2 FUNDAMENTALS OF GEOMETRICAL OPTICS[1]

2.1 GENERAL

2.1.1 Geometrical optics. The term geometrical optics is applied to that branch of physics which deals with the propagation of light in terms of rays. These rays are considered as straight lines in homogeneous media. Geometrical optics, however, does not include some of the wave aspects of light propagation and hence does not take into account interference or diffraction effects. It is the starting point of the design of all optical systems; often it is the end point. It offers a means of progressing from graphical representations to numerical methods of analysis, and of arriving at solutions which in most cases are sufficiently accurate. One purpose of this text is to describe the laws and principles of geometrical optics and to show their application to the design of optical elements and systems.

2.1.2 Wave surfaces and rays. A basic problem in the design of optical systems is the calculation of wave surfaces as they progress through the various optical media. In geometrical optics this calculation is approximated by considering a relatively small number of rays, and then tracing these rays through the system. The actual passage of the rays is computed using analytic geometry procedures and two simple laws, the law of reflection and the law of refraction.

2.1.3 Direction of rays. The rays are perpendicular to the wave surfaces if the radiation is passing through a medium which is optically isotropic. The position of a wave surface (often called a wavefront) with respect to a point source may be determined at any time by the following procedure. From the point source equal optical path lengths are laid off along the rays. The surface that passes through these end points and is normal to the rays is a wavefront. (The optical path length corresponding to a physical path length is the product of the physical path length and the index of refraction.) In birefringent material the ray directions are not necessarily normal to the wave surfaces. The path of a ray of light traveling in a homogeneous medium is a straight line. When the ray is incident upon a surface separating two optically different media, it is reflected and refracted. This usually results in an abrupt change in the direction of the ray.

2.1.4 Angles of incidence, reflection, and refraction. If a normal is erected to the surface separating two media at the point where the ray is incident, the angles which the normal makes with the incident, refracted, and reflected rays are termed, respectively, the angles of incidence, refraction, and reflection. The laws of refraction and reflection, which state the relations existing between these angles, are two of the fundamental laws upon which optical design is based. The third law, mentioned above, states that a ray in a homogeneous medium travels in a straight line.

2.2 THE LAW OF REFRACTION

2.2.1 Diagram for refraction. Figure 2.1 shows a ray of light refracted at an interface between two different homogeneous materials characterized by n_0 and n_1, which are the respective indices of refraction of of the materials. The interface is shown as a straight line representing the intersection of a plane surface with the plane of the paper. This is a special case of the general situation in which the interface is a curved surface. In addition to the refracted ray, shown in Figure 2.1, in general there will also be a reflected ray. This has been omitted in the figure only for the purpose of clarification. For most cases where refraction is the aim, the reflected rays account for less than 10% of the incident energy. Section 21.2 will discuss the calculation of the reflected energy.

2.2.2 Sign convention. The following sign convention will be used for the angles of incidence, refraction, and reflection. If the ray must be rotated clockwise through the acute angle to bring it into coincidence with the normal to the surface, the angle is called positive. The angles I and I' in Figure 2.1 are both positive.

2.2.3 Statement of the law of refraction. The law of refraction is stated in two parts:

(1) The incident ray, the refracted ray, and the normal to the surface all lie in a single plane.

(2) The sines of the angles of incidence and refraction are related by the equation

$$n_0 \sin I = n_1 \sin I' . \tag{1}$$

2.2.4 Vector form of the law of refraction.

2.2.4.1 In solving many three dimensional refraction problems it is convenient to express the law of refraction in vector form. This is accomplished by describing the incident ray direction by a vector of unit

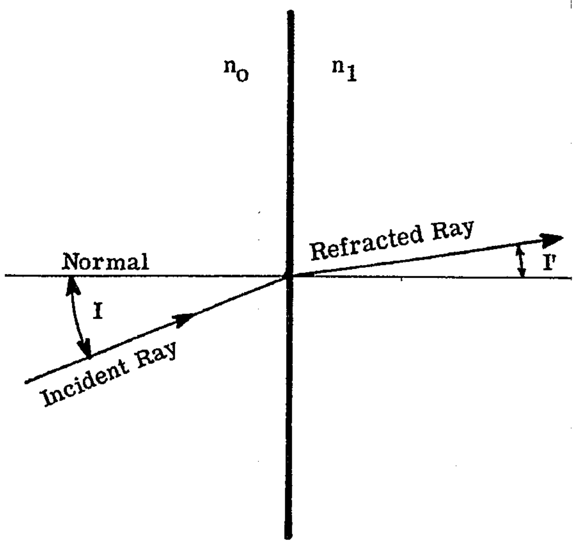

Figure 2.1 - Illustration of refraction.

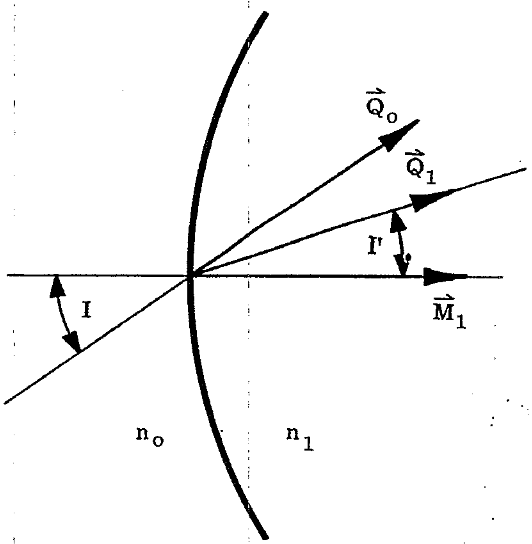

Figure 2.2 - Unit vectors for ray directions.

length $\vec{Q}_o$, the refracted ray by a unit vector $\vec{Q}_1$, and the normal by a unit vector $\vec{M}_1$. Figure 2.2 shows the relationship between these unit vectors and the ray directions. The vector $\vec{M}_1$ lies along the normal in the direction incident medium to refractive medium.

2.2.4.2 The vector product (cross product) of the two vectors $\vec{Q}_o$ and $\vec{M}_1$ is a vector of magnitude

$$\vec{Q}_o \times \vec{M}_1 = |Q_o|\,|M_1| \sin I = \sin I$$

because the angle between these vectors is I and they are each of unit length. The vector whose magnitude is sin I is perpendicular to the plane containing angle I (the plane of Figure 2.2), and directed perpendicularly into the plane of the paper. Similarly, $\vec{Q}_1 \times \vec{M}_1 = \sin I'$, and this is a vector parallel to $\vec{Q}_o \times \vec{M}_1$, because the refracted ray lies in the plane determined by the normal and the incident ray.

2.2.4.3 We have established the parallelism of the two vectors whose magnitudes are sin I and sin I' . By Equation (1) their magnitudes are in the ratio of the indices. Hence

$$\frac{\sin I}{\sin I'} = \frac{\vec{Q}_o \times \vec{M}_1}{\vec{Q}_1 \times \vec{M}_1} = \frac{n_1}{n_o},$$

and the vector form of the law of refraction may be written as

$$n_o(\vec{Q}_o \times \vec{M}_1) = n_1(\vec{Q}_1 \times \vec{M}_1). \qquad (2)$$

Equation (2) indicates, as all vector equations do, that the vector given by the left hand side equals in magnitude and direction the vector given by the right hand side.

2.2.4.4 Equation (2) can be written in another form by absorbing the scalar quantities n_o and n_1. Replacing the two vectors $n_o\vec{Q}_o$ and $n_1\vec{Q}_1$ by $\vec{S}_o$ and $\vec{S}_1$, respectively, we have

$$\vec{S}_o \times \vec{M}_1 = \vec{S}_1 \times \vec{M}_1,$$

and

$$(\vec{S}_1 - \vec{S}_o) \times \vec{M}_1 = 0.$$

Since neither $\vec{M}_1$ nor $(\vec{S}_1 - \vec{S}_0)$ is zero, these two vectors must be parallel or anti-parallel. Therefore we can define a quantity Γ (sometimes called the astigmatic constant) by writing

$$\vec{S}_1 - \vec{S}_0 = \Gamma \vec{M}_1 . \tag{3}$$

2.2.4.5 Having found the direction of $(\vec{S}_1 - \vec{S}_0)$, we now want to determine its magnitude, Γ. From the definitions of $\vec{S}_0$ and $\vec{S}_1$, and because $\vec{Q}_0$ and $\vec{Q}_1$ are unit length, $\vec{S}_0$ and $\vec{S}_1$ are two vectors of length n_0 and n_1, in the directions of the incident and refracted rays respectively. The difference, $\vec{S}_1 - \vec{S}_0$, between these vectors is indicated in Figure 2.3. The length of $\vec{S}_1 - \vec{S}_0$ is the difference between the projections of $\vec{S}_1$ and $\vec{S}_0$ on $\vec{M}_1$. For the case illustrated, $n_1 > n_0$ and therefore $\cos I' > \cos I$. Hence, since Γ is a positive number for Figure 2.3,

$$\Gamma = n_1 \cos I' - n_0 \cos I = -n_0 \cos I + n_1 \left[\left(\frac{n_0}{n_1} \cos I \right)^2 - \left(\frac{n_0}{n_1} \right)^2 + 1 \right]^{1/2} . \tag{4}$$

Equations (3) and (4) are used in the derivation of the skew ray formulae included in Section 5.

2.3 THE LAW OF REFLECTION

2.3.1 Diagram for reflection. Figure 2.4 shows a ray reflected from a surface. Just as in Figure 2.1, the interface is shown as a straight line, although in general it is a curve. Generally, there will also be a refracted ray which is more or less absorbed as it traverses the medium to the right of the interface. For clarity, only the incident and reflected rays are shown. The calculation of the refracted energy is discussed in Section 21.2.

2.3.2 Statement of the law of reflection. The law of reflection is also stated in two parts:

(1) The incident ray, the reflected ray, and the normal to the surface all lie in the same plane.

(2) The angle of incidence is numerically equal to the angle of reflection.

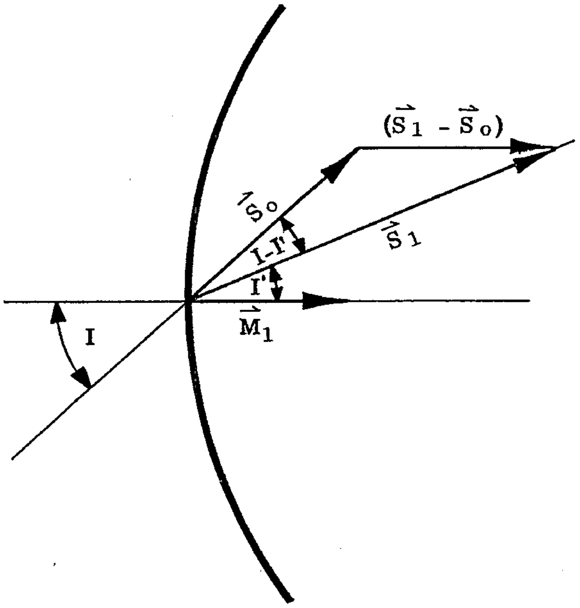

Figure 2.3 – Relation between $\vec{S}_0$, $\vec{S}_1$, and their difference.

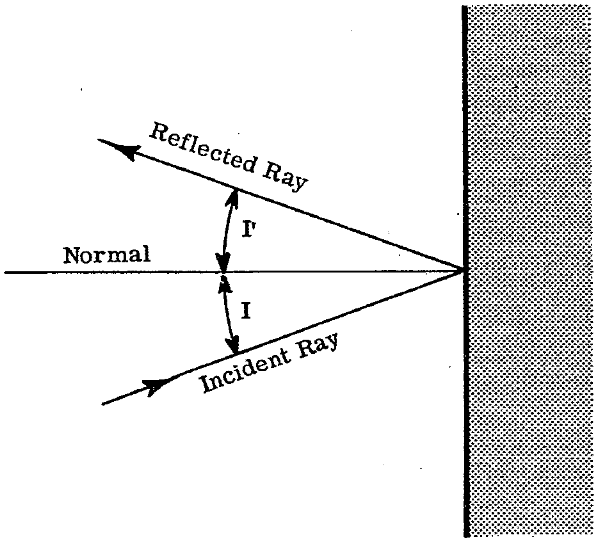

Figure 2.4 - Illustration of reflection.

Note that if I' is labelled as shown in Figure 2.4 , then I' is negative while I is positive according to the sign convention. The law of reflection then is

$$I = -I' . \tag{5}$$

2.3.3 Unification of the laws of reflection and refraction. A very convenient way to unify the laws of reflection and refraction is to use the single equation (1) for the law of refraction and to say that in the case of reflection

$$n_1 = -n_o . \tag{6}$$

With this convention, Equation (1) leads directly to Equation (5) . This convention will be used later to provide a completely unified treatment of reflection and refraction problems.

2.4 TOTAL INTERNAL REFLECTION

2.4.1 The critical angle. An inspection of Equation (1) shows that if $n_1 < n_o$, and I' is 90°, the angle of incidence then would be given by

$$\sin I_C = \frac{n_1}{n_o} , \tag{7}$$

where I_C is called the critical angle. If the angle of incidence exceeds the critical angle, the reflected ray has associated with it all the incident energy, as though the interface were a perfect mirror. This effect is used to an advantage in the design of prism systems to obtain reflectivity with very little loss of energy. (See Section 13).

2.4.2 Table of critical angles and indices. Table 2.1 lists the critical angle* corresponding to various indices of refraction. These data are useful in the design of prism systems, where it is necessary to be sure that the prism totally reflects all the desired rays.

n	I_c (radians)	n	I_c (radians)	n	I_c (radians)
1.50	0.729728	1.57	0.690526	1.64	0.655753
1.51	0.723820	1.58	0.685308	1.65	0.651099
1.52	0.718020	1.59	0.680177	1.66	0.646517
1.53	0.712324	1.60	0.675132	1.67	0.642005
1.54	0.706730	1.61	0.670168	1.68	0.637562
1.55	0.701234	1.62	0.665286	1.69	0.633186
1.56	0.695834	1.63	0.660481	1.70	0.628875

Table 2.1 - Table of critical angles (n vs I_c).

2.5 INDEX OF REFRACTION

2.5.1 Absolute index of refraction. It is appropriate at this time to discuss the meaning of index of refraction, referred to as n . The absolute refractive index of a material is defined as the ratio of the velocity of light in a vacuum to that in the material,

$$n_o = \frac{v_{vac}}{v_o} . \tag{8}$$

2.5.2 Relative index of refraction. In practice the absolute index of refraction is never directly measured. Instead the velocity in the material is compared to the velocity in air. From this comparison the relative index of refraction can be determined. The relative index of one material with respect to another is equal to the ratio of the absolute indices. For example, the relative index of a substance with respect to air is

$$(n_o)_{rel} = \frac{n_o}{n_{air}} = \frac{v_{vac}/v_o}{v_{vac}/v_{air}} = \frac{v_{air}}{v_o} .$$

---

* As indicated here the angle is expressed in radians. In the future, if an angle is given in radians, the word "radian" will be omitted; if the angle is given in degrees, the degree sign (°) will be used.

Equation (1) , which is the basic equation applying to a ray as it traverses a boundary, can be applied without knowing the absolute indices n_0 and n_1. Only the relative index, n_1/n_0, is needed. Hence all refraction problems involve only a ratio of two indices and it is not necessary to know the absolute index of optical materials. Therefore, unless specifically stated, the indices of refraction of optical materials relative to air are used, and it is these relative indices which are measured. (See Section 25.7.3) . In problems involving vacuum the absolute index of refraction of air must be used to calculate the absolute index of the material.

2.5.3 Table of refractive indices. The index of refraction of several optical materials is shown in Table 2.2. Except for silicon, where the index applies to the infrared, the indices are for the visible spectrum. Detailed refractive index data on optical glasses are available in catalogs from glass manufacturers. (See paragraph 2.7.9) . Materials other than glass are available and are used for optical elements. Refractive index and other data on these materials are discussed in Section 17 . It should be noted that the indices given in Table 2.2 , as well as in other references, are not only functions of wavelength, which is discussed in Section 2.6 , but are also functions of temperature and pressure. The pressure dependence becomes of major importance in the case of gases; sometimes a particular gas at relatively high pressure is used to enclose part or all of an optical system.

Material	n
Vacuum	1.
Air	1.0003
Water	1.33
Fused quartz	1.46
Borosilicate crown glass	1.51
Ordinary crown glass	1.52
Canada balsam	1.53
Light flint	1.57
Dense barium crown	1.62
Extra dense flint	1.72
Silicon (in the infrared)	3.4

Table 2.2 - Refractive indices of various materials.

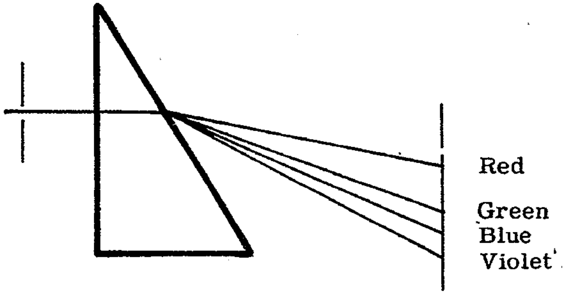

Figure 2.5 - Beam of white light passing through a dispersing prism.

2.6 DISPERSION OF LIGHT

2.6.1 General. It was shown by Newton that white light is not to be considered as a fundamental type, but is rather a composite mixture which can be separated into a range of colors, - that is a spectrum - , by passage through a prism as shown schematically in Figure 2.5 . According to the wave theory of light, each color corresponds to a definite frequency of vibration or, when the light is traveling in a vacuum, to a definite wavelength (λ). The shorter waves correspond to the violet end of the spectrum; the longer, to the red. Further investigation has shown that the radiation spectrum extends to longer wavelengths beyond the red, the infrared (IR) region, and to shorter waves beyond the violet, the ultraviolet (UV) region.

2.6.2 Variation of index with wavelength.

2.6.2.1 Since index is inversely proportional to the velocity of light in a given medium, and since this velocity is not constant for all colors, the index is a function of the color of the light. The color may be specified either by stating the frequency or wavelength in vacuum; hence, the index may be considered a function of either frequency or wavelength. Which functional dependence is used depends on the specific problem involved. In geometrical optics, since spectrum lines are used to measure indices, and since these lines are indicated by wavelength (instead of frequency), it is customary to use the functional dependence on wavelength.

2.6.2.2 For a given refracting medium, the absolute refractive index takes on a different value for each wavelength. In all practical cases it is higher for short wavelengths, and lower for long ones. Thus in Figure 2.5 a ray of composite light is incident normally on the first surface. Since the angle of incidence on this surface is zero, the angle of refraction is also zero and the ray is undeviated. At the second surface, however, the light is deviated, the blue ray being bent more than the red. This unequal refraction is called dispersion. The variation of index with wavelength, for most optical materials in the wavelength region where they are used, is such that the index decreases as the wavelength increases. The index varies approximately linearly with $1/\lambda^2$ where λ is the wavelength of the radiation.

2.6.3 Fraunhofer lines. In optical design work, the indices of refraction of the media to be used must be known in the wavelength region in which the device is to be used. (Methods of measuring index will be discussed in Section 24.6). Within the region, the choice of wavelengths at which measurements are made depends partly on convenience in measurement, and partly on custom. The section on glass characteristics applies generally to the visible region. Similar considerations apply to the ultraviolet and infrared regions, but the use of specific wavelengths for reference in those regions is not yet so well established. The range of visible wavelengths runs from about 0.380μ to about 0.740μ. (See Section 4.5). Within this region several reference wavelengths are used which, for historical reasons, are known as Fraunhofer lines, and are customarily denoted by letters assigned to them in a system originated by Joseph von Fraunhofer in his studies of the solar spectrum. In Table 2.3 are given the wavelengths of light of some of the Fraunhofer lines, and the elements from which the lines result. Also included are two additional lines, one in the near infrared, the other in the near ultraviolet, which are being used as standard wavelengths for index measurements.

Color of light	Line	Wavelength, Microns	Element
Infrared		1.0140	Hg
Red	A'	0.7665	K
Red	C	0.6563	H
Yellow	D	0.5893	Na
Yellow	d	0.5876	He
Green	e	0.5461	Hg
Light Blue	F	0.4861	H
Blue	g	0.4358	Hg
Dark Blue	G'	0.4340	H
Violet	h	0.4047	Hg
Ultraviolet		0.3650	Hg

Table 2.3-Fraunhofer and other standard lines.

2.7 CHARACTERISTICS OF OPTICAL GLASS

2.7.1 Reference indices. In designing chromatically corrected systems, it is necessary to make provision for the variation of the index of refraction with wavelength. This will be expanded in later sections., but for now it is important to be aware of the terms and quantities which are usually sufficient to describe the properties of an optical medium in the visible spectrum. In this and in the following paragraphs, reference should be made to specification MIL-G-174, Optical Glass, to become acquainted with approved standard requirements for the military. It is impractical to treat simply the infinite number of indices corresponding to all the wavelengths in white light. Common practice is to select a convenient wavelength near the middle of the eye's sensitive range, using one which can be easily and accurately reproduced. The refractive index of the material at this wavelength is then used as a basic reference both in design and in material designation. The material's refractive index for yellow light corresponding to the mean wavelength of the two sodium D lines is usually used in the United States and is designated n_D. European practice is to use n_d, the index corresponding to the yellow helium line. Similarly, the terms n_F and n_C are the indices of refraction for the F and C lines of hydrogen and provide reference indices in the blue and red regions.

2.7.2 Abbe constant. A commonly used expression for identifying chromatic properties is the Abbe constant, which is defined as

$$\nu = \frac{n_D - 1}{n_F - n_C} .$$

The symbol V , rather than the Greek ν is frequently used; however ν will be used in this text. The Abbe constant is named for its inventor, the German scientist Ernst Abbe. It is often called the nu value or the vee number. The numerator, $n_D - 1$, is called the refractivity for the sodium D lines.

2.7.3 Partial dispersion. The difference between any two indices for a given substance, corresponding to two different wavelengths, is called the partial dispersion. Hence $n_D - n_C$ is the partial dispersion for the D and C lines. The particular partial dispersion, $n_F - n_C$, is called the mean dispersion because it covers approximately the visual range of wavelengths. Use is sometimes made of a partial dispersion ratio, for example, $(n_D - n_C) / (n_F - n_C)$.

2.7.4 Glass type number. It has become common practice to identify a glass by the type number, which is a six-digit number. The first three digits of the type number are the first three rounded digits of the refractivity, $(N_D - 1)$, and the last three digits of the type number are the first three rounded digits of the ν- value of the glass. A glass with $n_D = 1.51250$ and $\nu = 60.5$ would have a type number of 513605.

2.7.5 Staining. In addition to the quantities involving refractive indices, which have been mentioned above, additional optical characteristics must be considered in optical design. One of these, surface staining, obviously affects the transmittance; such staining is accelerated by the presence of acidic atmospheres, for example caused by carbon dioxide or perspiration. Staining can be measured quantitatively by the time required to form a film one quarter of a wavelength thick when the sample is immersed in nitric acid under controlled conditions of concentration and temperature.

2.7.6 Dimming. A characteristic somewhat related to staining is surface dimming, which occurs when the polished sample is exposed to moist air. It can be measured quantitatively by exposing the sample to a 100% relative humidity atmosphere at a given temperature for a specified time, and classifying the appearance of the surface.

2.7.7 Bubbles. All glasses contain some bubbles, or inclusions, varying in size and number according to the glass type. A glass sample is classified according to the number of bubbles in a specified volume of material. If a bubble is less than 0.02 mm in diameter (or some other standard value), it is not counted as it is considered invisible.

2.7.8 Table of optical glass characteristics. Table 2.4 lists the quantities described above in identifying glass. The glass type number is given in both the extreme left and extreme right hand columns. The second column at the left gives the ν - number. There follow eleven columns giving the refractive index for the corresponding wavelengths. The next column gives the mean dispersion. There follow six columns listing two numbers for each glass type. The one in large type is a partial dispersion, the other a partial dispersion ratio. The specific gravity is listed in the next column; as the metal parts of optical instruments become more and more fabricated of light alloys, the glass weight becomes an important factor and must be considered in overall optical design. The next column gives the staining time in hours, and adjacent to it is listed the stain test class. In the next column is given the dimming test class number, running from 1 (not visibly dimmed) to 5 (dimming interfering with clear vision). The bubble code is given in the next to the last column; the code runs from 1 (few bubbles) to 4 (many bubbles). The letter P following a glass type indicates that this type is available in a form which makes it resistant to gamma rays and X-rays. The term fine annealed indicates that permanent strain on cooling has been virtually eliminated.

2.7.9 Availability of glass tables. Designers, or interested students should obtain from glass manufacturers the latest catalog information. Some suggested sources are: (1) in the United States, Bausch and Lomb, Rochester, New York; Corning Glass Works, Corning, New York; Eastman Kodak Co., Rochester, New York; Hayward Glass Co., Whittier, California; Pittsburgh Plate Glass Co., Pittsburgh, Pennsylvania; and (2) abroad, Chance-Pilkington Optical Works, St. Asaph, England; Tozai Boeki Kaisha, Ltd., No. 13, 4-Chome, Shiba-Tamuracho, Minatoku, Tokyo, Japan; Minex, P.W.O. Works, Jelenia Góra, Poland; Ohara Optical Glass Manufacturing Co., Sagamihara, Kanagawa, Japan; Parra-Mantois, Le Vésinet, France; Schott Glass Works, Mainz, West Germany; Schott Glass Works, Jena, East Germany. Catalogs of Russian manufacturers are published by Gosudarstvennoe Isdatelstvo, Moscow, USSR. Additional U.S. companies and representatives of foreign companies are listed in the Optical Industry Directory (See page 1-5).

CHARACTERISTICS — OPTICAL GLASS

Indices Given are for "Fine Annealed" Glass

TYPE	V $\frac{n_D-1}{n_F-n_C}$	1014.0	$n_{A'}$ Potassium 766.5	n_C Hydrogen 656.3	n_D Sodium 589.3	n_d Helium 587.6	n_e Mercury 546.1	n_F Hydrogen 486.1	n_g Mercury 435.8	$n_{G'}$ Hydrogen 434.1	n_h Mercury 404.7	365.0
BOROSILICATE CROWN												
498670	**67.0**	*	1.49316	1.49577	**1.49800**	1.49808	1.49984	1.50320	1.50717	1.50732	1.51048	*
506596 (See Note 1)	**59.6**	*	1.50058	1.50347	**1.50600**	1.50609	1.50811	1.51196	1.51656	1.51675	1.52039	*
511635	**63.5**	1.5017	1.50578	1.50860	**1.51100**	1.51107	1.51300	1.51665	1.52096	1.52114	1.52454	1.53057
517645 517645P	**64.5**	1.5076	1.51179	1.51461	**1.51700**	1.51707	1.51899	1.52262	1.52690	1.52709	1.53043	1.53644
CROWN												
513605	**60.5**	1.5030	1.50708	1.50999	**1.51250**	1.51258	1.51459	1.51846	1.52304	1.52324	1.52685	1.53332
518596	**59.6**	1.5083	1.51242	1.51544	**1.51800**	1.51807	1.52015	1.52413	1.52886	1.52906	1.53279	1.53951
523586	**58.6**	1.5130	1.51729	1.52036	**1.52300**	1.52307	1.52520	1.52929	1.53415	1.53435	1.53819	1.54505
524595	**59.5**	*	1.51838	1.52140	**1.52400**	1.52408	1.52618	1.53021	1.53500	1.53520	1.53898	*
LIGHT BARIUM CROWN												
541599 541599P	**59.9**	1.5309	1.53522	1.53833	**1.54100**	1.54109	1.54323	1.54736	1.55226	1.55246	1.55633	1.56326
573568	**56.8**	*	1.56614	1.56954	**1.57250**	1.57259	1.57498	1.57962	1.58514	1.58538	1.58975	*
573574 573574P	**57.4**	1.5616	1.56619	1.56956	**1.57250**	1.57259	1.57497	1.57953	1.58497	1.58521	1.58951	1.59723
DENSE BARIUM CROWN												
588612	**61.2**	*	1.58184	1.58513	**1.58800**	1.58811	1.59036	1.59474	1.59992	1.60015	1.60424	*
611572	**57.2**	1.5993	1.60423	1.60785	**1.61100**	1.61109	1.61364	1.61853	1.62438	1.62462	1.62923	1.63754
611588	**58.8**	*	1.60439	1.60793	**1.61100**	1.61109	1.61357	1.61832	1.62396	1.62421	1.62867	*
612595	**59.5**	*	1.60544	1.60896	**1.61200**	1.61209	1.61455	1.61924	1.62484	1.62508	1.62946	*
617549	**54.9**	1.6048	1.60995	1.61370	**1.61700**	1.61710	1.61977	1.62493	1.63115	1.63142	1.63634	1.64516
617551	**55.1**	*	1.50994	1.61371	**1.61700**	1.61710	1.61976	1.62490	1.63104	1.63130	1.63617	*
620603	**60.3**	*	1.61342	1.61696	**1.62000**	1.62011	1.62255	1.62724	1.63282	1.63306	1.63748	*
623569	**56.9**	*	1.61606	1.61978	**1.62300**	1.62309	1.62571	1.63073	1.63675	1.63701	1.64171	*
638555	**55.5**	*	1.63074	1.63461	**1.63800**	1.63810	1.64084	1.64611	1.65243	1.65271	1.65772	*
651558	**55.8**	*	1.64362	1.64757	**1.65100**	1.65109	1.65389	1.65924	1.66563	1.66591	1.67097	*

Note 1: Available in Condenser quality only.
*Data not currently available.

n_F-n_C	n_C-n_A $\frac{n_C-n_A}{n_F-n_C}$	n_D-n_C $\frac{n_D-n_C}{n_F-n_C}$	n_E-n_D $\frac{n_E-n_D}{n_F-n_C}$	n_F-n_E $\frac{n_F-n_E}{n_F-n_C}$	$n_{G'}-n_F$ $\frac{n_{G'}-n_F}{n_F-n_C}$	$n_h-n_{G'}$ $\frac{n_h-n_{G'}}{n_F-n_C}$	Specific Gravity	Staining Time at 25°C—Hrs	Stain Test Class	Dimming Test Class	Bubble Code	TYPE
BOROSILICATE CROWN												
0.00744	0.00261 0.351	0.00223 0.300	0.00184 0.247	0.00336 0.452	0.00412 0.554	0.00316 0.425	2.44	+100	1	2.5	1	498670
0.00849	0.00289 0.340	0.00253 0.298	0.00211 0.248	0.00385 0.454	0.00479 0.564	0.00364 0.429	2.47	+100	1	2.5	2	506596
0.00805	0.00282 0.350	0.00240 0.298	0.00200 0.248	0.00365 0.453	0.00449 0.558	0.00340 0.422	2.48	+100	1	1.0	1	511635
0.00801	0.00282 0.352	0.00239 0.298	0.00199 0.248	0.00363 0.453	0.00447 0.558	0.00334 0.417	2.53	+100	1	1.0	1	517645 517645P
CROWN												
0.00847	0.00291 0.344	0.00251 0.296	0.00209 0.247	0.00387 0.457	0.00478 0.564	0.00361 0.426	2.51	+100	1	1.5	1	513605
0.00869	0.00302 0.348	0.00256 0.295	0.00215 0.247	0.00398 0.458	0.00493 0.567	0.00373 0.429	2.54	+100	1	3.0	3	518596
0.00893	0.00307 0.344	0.00264 0.296	0.00220 0.246	0.00409 0.458	0.00506 0.567	0.00384 0.430	2.53	+100	1	3.0	4	523586
0.00881	0.00302 0.343	0.00260 0.295	0.00218 0.248	0.00403 0.457	0.00499 0.567	0.00378 0.428	2.53	+100	1	3.0	1	524595
LIGHT BARIUM CROWN												
0.00903	0.00312 0.346	0.00267 0.296	0.00223 0.247	0.00413 0.457	0.00512 0.567	0.00387 0.429	2.84	+100	1	2.5	1	541599 541599P
0.01008	0.00340 0.337	0.00296 0.294	0.00248 0.246	0.00464 0.460	0.00576 0.571	0.00437 0.434	3.20	22	2	3.0	3	573568
0.00997	0.00337 0.338	0.00294 0.295	0.00247 0.248	0.00456 0.457	0.00568 0.570	0.00430 0.431	3.21	9	3	2.0	3	573574 573574P
DENSE BARIUM CROWN												
0.00961	0.00329 0.342	0.00287 0.298	0.00236 0.245	0.00438 0.455	0.00541 0.562	0.00409 0.425	3.29	.03	5	4.0	1	588612
0.01068	0.00362 0.339	0.00315 0.295	0.00264 0.247	0.00489 0.458	0.00609 0.570	0.00461 0.432	3.57	0.033	5	2.0	4	611572
0.01039	0.00354 0.341	0.00307 0.295	0.00257 0.247	0.00475 0.457	0.00589 0.567	0.00446 0.429	3.58	0.017	5	3.0	4	611588
0.01028	0.00352 0.342	0.00304 0.296	0.00255 0.248	0.00469 0.456	0.00584 0.568	0.00438 0.426	3.45	.02	5	4.0	1	612595
0.01123	0.00375 0.334	0.00330 0.294	0.00277 0.247	0.00516 0.459	0.00649 0.578	0.00492 0.438	3.66	0.050	5	1.5	4	617549
0.01119	0.00377 0.337	0.00329 0.294	0.00276 0.247	0.00514 0.459	0.00640 0.572	0.00487 0.435	3.36	0.2	4	3.5	3	617551
0.01028	0.00354 0.344	0.00304 0.296	0.00255 0.248	0.00469 0.456	0.00582 0.566	0.00442 0.430	3.58	**	5	4.5	3	620603
0.01095	0.00372 0.340	0.00322 0.294	0.00271 0.247	0.00502 0.458	0.00628 0.573	0.00470 0.429	3.58	**	5	4.0	1	623569
0.01150	0.00387 0.337	0.00339 0.295	0.00284 0.247	0.00527 0.458	0.00660 0.574	0.00501 0.436	3.59	**	5	4.0	3	638555
0.01167	0.00395 0.338	0.00343 0.294	0.00289 0.248	0.00535 0.458	0.00667 0.572	0.00506 0.434	3.81	**	5	4.0	1	651558

** Dissolves in HNO_3

Courtesy of **BAUSCH & LOMB OPTICAL CO.**

Table 2.4 - Excerpt from commercial glass catalog.