Name Solutions

Closed book; closed notes. Time limit: 120 minutes.
An equation sheet is attached and can be removed. A spare raytrace sheet is also attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
As usual, only the magnitude of a magnification or magnifying power may be given.
If a method of solution is specified in the problem, that method must be used.
Raytraces must be done on the raytrace form. Be sure to indicate the initial conditions for your rays.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Provide your answers in a neat and orderly fashion.
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.
Note: On some quantities, only the magnitude of the quantity is provided. The proper sign convention must be applied.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (5 points) A field lens is added to a Keplerian telescope. What is the effect of the field lens on each of the following?

a) MP No Change

b) Eye Relief Reduced

c) Exit Pupil Diameter No Change

d) Field of View Increased

e) Telescope Length No Change

f) Image Orientation No Change

2) (15 points) An aberration-free beam is focused through a thick glass plate (a plane-parallel plate) of thickness t in air. The plate has an index of refraction of n_d and an Abbe number of ν.

a) Derive an expression for the longitudinal (or axial) chromatic aberration introduced by the glass plate in terms of t, n_d and ν. The longitudinal chromatic aberration is defined to be the distance between the F and C foci.

A useful approximation to use is $n_F n_C \approx n_d^2$

Image Shift $= d = \left(\frac{n-1}{n}\right) t$

$$d_F = \left(\frac{n_F - 1}{n_F}\right) t$$

$$d_C = \left(\frac{n_C - 1}{n_C}\right) t$$

$$d_d = \left(\frac{n_d - 1}{n_d}\right) t$$

Note:

$d_F > d_C$

because $n_F > n_C$

Axial Chromatic:

$$d_F - d_C = \left(\frac{n_F - 1}{n_F}\right) t - \left(\frac{n_C - 1}{n_C}\right) t$$

$$d_F - d_C = \frac{n_C(n_F - 1) - n_F(n_C - 1)}{n_F n_C} t$$

$n_F n_C \approx n_d^2$

$$d_F - d_C = \frac{n_F - n_C}{n_d^2} t$$

$$d_F - d_C = \frac{n_F - n_C}{n_d - 1} \frac{n_d - 1}{n_d^2} t$$

$$\underline{d_F - d_C = \frac{1}{\nu} \frac{n_d - 1}{n_d^2} t}$$

Continues...

This result can be rewritten in terms of d_d:

$$d_F - d_C = \frac{1}{\nu}\frac{d_d}{n_d} \qquad d_d = \left(\frac{n_d - 1}{n_d}\right) t$$

b) A thin lens of 100 mm focal length is made out of LLF1 glass (Glass Code: 548458). A 100 mm thick glass plate is also made out of LLF1 glass. Determine the axial chromatic aberration introduced by each of these elements.

In these two situations, is the relative order of the F and C foci the same or reversed?

The order is reversed:

- F (or blue) focuses closer to the lens
- F has the largest shift due to the plate.

LLF1: $n_d = 1.548 \qquad \nu = 45.8$

Lens $\qquad \frac{\delta f}{f} = \frac{1}{\nu} \qquad \delta f = f_C - f_F = f/\nu = 100\text{ mm}/45.8$

$$\delta f = 2.18\text{ mm}$$

Plate $\qquad d_F - d_C = \frac{1}{\nu}\frac{n_d - 1}{n_d^2}\,t = \frac{1}{45.8}\left(\frac{.548}{1.548^2}\right) 100\text{ mm}$

$$d_F - d_C = 0.50\text{ mm}$$

Lens: 2.18 mm Plate: 0.50 mm Order: Reversed

3) (15 points) A moving train is being photographed by a fixed camera. The camera does not move during the exposure. The train is at a distance of 100 m, and the focal length of the camera lens is 50 mm. The speed of the train is 10 m/s and the train motion is perpendicular to the optical axis of the camera.

a) Using reasonable approximations, what is the slowest shutter speed that can be used so that the motion blur on the detector is 5 μm?

$$v = 10 \text{ m/sec}$$

$$m = \frac{z'}{z} \approx -\frac{f}{z} = -\frac{50\text{mm}}{100\text{m}} = -.0005$$

Velocity of the image:

$$v' = |m| v = 0.005 \text{ m/sec}$$

The image blur equals the image velocity times the exposure duration Δt:

$$\text{Blur} = v' \Delta t = 5\mu m$$

$$\Delta t = \frac{5 \times 10^{-6} m}{0.005 m/sec} = 0.001 \text{ sec}$$

$$\text{Shutter Speed} = 0.001 \text{ sec} = \frac{1}{1000} \text{ sec}$$

Shutter speed = 0.001 sec

Continues...

b) The train is being photographed on a sunny day, and the detector requires an exposure of 0.002 J/m^2 in order to produce a good image. What f/# is should be used to obtain this exposure given this shutter speed? Use a typical value for the solar irradiance on the surface of the earth, and make any other reasonable assumptions.

Sunny Day : $E_0 = 1000\ W/m^2$

$$L_0 = \frac{\rho E_0}{\pi} \qquad \rho = 0.18$$

$$E' = \frac{\pi L_0}{4(f/\#)^2} = \frac{\rho E_0}{4(f/\#)^2}$$

Required Exposure : $H' = 0.002\ J/m^2$

$$E' = H'/\Delta t \qquad \Delta t = 0.001\ sec$$

$$E' = 2\ W/m^2$$

Equating Incident with Required

$$E' = \frac{\rho E_0}{4(f/\#)^2} = 2\ W/m^2$$

$$(f/\#)^2 = \frac{\rho E_0}{4(2\ W/m^2)} = \frac{(0.18)(1000\ W/m^2)}{4(2\ W/m^2)}$$

$$(f/\#)^2 = 22.5$$

$$f/\# = 4.7$$

f/# = <u>f/4.7</u>

4) (25 points) A system is comprised of three thin lenses in air. The following partially-completed raytrace of the system is the starting point for this problem, and will be completed during various parts of the problem. Extra lines/rays are provided.

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace. The image size and location must be determined from the marginal and chief rays associated with the object. Gaussian imaging methods may not be used for any portion of this problem. Be sure to clearly label your rays on the raytrace form.

	Object	EP	Lens 1	Lens 2	Lens 3	XP	Image F′
Surface	0	1	2	3	4	5	6
f			100 mm	-100 mm	100 mm		
$-\phi$			-.01	.01	-.01		
t			-66.67	40 mm	60 mm	-150	197.4

Potential Chief Ray:

	0	1	2	3	4	5	6
$\tilde{\tilde{y}}$		0	-4.0	0.0 mm	6.0	0	
$\tilde{\tilde{u}}$	.06	.06	0.1	0.1	.04		

Potential Marginal Ray

197.4

	0	1	2	3	4	5	6
$\tilde{y}$	1.0	1.0	1.0	0.6	0.36	1.5	0
$\tilde{u}$	0	0	-0.01	-0.004	-0.0076	-0.0076	

Marginal Ray - Scale by 16.67

	0	1	2	3	4	5	6
y	16.67	16.67	16.67	10.0	6.0	25.0	0
u	0	0	-.1667	-.06667	-.1267	-.1267	

Chief Ray - Scale by 3.543

	0	1	2	3	4	5	6
y		0	14.17	0	21.26	0	27.98
u	.2126	.2126	.3543	.3543	.1417	.1417	

	0	1	2	3	4	5	6
y							
u							

	0	1	2	3	4	5	6
y							
u							

a) Which element serves as the System Stop (circle one)?

The potential chief ray goes through the center of the stop

Lens 1 Lens 2 Lens 3

b) Determine the Entrance and Exit Pupil locations.

Extend the potential chief ray into object and image space. $\bar{y} = 0$ at EP, XP.

EP: Located 66.67 mm to the Right of the first lens.

XP: Located 150 mm to the Left of the third lens.

c) Determine the system Focal Length and its Back Focal Distance (BFD).

Trace a potential marginal ray $\tilde{y}_0 = 1.0$ from infinity (parallel to the axis).

$\tilde{u}' = -.0076$

$\phi = -\tilde{u}'/\tilde{y}_1 = 0.0076$

$f = 1/\phi = 131.58$

f = 131.58 mm

BFD = 47.4 mm

BFD = (L3 → XP) + (XP → F')

BFD = -150.0 + 197.4 = 47.4 mm

d) The System Stop has a diameter of 20 mm. Determine the diameters of the Entrance Pupil and the Exit Pupil.

Scale the marginal ray to a radius of 10 mm at the stop

$$\text{Scale Factor} = \frac{10\text{mm}}{\tilde{y}_{L2}} = \frac{10\text{mm}}{0.6\text{mm}} = 16.67$$

$a_{EP} = 16.67$ mm

$a_{XP} = 25.0$ mm

Entrance Pupil Diameter = 33.33 mm

Exit Pupil Diameter = 50.0 mm

Continues…

e) For distant objects, the system has an unvignetted Field of View of +/- 12 deg. What is the image height in the image plane for this FOV? What are the required Lens Diameters to support this FOV?

$\bar{u}_0 = \tan(12°) = 0.2126$

Scale the potential chief ray to this object space values

Scale Factor $= \bar{u}_0 / \hat{\bar{u}}_0 = \dfrac{0.2126}{0.06} = 3.543$

Extend the chief ray to the image plane to obtain the image height.

$\bar{y}' = \underline{27.98 \text{ mm}}$

For Unvignetted: $a = |y| + |\bar{y}|$

Lens 1: $y_1 = 16.67$ $\bar{y}_1 = 14.17$ $a_1 = 30.84$

Lens 2: $y_2 = 10.0$ $\bar{y}_2 = 0$ $a_2 = 10.0$

Lens 3: $y_3 = 6.0$ $\bar{y}_3 = 21.26$ $a_3 = 27.26$

Image Height = +/- 27.98 mm

Lens 1 Diameter = 61.68 mm

Lens 2 Diameter = 20.0 mm

Lens 3 Diameter = 54.52 mm

5) (15 points) A thick lens in air has the following specifications:

R1 = -25 mm R2 = 50 mm
t = 20 mm n = 1.55

A second optical system produces a real image that is projected into this lens. This image serves as a virtual object for the thick lens. The virtual object has a height of +/- 5 mm and is located 15 mm to the right of the first surface of the thick lens.

Determine the size and location of the image produced by the thick lens.

NOTE: This problem is to be worked using raytrace methods only. Two rays must be traced: One for image location and one for image size. Gaussian imaging methods may not be used for any portion of this problem. Be sure to clearly label your rays on the raytrace form.

Construct two object-space rays to represent the virtual object:

- one through the axial object position (arbitrary angle: 0.1 used)
- one through the top of the object (arbitrary angle: 0 used)

Transfer these rays to the first vertex.

Transfer distance = -15 mm

There is no refraction associated with this transfer.

Refract through both surfaces to the image plane.

Continues with a raytrace form on the next page…

$C1 = 1/R_1 = -.04$ $C2 = 1/R_2 = 0.02$

		Object			Image			
	Surface	0	1	2	3	4	5	6
	C		-.04	.02				
	t	-15	20	?				
	n	1.0	1.55	1.0				
	−ϕ		0.022	0.011				
	t/n	-15	12.903	? 10.59				
Axial Point	y	0	-1.5	-0.6355	0			
	nu	0.1*	0.067	0.0600				
	u							
Top of Object	y	5	5	6.4193	8.332			
	nu	0*	0.110	0.1806				
	u							

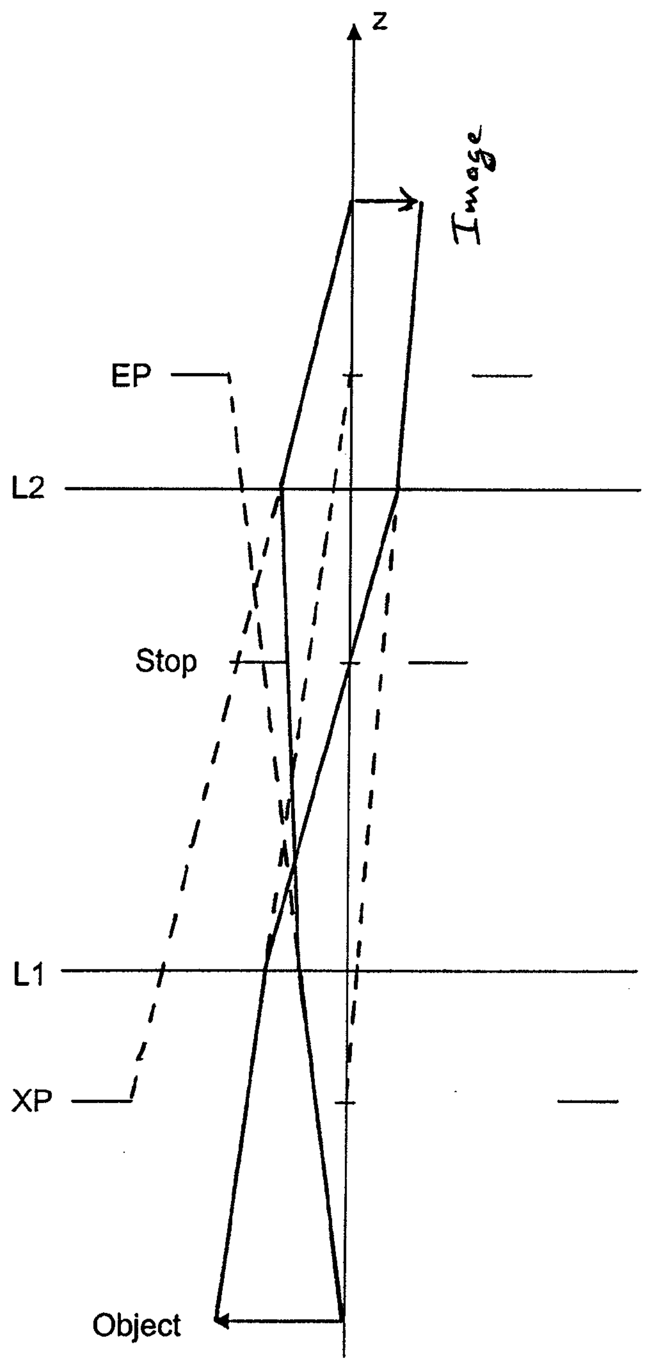
* arbitrary

Surface 2 to Image = 10.59 mm

Image Height = 8.332 mm

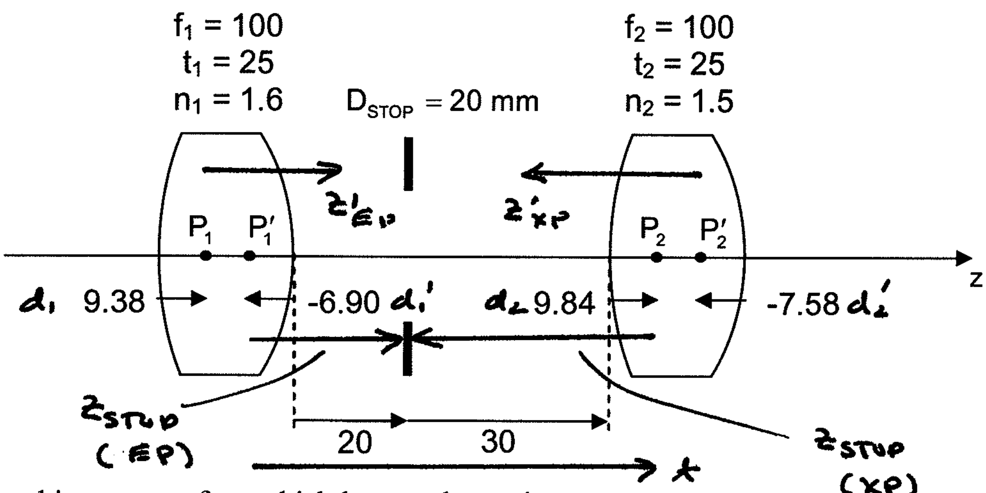
Image size = +/- 8.33 mm; Located 10.59 mm to the R of the rear vertex.

6) (10 points) This diagram shows an optical system consisting of two refracting surfaces and an object. Also shown are the locations and sizes of the stop, the entrance pupil (EP) and the exit pupil (XP). Show the paths of the marginal and chief rays through the system along with the location and size of the image. No calculations or equations are required or allowed. **Please use a straight edge!**



7) (20 points) Two thick lenses in air are combined into a single imaging system. Both lenses are 25 mm thick and both lenses have a focal length of 100 mm, however the index of the first lens is 1.6 and the index of the second lens is 1.5. The vertex-to-vertex spacing of the lenses is 50 mm. The principal plane locations for the two individual lenses with respect to surface vertices are shown in the figure. All units are in mm.

A 20 mm diameter stop is located between the two thick lenses. The stop is 20 mm to the right of the rear vertex of the first lens. The vertex-to-vertex separation of the two lenses is 50 mm. All units are in mm.



a) For this system of two thick lenses, determine:

System Focal Length
Location of the Rear Principal Plane of the system relative to the rear vertex of the second lens
Back Focal Distance

b) Determine the entrance pupil and exit pupil locations and diameters. The entrance pupil is to be located relative to the front vertex of the first lens, and the exit pupil is to be located relative to the rear vertex of the second lens.

NOTE: Only Gaussian methods may be used for this problem.

System Focal Length = 75.0 mm BFD = 17.2 mm

System P': Located 57.8 mm to the L of the rear vertex of the second lens.

EP: D_{EP} = 27.4 mm; Located 46.2 mm to the R of the front vertex of the first lens.

XP: D_{XP} = 33.2 mm; Located 73.8 mm to the L of the rear vertex of the second lens.

Method of Solution:

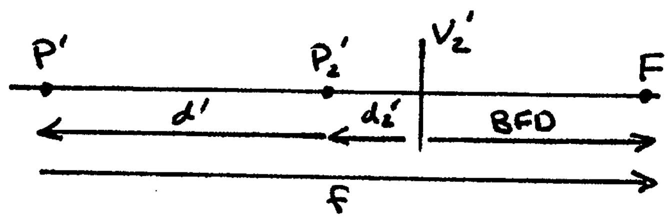
$\phi_1 = \phi_2 = 0.01\ /mm \qquad t: P_1' \rightarrow P^2 \qquad n = 1$

$t = 50 - d_1' + d_2 = 50 + 6.90 + 9.84 = 66.74$

$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 t = 0.0133/mm \qquad n' = 1$

$f = 1/\phi = \underline{75.0\ mm}$

$\delta' = d' = -\frac{\phi_1}{\phi} t = -50.2\ mm \quad (\text{from } P_2')$



$d_2' = -7.58\ mm$

$\overline{V_2'P'} = d' + d_2' = \underline{-57.8\ mm}$

$BFD = f + d' + d_2' = \underline{17.2\ mm}$

$\underline{XP}$: Image through second lens

$z_{STOP} = -30\ mm - d_2 = -39.84\ mm \quad (\text{from } P_2)$

$\frac{1}{z'_{XP}} = \frac{1}{z_{STOP}} + \frac{1}{f_2}$

$z'_{XP} = -66.2\ mm \quad (\text{from } P_2')$

$s'_{XP} = z'_{XP} + d_2' = \underline{-73.8\ mm} \quad (\text{from } V_2')$

Continues…

$$m_{XP} = \frac{z'_{XP}}{z_{STOP}} = \frac{-66.2\ mm}{-39.84\ mm} = 1.66$$

$$D_{XP} = m_{XP}\, D_{STOP} = \underline{33.2\ mm}$$

<u>EP</u>: Image stop through first lens

Light $R \Rightarrow L$ $n = n' = -1$

$$z_{STOP} = 20\,mm - d_1' = 26.90\ mm \quad (\text{from } P_1')$$

$$\frac{-1}{z'_{EP}} = \frac{-1}{z_{STOP}} + \frac{1}{f_1}$$

$$z'_{EP} = 36.8\ mm \quad (\text{from } P_1)$$

$$S'_{EP} = z'_{EP} + d_1 \qquad d_1 = 9.38\ mm$$

$$S'_{EP} = \underline{46.2\ mm} \quad (\text{from } V_1)$$

$$m_{EP} = \frac{z'_{EP}/n'}{z_{STOP}/n} = \frac{36.8\ mm}{26.9\ mm} = 1.37$$

$$D_{EP} = m_{EP}\, D_{STOP} = \underline{27.4\ mm}$$