0.25	-0.025	-0.025	0	
$\bar{y}$	1.0	0	0	-10.00	-11.42	0
$\bar{u}$	-0.0568	-0.0568	-0.0568	-0.0568	0.400	
ν_{F-C}	∞	∞	60	∞	60	
a	0	0	-0.02017	0	-0.00026	Σ a = -0.02043
b	0	0	0	0	-0.00476	Σ b = -0.00476
α TAch						= 0.0327
α Tch						= 0.0076

Table 7.2 - Calculations on a simple afocal microscope. All lengths are in mm.

	Axial Ray	Oblique Ray
Objective Lens	1.82	∞
Eyelens	20	1.09

Table 7.3 - f - numbers of lenses shown in Table 7.2.

7.3.7 Chromatic aberrations of a simple afocal microscope.

7.3.7.1 Before deciding which of these alternatives is preferable, consider the calculation for axial and lateral color for the system shown in Table 7.2. Since this is an afocal system, the axial beam emerges parallel to the axis, and $u_{k-1} = 0$. Under this condition, it is not possible to use Equations 6-(37) and 6-(38), for the color surface contributions. When $u_{k-1} = 0$ it is necessary to substitute the differential du from Paragraph 6.10.2.2 into Equation 6-(33) and obtain the following equations for the angular chromatic aberrations,

$$du_{k-1} = \alpha \text{ TAch} = \frac{1}{y_k \; n_{k-1}} \sum_{j=1}^{j=k-1} a \tag{3}$$

and

$$d\bar{u}_{k-1} = \alpha \text{ Tch} = \frac{1}{y_k \; n_{k-1}} \sum_{j=1}^{j=k-1} b \tag{4}$$

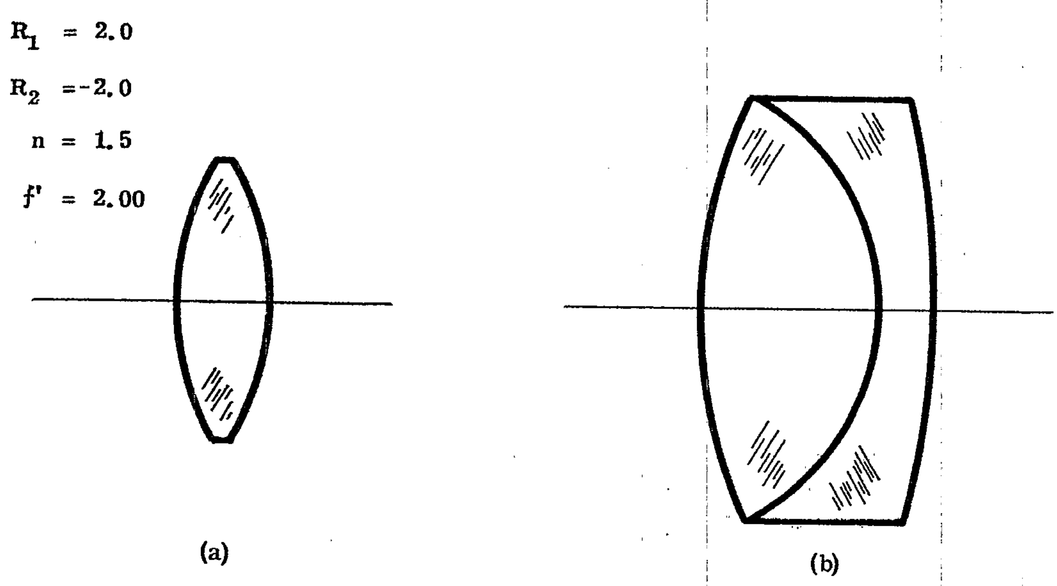

Figure 7.3 - (a) is an $f/1$ single lens; (b) is an $f/1.8$ achromatic doublet.

	Object Plane	Entrance Pupil Plane	Lens (a) Objective	Lens (c)	Lens (b)	Exit Pupil Plane
Surface	0	1	2	3	4	5
$-\phi$	0	0	-0.0625	-0.02662	-0.06662	0
t	17.6	0	151	30.02	8.571	
y	0	4.4	4.4	0.625	-0.625	-0.625
u	0.25	0.25	-0.025	-0.04164	0	
$\bar{y}$	1.0	0	0	-8.580	-3.428	0
$\bar{u}$	-0.0568	-0.0568	-0.0568	0.1716	0.400	
ν_{F-C}	∞	∞	60	60	60	
a	0	0	-0.02017	-0.00017	-0.00043	Σ a = -0.0208
b	0	0	0	0.00238	-0.00238	Σ b = 0
α TAch						0.0332
α Tch						0

Table 7.4 - Calculations on an afocal microscope with a double lens eyepiece. All lengths are in mm.

These equations are analogous to Equations 6-(37) and 6-(38). The results show that the simple microscope is afflicted with 0.0327 radians of axial color and 0.0076 radians of lateral color. Almost all the axial color is due to the objective. The lateral color is due entirely to the eyelens. The normal observer can detect as little as 0.0003 radians of color fringing assuming that the minimum angle of resolution is about 1' of arc. It is then clear that both the axial and lateral color exceed noticeable amounts of chromatic aberration.

7.3.7.2 The objective lens can be corrected for axial color by making it a doublet. Equations 6-(44) and 6-(45) are used to compute the powers of the separate components. Figure 7.3 (b) shows an $f' = 10$ objective with an f - number of 1.82. It turns out that these curves are again too sharp and the monochromatic aberrations will be difficult to correct. In order to correct the monochromatic aberrations then, it is necessary to flatten the surfaces by dividing the lens into two doublets, each working at $f/3.64$. To do this we divide the entire $|\Delta u| = 0.275$ into two equal parts, each of 0.1375. Each doublet will now work at the same f - number. This value of the f - number $(= 0.5/|\Delta u|)$ will not necessarily equal the value for an infinite object $(= f'/D)$.

7.3.7.3 The chromatic aberration in the eyelens can be corrected in the same way by splitting this lens into two lenses each working at $f/2.18$ and then by achromatizing each part. There is, however, another method which is sometimes used in eyepiece design. A single positive lens may be placed in front of the image plane 3 and adjusted to help refract the chief ray. For such a lens $\bar{y}$ and y will have opposite signs so according to Equation 6-(41) the lens should give a positive lateral color contribution. A lens such as this has been worked out in Table 7.4. The procedure for designing this system was as follows. The extra lens (c) was inserted to the left of the image plane at a position where $y_3 = 0.625$, the same value as the final height of the axial ray but of opposite sign. The chief ray was then traced to the (c) lens intersecting it at $\bar{y} = -8.580$.

7.3.7.4 Since this extra lens is to be used, it should help bend the chief ray. In Table 7.2 the chief ray was bent from -0.0568 to 0.400 by the (b) lens, a total bending of 0.4568. With the (c) lens added, the (b) and (c) lenses should each bend the chief ray by 0.2284. Therefore $\bar{u}_3$ between the (c) and (b) lenses should be 0.1716. This determines ϕ_3, the power of the (c) lens. With ϕ_3 known, u_3 is determined and then t_3 is set so that $y_4 = -0.625$. Thus ϕ_4 is defined. Now the lateral color contribution of a thin lens is proportional to $y\bar{y}\phi/\nu$. $\bar{y}\phi$ is equal to the bending $|\Delta\bar{u}|$ experienced by the chief ray. By making the (b) and (c) lenses refract the chief ray equally and by making $y_3 = -y_4$, the lateral color contributions of the (b) and (c) lenses exactly cancel each other since the ν-values are the same. The axial color of the system is only slightly more under-corrected than the original system in Table 7.2. This chromatic aberration can be eliminated completely by introducing over-correction in the objective lens (a).

7.3.8 Additional effects of adding a field lens.

7.3.8.1 Lens (c) is referred to as a field lens of the eyepiece. (The introduction of a lens near the position of the image due to the objective increases the field of view.) This extra lens (c) has helped the system significantly. The (b) and (c) lenses are $f/2.18$ now and are far more reasonable lenses. There is at this point sufficient reason to expect that this microscope could be corrected to give good imagery. In Section 8 it will be shown that the monochromatic aberrations at an object height of $y_o = 1$ are rather large, so that the final optical design will probably have to have a smaller object field.

7.3.8.2 It should also be noted that the introduction of the (c) lens caused a marked reduction (by a factor of 3) in the distance between the eyelens and the exit pupil. In Table 7.4 this distance is only 8.57 millimeters. This distance, called the eye relief, is too short for comfortable viewing, so some other arrangement of lenses should be found. Without introducing a serious amount of lateral color the (c) lens could be designed with less power. The chief ray would then strike the (b) lens at a larger aperture resulting in an increased distance to the exit pupil. With an eyepiece of this type, the lateral and axial color for the object is fully corrected. However the eyepiece is not color corrected for the plane between the two lenses (b) and (c) where an intermediate image is formed. If cross hairs, or a reticle, is placed in this position it is in effect viewed only by the single eyelens. The reticle will be imaged with lateral color since the single lens is not achromatized. If a reticle is to be used, it is advisable to use an eyepiece that is also color corrected for the intermediate image plane.

7.4 THE TELESCOPE

7.4.1 General. A telescope may be considered as a special case of the microscope, with this slight difference. In the microscope, one compares the visual angle subtended by the image, as viewed through the instrument, with the visual angle subtended by the object at the unaided eye. It is assumed that the observer can place the object at the distance V from the eye. In the telescope it is assumed that the object is inaccessible to the observer. Therefore, in a telescope one compares the visual angles, assuming the observer is always at a fixed distance with respect to the object. This is illustrated in Figure 7.4.

7.4.2 Magnifying power. An object of height $\bar{y}_o$ is located a distance L from an observer. (L is always considered to be positive.) The angle α subtended by the object is $-\bar{y}_o/L$. With the instrument in place, the object is at a distance of z from the first focal point of the objective. A ray from the top of the object, $\bar{y}_o$, passing through the first focal point of the objective strikes the objective at a height $\bar{y}_1 = -\bar{y}_o f_o/z$. This ray then is parallel to the optical axis until it strikes the eyepiece. It then refracts to the second focal point of the eyepiece at an angle with the axis of $\beta = (y_o/z)(f_o/f'_e)$. If the eyepiece is adjusted so that $A = \infty$, or if the eye is located at the second focal point of the eyelens, then β is the apparent angle subtended by the object. Then,

$$MP = -\frac{f_o}{f'_e}\frac{L}{z} = m_o \frac{L}{f'_e} . \tag{5}$$

This equation actually applies to the microscope by making $L = V$. If L becomes very large as it does for most applications in which telescopes are used, then L/z approaches 1 and, $MP = -f_o/f'_e$. This is the formula usually given for a telescope. At a value of L where L/z is not unity, the MP is increased. Thus it is possible to obtain a magnifying power greater than unity even if $f_o = f'_e$. This makes an interesting optical device. It has unit MP for objects at infinity but greater than unit MP for objects at finite distances.

7.4.3 Objective and eyepiece design. The optics of the objective and eyepiece for the telescope are similar to that of the microscope. The entrance pupil is usually placed at the objective. The eyepiece is usually split into two or more lenses in order to correct for the lateral color. The extra lenses also allow for a wider field of view than one could achieve with a single lens.

7.5 OPTICAL RELAY SYSTEMS. PERISCOPES

7.5.1 Image orientation.

7.5.1.1 In the case of the afocal magnifier, the expression for the MP is V/f'. Since V is always con-

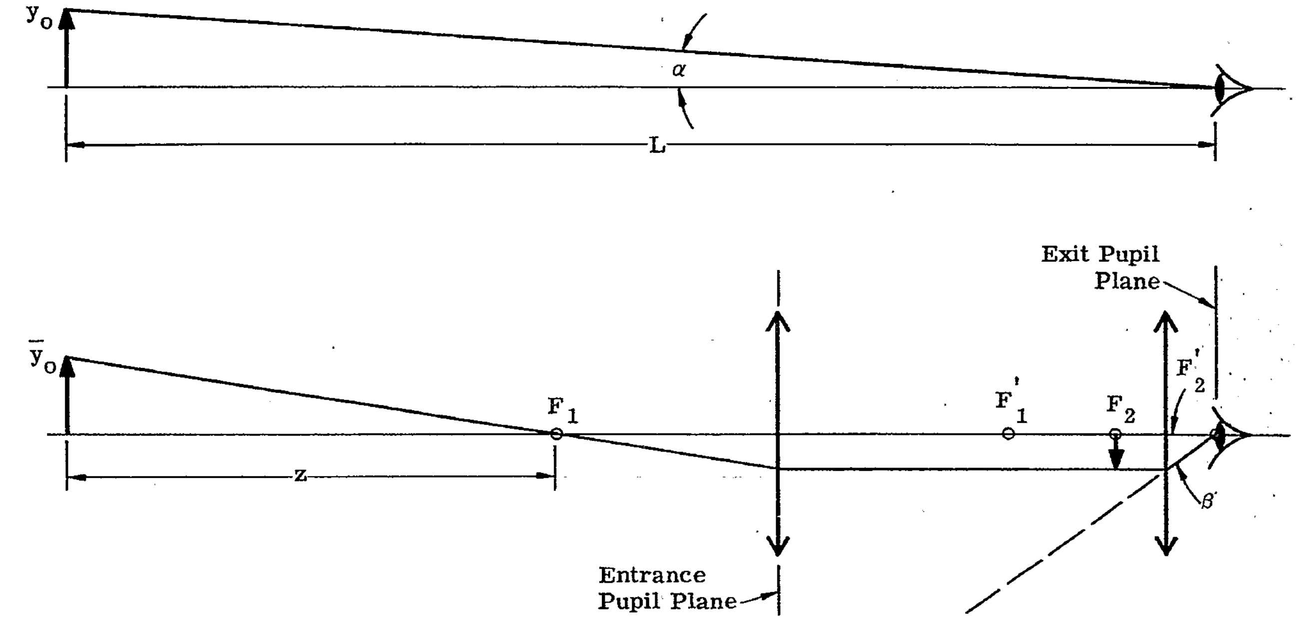

Figure 7.4. The optics of a simple telescope

sidered to be a positive number, the formula indicates that the MP is positive for a positive lens. Positive MP means that the virtual image is in the same orientation as the object.

7.5.1.2 Negative MP or negative magnification means that the image $\bar{y}_k$ is the negative of the object $\bar{y}_o$, in other words the object is inverted; if the optical system is a centered spherical optical system $\bar{x}_k$ will be the negative of $\bar{x}_o$. This means that the object $\bar{y}_o$ $\bar{x}_o$ appears as $\bar{x}_k$ $\bar{y}_k$. Or a letter R will appear as ꓤ. The image is said to be inverted but right handed. This means it is upside down but readable. It appears as a normal R by turning the paper through 180° in its plane.

7.5.1.3 An erect, left-handed image, such as occurs in plane mirrors, would appear as Я. All left-handed images, whether erect or inverted, are unreadable by rotation in the plane only. Left-handed images are sometimes referred to as perverted images. Also see Section 13.

7.5.2 Image inversion for microscopes and telescopes. From Equations (2) and (5) it is seen that a simple microscope and telescope give a negative MP because m_o for an objective is negative. Therefore, these instruments provide a right-handed inverted image. In the microscope it seldom matters if the object is inverted, but in telescopes it is very disturbing to see turned upside down, objects which we are used to seeing erect. Therefore, for telescopes, some means for erecting the image is usually provided. This can be done using prisms or extra lenses. The use of prisms will be described in Section 13. A brief analysis of methods of image erection by lenses will be discussed in the next paragraph.

7.5.3 Image erection by lenses. It is possible to use a second objective in a microscope or telescope to re-image the first image before it is viewed by the eyepiece. This is illustrated in Figure 7.5. The magnifying power for such a system is given by the expression, $MP = m_1 \ m_2 \ L/f'_e$. This procedure can, of course, be carried on with several re-imaging stages if it is desirable to have a long system as in periscopic designs. Since L/f'_e is positive, and each m due to the relaying objectives is negative, it is clear then that if there are an odd number of real images, the MP is negative, while for an even number of real images the MP is positive. A positive overall MP means the image is erect and right-handed.

7.5.4 Field lenses for periscopes. Inspection of Figure 7.5 shows that the size of the object which may be seen through the instrument is definitely limited by the size and permissible f - number of the second

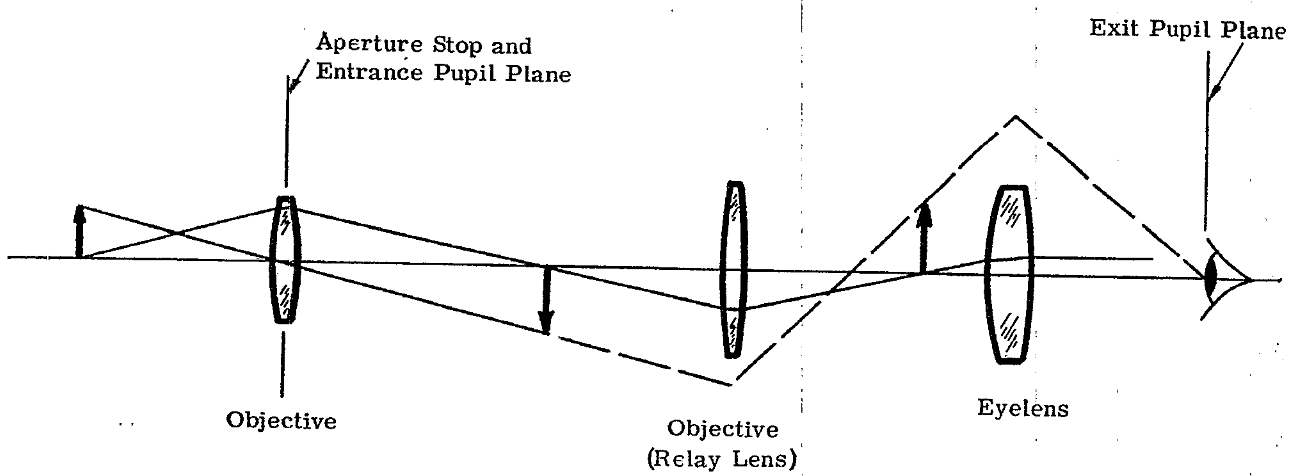

Figure 7.5 - An optical relay system or periscope.

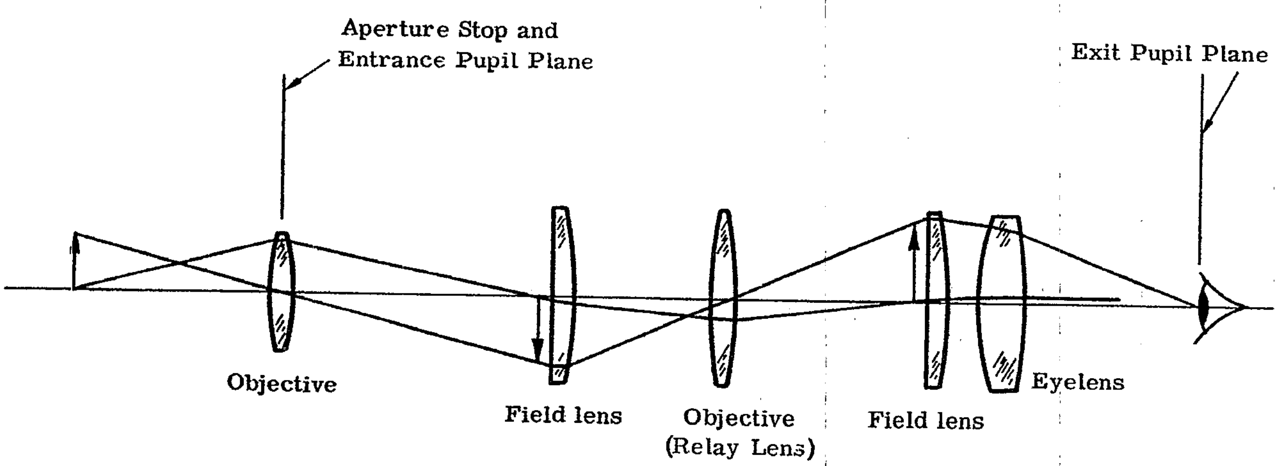

Figure 7.6 - A periscope with field lenses.

objective. The field of view can be increased significantly by introducing extra lenses to help refract the chief ray. This is the same situation described with the eyepiece in Table 7.4. Extra lenses can be introduced at the position of the objective or, if it is desirable to keep the diameter of the system small, the extra lenses can be added near the intermediate images. If the lenses are added near the intermediate images, and therefore referred to as field lenses, they act principally on the chief rays and their primary purpose is to help increase the field of view. Figure 7.6 shows an erecting telescope using field lenses to help increase the field of view.

7.5.5 Position of the aperture stop. The drawing (Figure 7.6) indicates that the first objective is the aperture stop. As drawn, the second objective (relay lens) is larger in diameter than necessary. It could be reduced until the axial ray passed through the margin of the lens, as it does at the first objective. If the diameter of the relay lens were further reduced, this lens would become the aperture stop and the first objective would be too large. In practice the diameters are adjusted so that both objectives are aperture stops.

7.6 THE GALILEAN TELESCOPE

7.6.1 Use of negative eyepiece. In the telescope with $f'_o \geqq |f_e|$ it is possible to have a positive or negative focal length eyepiece. If the eyepiece focal length is negative, Equation (5) shows that the MP is positive. The image would therefore be erect. Such a system has very interesting possibilities. A sketch of a telescope of this type is shown in Figure 7.7.

7.6.2 Analysis of the simple Galilean telescope.

7.6.2.1 The exit pupil and aperture stop of this system is usually the pupil of the eye. The entrance pupil is actually located behind the observer's eye, and the size of the objective determines the size of the field of view. The objective is therefore the field stop. A system of this type is worked out in Table 7.5. The table shows the sizes and positions of the entrance and exit pupils. The object field of view (sometimes called the real field) $-\beta\phi_o/\phi_e$ depends on β. In order to obtain an image field of view (sometimes called the apparent field) of β, the $\bar{y}_2$ on the objective lens must be,

$$\bar{y}_2 = \beta \left[MPd - t_2 \right] .$$

Therefore, for a given diameter of objective lens, the field of view is determined.

7.6.2.2 For the case of $d = 0$, we have $\bar{y}_2 = -\beta t_2$. Since $f'_o/2\bar{y}_2 = (f\text{ - number})_o$ at which the objective is working for the chief ray,

$$\beta = -\frac{f'_o}{2t_2\,(f\text{ - number})_o}$$

and

$$\alpha = \frac{\beta}{MP} = -\frac{f'_o}{2\,MP\,t_2\,(f\text{ - number})_o} .$$

For MP large compared to unity, the focal length of the objective is large compared to the focal length of the eyepiece. Assuming that $f'_o + f_e = t_2$ can be replaced by f'_o, we have

$$\beta = \frac{1}{2\,(f\text{ - number})_o}$$

and

$$\alpha = \frac{1}{MP\,2\,(f\text{ - number})_o} .$$

These equations show that for a large MP the field of view can be made large only by decreasing $(f\text{ - number})_o$. For example, if $MP = 10$, then $\alpha = 0.05$ radian if the objective is $f/1$. An $f/1$ lens is very difficult to make. The usual doublet achromat would have only an $f/3$ aperture. For such a doublet objective $\alpha = 0.017$ radian or 0.95°. Then $\beta = 9.5°$, which is a very small apparent field of view.

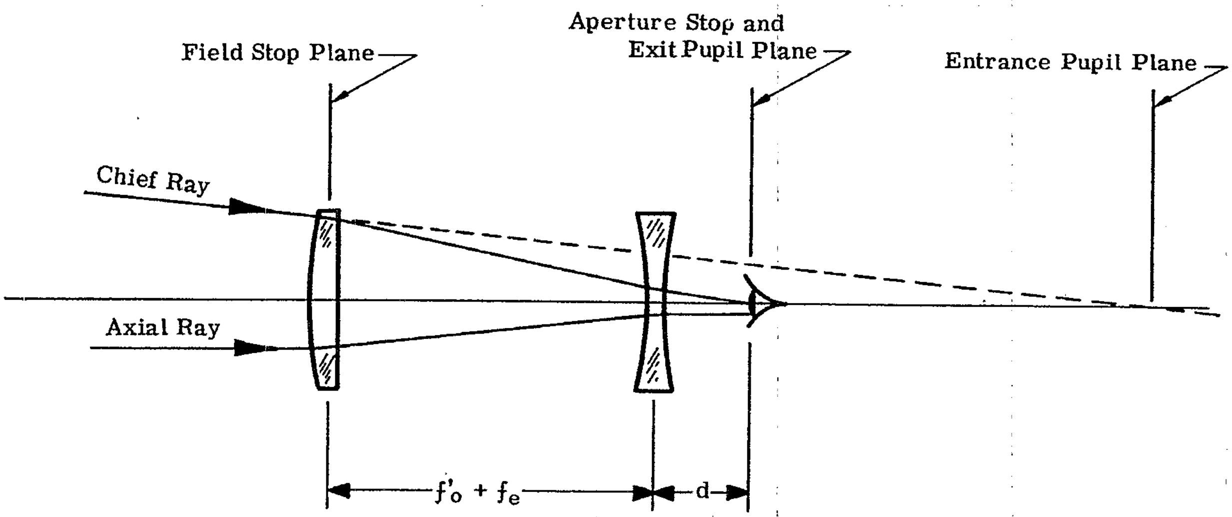

Figure 7.7 - The Galilean telescope

Surface	Object 0		Entrance Pupil 1		Objective 2		Eyepiece 3		Exit Pupil 4
$-\phi$	0		0		$-\phi_o$		$-\phi_e$		0
t		∞		$-MP\left[dMP - t_2\right]$		$(f'_o + f_e)$		d	
y	0		$-r\,\frac{\phi_e}{\phi_o}$		$-r\,\frac{\phi_e}{\phi_o}$		r		r
u		0		0		$r\phi_e$		0	
$\bar{y}$	∞		0		$\beta\left[dMP - t_2\right]$		$-d\beta$		0
$\bar{u}$		$-\beta\,\frac{\phi_o}{\phi_e}$		$-\beta\,\frac{\phi_o}{\phi_e}$		$\beta - \phi_e d\beta$		β	

Table 7.5 - Calculations for a Galilean telescope.