Name Solutions

Closed book; closed notes. Time limit: 2 hours.
An equation sheet is attached and can be removed. Spare raytrace sheets are attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
As usual, only the magnitude of a magnification or magnifying power may be given.
If a method of solution is specified in the problem, that method must be used.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.
Note: On some quantities, only the magnitude of the quantity is provided. The proper sign convention must be applied.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (10 points) A 3X Galilean telescope has an objective lens with a focal length of 150 mm. Complete the design by determining the focal length of the eye lens and the separation of the two lenses.

$$MP = -\frac{f_{obj}}{f_{eye}} = 3 \qquad f_{obj} = 150\ mm$$

$$f_{eye} = -50\ mm$$

$$t = f_{obj} + f_{eye} = 100\ mm$$

Separation = 100 mm $\qquad f_{EYE}$ = -50 mm

2) (10 points) A thin lens in air is made of F2 glass (620364). The lens has a focal length of 100 mm for d-light. The partial dispersion ratio for the glass is

$$P_{d,C} = \frac{n_d - n_C}{n_F - n_C} = 0.294 \qquad \nu = \frac{n_d - 1}{n_F - n_C}$$

Determine the focal lengths at the F and C wavelengths.

$620364 \rightarrow \quad n_d = 1.620 \quad \nu = 36.4$

There are several solution methods

$$\nu = \frac{n_d - 1}{n_F - n_c} = \frac{.620}{n_F - n_c} = 36.4 \qquad n_F - n_c = 0.0170$$

$$P_{d,c} = \frac{n_d - n_c}{n_F - n_c} = \frac{n_d - n_c}{0.0170} = 0.294 \qquad n_d - n_c = 0.0050$$

$$n_F - n_d = 0.0120$$

$$n_c = 1.615 \qquad n_F = 1.632$$

$$\phi_d = \frac{1}{100\,mm} = .01/mm = (n_d - 1)\Delta C = .620\,\Delta C$$

$$\Delta C = 0.01613/mm$$

$$\phi_c = (n_c - 1)\Delta C = .615\,\Delta C = 0.009920/mm$$

$$f_c = 1/\phi_c = \underline{100.81\ mm}$$

$$\phi_F = (n_F - 1)\Delta C = .632\,\Delta C = 0.01019/mm$$

$$f_F = 1/\phi_F = \underline{98.09\ mm}$$

f_F = 98.09 mm $\qquad f_C$ = 100.81 mm

2) (10 points) A thin lens in air is made of F2 glass (620364). The lens has a focal length of 100 mm for d-light. The partial dispersion ratio for the glass is

$$P_{d,C} = \frac{n_d - n_C}{n_F - n_C} = 0.294 \qquad \nu = \frac{n_d - 1}{n_F - n_C}$$

$\nu = 36.4$

Determine the focal lengths at the F and C wavelengths.

Alternate 1: $\frac{\delta\phi}{\phi_d} = \frac{1}{\nu}$ $\phi_d = 1/f_d = 0.01/mm$

$\delta\phi = \phi_F - \phi_C = 0.0002747$

$P_{d,C}$ is the fraction of this change between C and d.

$\phi_d - \phi_C = P\,\delta\phi = 0.294(0.0002747) = 0.00008077/mm$

$\phi_F - \phi_d = \delta\phi - 0.00008077 = 0.0001939/mm$

$\phi_C = \phi_d - 0.00008077 = 0.009919/mm$

$\underline{f_C = 100.81}$ mm

$\phi_F = \phi_d + 0.0001939 = 0.0101939/mm$

$\underline{f_F = 98.09}$ mm

Alternate 2: $\frac{\delta f}{f_d} = \frac{1}{\nu}$ $f_d = 100mm$

$\delta f = f_C - f_F = 2.75$ mm

$P_{d,C}$ is the fraction of this change between C and d:

$f_C - f_d = 0.294(2.75) = 0.808$ mm

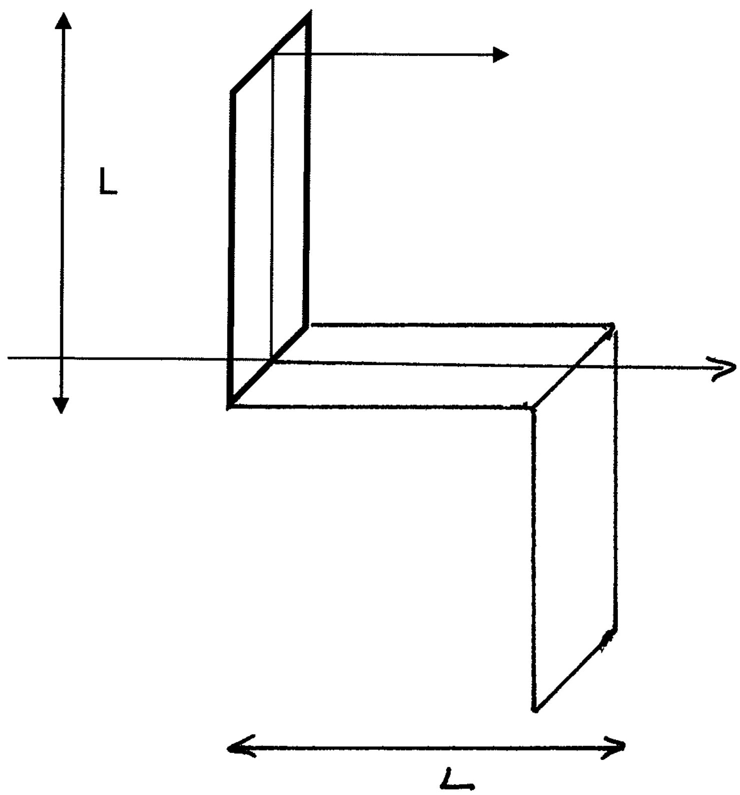
$f_d - f_F = \delta f - 0.808 = 1.942$ mm

$f_C = \underline{100.81}$ mm $f_F = \underline{98.06 \text{ mm}}$*

f_F = 98.09 mm f_C = 100.81 mm

* difference is due to the approximation $\phi_C \phi_F \approx \phi_d^2$

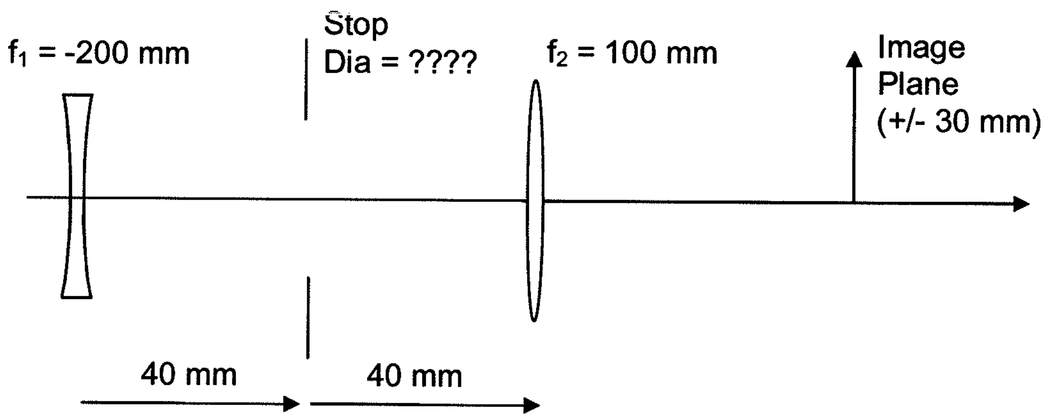
3) (10 points) Draw the tunnel diagram for this prism with the ray path shown. If the total length of the prism (apex-to-apex) is L as shown, what is the length of the tunnel diagram?



Tunnel Diagram Length = L

4) (30 points) The following diagram shows the design of an objective that is comprised of two thin lenses in air. The system stop is located between the two lenses.

The system operates at f/4.
The object is at infinity.
The maximum image size is +/- 30 mm.



This is a reverse telephoto objective
BFD > f

Determine the following:
- Entrance pupil and exit pupil locations and sizes.
- System focal length and back focal distance.
- Stop diameter.
- Angular field of view (in object space).
- Required diameters for the two lenses for the system to be unvignetted over the specified maximum image size.

NOTE: This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem. The field of view must be determined from the chief ray.

Be sure to clearly label your rays on the raytrace form. Your answers must be entered below. Be sure to provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.).

Entrance Pupil: 33.33 mm to the R of the first lens. D_{EP} = 27.78 mm

Exit Pupil: 66.67 mm to the L of the second lens. D_{XP} = 55.56 mm

System Focal Length = 111.11 mm Back Focal Distance = 155.56 mm

Stop Diameter = 33.33 mm FOV = +/- 15.1 deg in object space

Lens 1 Diameter = 45.78 mm Lens 2 Diameter = 56.9 mm

	Obj	EP	L1	Stop	L2	XP	F' Image
Surface	0	1	2	3	4	5	6
f		-	-200	-	100	-	
$-\phi$			0.005		-.01		
t		-33.33	40	40	-66.67	222.22	

Potential Chief Ray

$\hat{\bar{y}}$		0	-40	0	40	0	
$\hat{\bar{u}}$		.12	.1*	.1*	.06		

Potential Marginal Ray

222.22

$\hat{y}$	1	1	1	1.2	1.4	2.00	0
$\hat{u}$	0	0	.005	.005	-.009	-.009	

Potential Chief Ray - Extended

$\hat{\bar{y}}$		0	-4.0	0	4.0	0	13.33
$\hat{\bar{u}}$		.12	.1	.1	.06	.06	

Marginal Ray - Scale Factor = 13.89

y	13.89	13.89	13.89	16.67	19.45	27.78	0
u	0	0	.0695	.0695	-.125	-.125	

Chief Ray - Scale Factor = 2.25

$\bar{y}$		0	-9.0	0	9.0	0	30
$\bar{u}$		.270	.225	.225	.135	.135	

y							
u							

y							
u							

Continues…

* arbitrary

Provide Method of Solution:

EP/XP Locations: Trace a potential chief ray starting at the center of the stop. The pupils are located where this ray crosses the axis in object/image space.

L1 → EP = 33.33 mm (R of L1)

L2 → XP = −66.67 mm (L of L2)

Focal Length/BFD: Trace a potential marginal ray parallel to the axis in object space ($y = 1$). The rear focal point is located where this ray crosses the axis.

$XP \rightarrow F' = 222.22$ mm

$BFD = (L2 \rightarrow XP) + (XP \rightarrow F') = -66.67 + 222.22$

$BFD = 155.56$ mm

$\phi = -\frac{u'}{y_1}$ $u' = -.009$ $y_1 = 1.0$

$\phi = .009/\text{mm}$ $f = 111.11$ mm

Extend the chief ray to the image plane F'.

Pupil Sizes: The system operates at f/4 $f = 111.11$ mm

$f/\# = 4 = f/D_{EP}$ $D_{EP} = 27.78$ mm

$r_{EP} = 13.89$ mm

Scale the marginal ray:

Scale Factor = 13.89

$r_{STOP} = 16.67$ mm $r_{XP} = 27.78$ mm

$D_{STOP} = 33.33$ mm $D_{XP} = 55.56$ mm

Continues…

Provide Method of Solution:

FOV: Scale the potential chief ray to an image height of 30 mm (from the potential ray value of 13.33 mm)

Scale Factor $= 30.0/13.33 = 2.25$

Object Space Chief Ray:

$\bar{u}_0 = 0.270$

$HFOV = \tan^{-1}(\bar{u}_0) = 15.1°$

$FOV = 30.2°$ or $+/- 15.1°$

Lens Diameters: For Unvignetted

$a \geq |y| + |\bar{y}|$

L_1 $y_1 = 13.89$ mm

$\bar{y}_1 = -9.0$ mm

$a_1 \geq 22.89$ mm

$D_1 \geq 45.78$ mm

L_2 $y_2 = 19.45$ mm

$\bar{y}_2 = 9.0$ mm

$a_2 \geq 28.45$ mm

$D_2 \geq 56.9$ mm

5) (20 points) An object-space telecentric system has a focal length of 100 mm. The stop diameter is 10 mm (i.e. stop radius equals 5 mm). An object is located 200 mm to the left of the lens. The object size is +/- 10 mm.

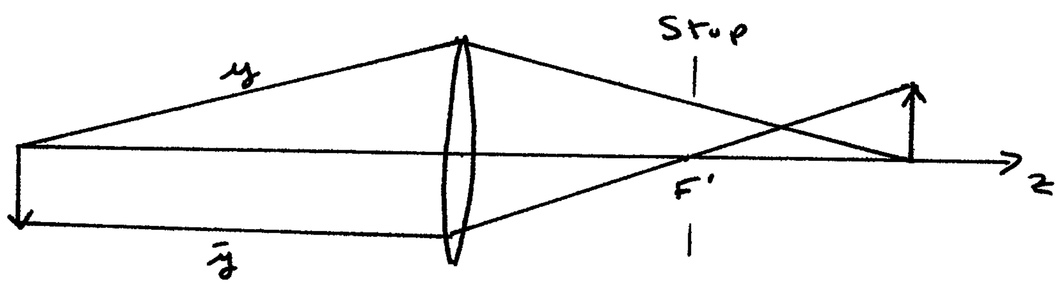
a) Sketch the system and determine the required lens diameter for the system to be unvignetted. **The method of solution is not specified.**

For object-space telecentricity, the stop must be located at the rear focal point of the lens.

Using ray trace: Obj → Lens = 200 mm

Lens → Stop = 100 mm

- Trace a potential marginal ray
- Scale to the stop radius = 5 mm
- Trace a chief ray parallel to the axis from the edge of the object



Lens Diameter: $a > (|y| + |\bar{y}|)$

$y = 10 \quad \bar{y} = -10$

$a \geq 20$ mm

Dia $\geq$ 40mm

Continues…

Surface	0 (Obj)	1 (Lens)	2 (Stop)	3 (Image)	4	5	6
f		100					
−φ		−.01					
t	200	100	100				
Potential Marginal							
$\tilde{y}$	0	2	1	0			
$\tilde{u}$	.01*	−.01	−.01				
Marginal - Scale by 5.0							
y	0	10	5	0			
u	.05	−.05	−.05				
Chief							
$\bar{y}$	−10	−10	0	10			
$\bar{u}$	0	.1	.1				
y							
u							

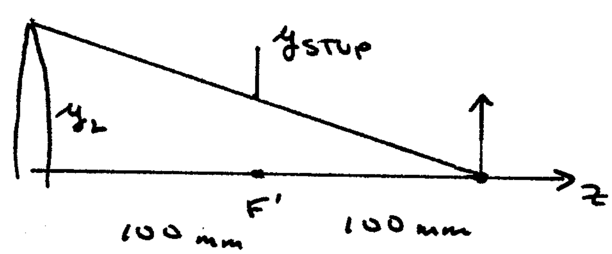
* arbitrary

The problem can also be done geometrically:

$z = -200$ mm $\quad z' = 200$ mm

The image size marginal ray:

$y_L = 2\, y_{STOP} = 10$ mm



The chief ray height at the lens must equal the object size by telecentricity. $\bar{y}_L = 10$

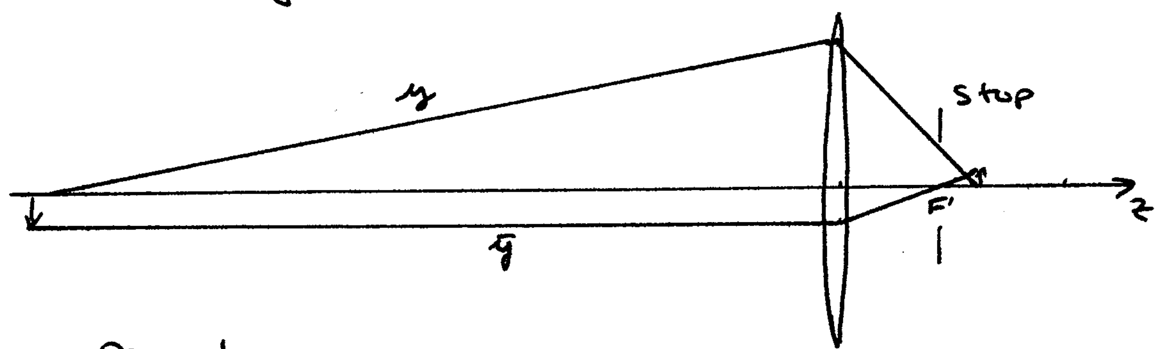
$a \geq |y| + |\bar{y}| = 20$

$D_{ia} = 40$

Lens Diameter = 40 mm

b) Repeat the problem when the system setup is changed for an object location 1000 mm to the left of the lens. Comment on the implications of this result for the use of object-space telecentric systems. **The method of solution is not specified.**

The ray trace solution is identical to part a – except that the object distance is 1000mm.



Lens Diameter:

$y = 50$ $\quad a \geq |y| + |\bar{y}|$

$\bar{y} = -10$ $\quad a \geq 60$ mm

Dia ≥ 120 mm

The image is smaller and closer to the stop.

	Obj	Lens	Stop	Image			
Surface	0	1	2	3	4	5	6
f		100					
−φ		−.01					
t	1000	100	11.1				
Potential Marginal							
$\hat{y}$	0	10	1	0			
$\hat{u}$	.01*	−.09	−.09				
Marginal - Scale by 5.0							
y	0	50	10	0			
u	.05	−.45	−.45				
Chief							
$\bar{y}$	−10	−10	0	1.11			
$\bar{u}$	0	.1	.1				
y							
u							

* arbitrary

Continues…

This part can also be done geometrically:

$z = -1000 \qquad z' = 111.11$

Lens → stop = 100 mm

Stop → Image = 11.1 mm

Image-Side Marginal Ray

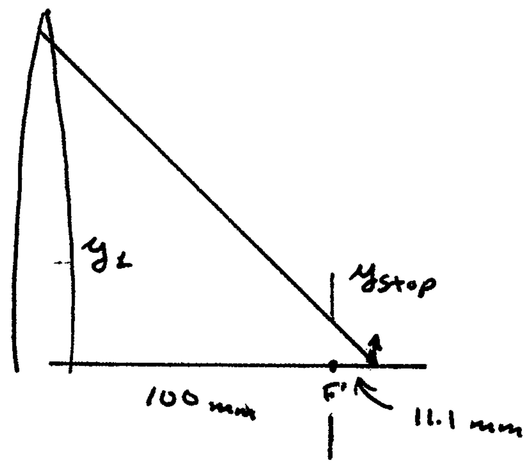
$$\frac{y_L}{111.11\,mm} = \frac{y_{STOP}}{11.1\,mm} = \frac{5\,mm}{11.1\,mm}$$

$y_L = 50\,mm$

$\bar{y}_L = 10\,mm$

$a \geq |y| + |\bar{y}| = 60\,mm$

Dia = $\underline{120\,mm}$



The chief ray height at the lens is the object size.

This requires an f/0.8 lens.

Lens Diameter = 120 mm

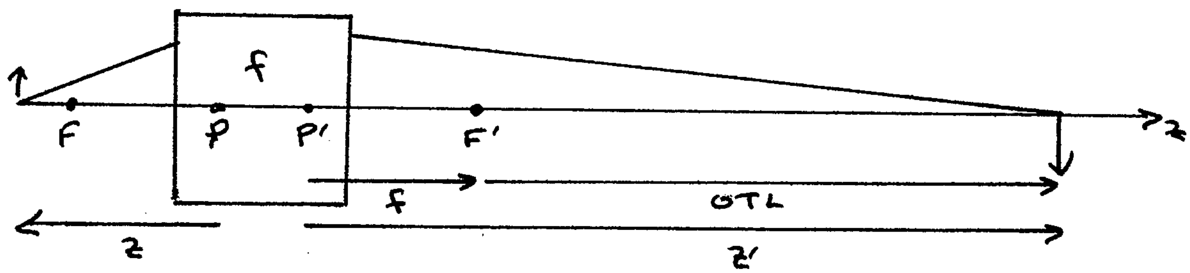
Comments: Increasing the object distance reduces the image size <u>and</u> moves the image closer to the stop. The image-space marginal ray angle increases and the required lens diameter increases to impractically fast f/#'s. As a result, object-space telecentric systems can only be used for <u>near</u> finite conjugate objects.

6) (20 points) A hemispherical lens is used as a 50X objective in a microscope with an optical tube length of 170 mm. The optical tube length is the distance from the rear focal point of the objective to the intermediate image presented to the eyepiece of the microscope.

The hemispherical lens has a radius R, with a thickness of R, and an index n = 1.500. The lens is used in the plano-convex configuration (the plano surface faces the object).

Determine the required radius of curvature for the hemispherical lens R and the object-side working distance provided by this objective. **This problem is to be done using Gaussian methods. No credit will be given for raytrace analysis.**

Treat the objective as a "black box" for imaging:



50X: $m_{obj} = -50 = z'/z$

$z = -z'/50$

$OTL = 170\ mm$

$z' = f + OTL = f + 170mm$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$\frac{1}{z'} = \frac{-50}{z'} + \frac{1}{f}$$

$z' = 51f = f + 170\ mm$

$50f = 170\ mm$

$\underline{f = 3.4\ mm}$

$z' = 173.4mm$

$z = -z'/50 = -3.47\ mm$

Alternate Newtonian Solution:

$$m_{obj} = -50 = -\frac{OTL}{f}$$

$$f = \frac{OTL}{50} = \frac{170\ mm}{50}$$

$\underline{f = 3.4\ mm}$

Continues...

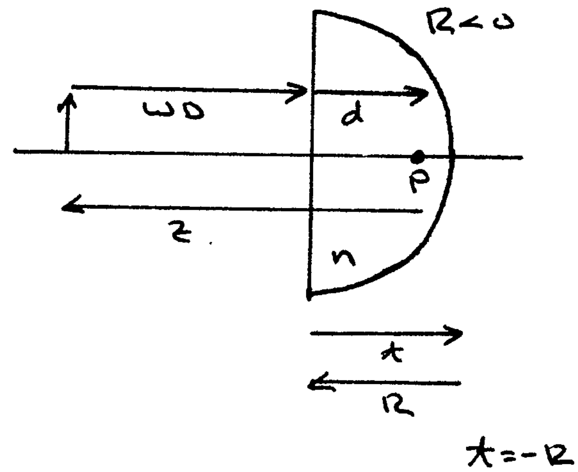
Now examine the hemispherical lens:

$\phi = \phi_2 = (1-n)/R$

$\phi = (n-1)/-R$

$n = 1.5$

$\phi = \frac{-1}{2R}$ $\quad f = -2R$



Front Principal Plane:

$d = \frac{\phi_2}{\phi}\frac{t}{n} = \frac{t}{n} = -\frac{R}{n}$

From imaging: $f = 3.4$ mm $\quad z = -3.47$ mm

$f = -2R$

$\underline{R = -1.7 \text{ mm}}$

$d = -\frac{R}{n}$

$\underline{d = 1.13 \text{ mm}}$

$WD = |z + d| = |-3.47 \text{ mm} + 1.13 \text{mm}|$

$WD = \underline{2.34 \text{ mm}}$

Radius of Curvature R = - 1.7 mm

Working Distance = 2.34 mm