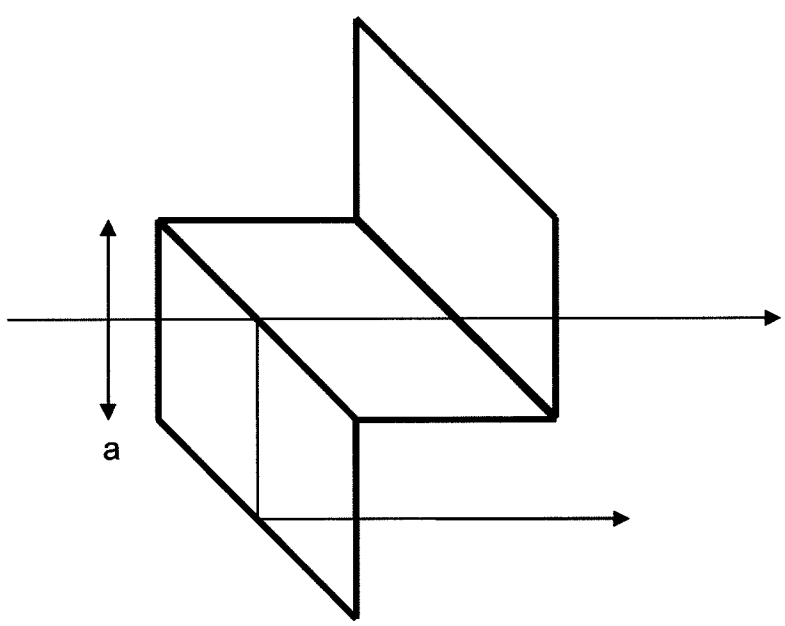
October 20, 2011 Lecture 18 Name Solutions

Closed book; closed notes. Time limit: 75 minutes.
An equation sheet is included. A spare raytrace sheet is also attached
Use the back sides if required.
Assume thin lenses in air if not specified.
If a method of solution is specified in the problem, that method must be used.
You must show your work and/or method of solution in order to receive credit or partial credit for your answer.
Only a basic scientific calculator may be used. This calculator must not have programming or graphing capabilities. An acceptable example is the TI-30 calculator. Each student is responsible for obtaining their own calculator.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (10 points) Draw the tunnel diagram for this prism with the ray path shown. The tunnel diagram must be drawn to the same scale as the prism drawing.

If "a" is the width of the entrance face of the prism, what is the total length of the tunnel diagram? The prism angles are 45° and the rays enter and exit at the center of the prism faces.



Length = 2a

2) (10 points) A 10x10 cm object is to be imaged onto a 10x10 mm detector with a 20 mm focal length thin lens. What is the overall object-to-image distance?

$10\text{ cm} \rightarrow 10\text{ mm}$ $\qquad m = -0.1$

$f = 20\text{ mm}$

$m = -0.1 = \frac{z'}{z}$ $\qquad z' = -0.1\,z$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} =$$

$$\frac{1}{-0.1\,z} = \frac{1}{z} + \frac{1}{20\text{mm}}$$

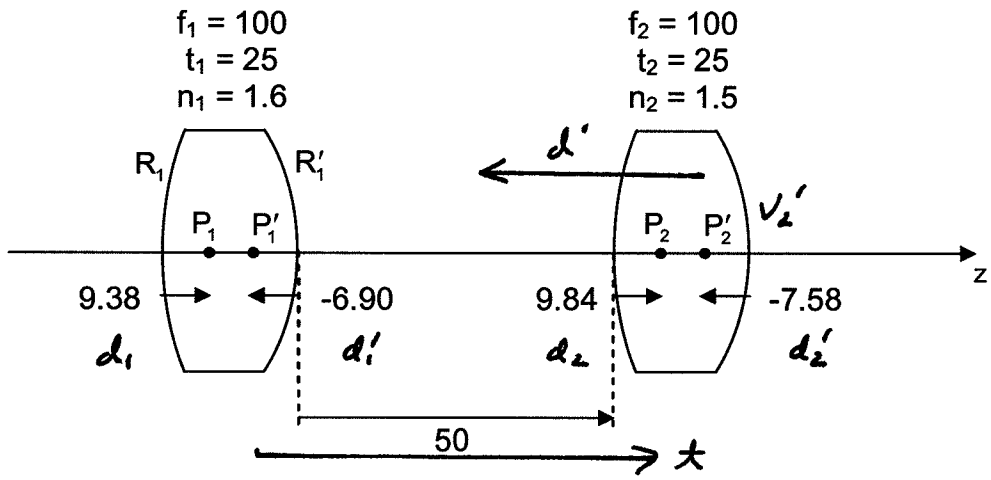
$$z = -220\text{ mm}$$

$$z' = 22\text{ mm}$$

$$L = z' - z = \underline{242\text{ mm}}$$

Object-to Image Distance = 242 mm

3) (25 points) Two thick lenses in air are combined into a single imaging system. Both lenses are 25 mm thick and both lenses have a focal length of 100 mm, however the index of the first lens is 1.6 and the index of the second lens is 1.5. The vertex-to vertex spacing of the lenses is 50 mm. The principal plane locations for the two individual lenses with respect to surface vertices are shown in the figure. All units are in mm.



NOTE: Only Gaussian methods may be used for this problem.

(a) (10 points) Determine the Radii of Curvature of both surfaces of the first lens ($n_1 = 1.6$). Do this for only the first lens of the system.

$f_1 = 100$ mm $\quad \phi_1 = 0.01/\text{mm}$

$t_1 = 25$ mm $\quad n_1 = 1.6 \quad \tau_1 = 15.625$ mm

$\delta_1 = d_1 = \frac{\phi_{S2}}{\phi}\tau_1 = 9.38$ mm $\qquad \delta_2 = d_2 = -\frac{\phi_{S1}}{\phi}\tau_1 = -6.90$

$\phi_{S2} = 0.00600/\text{mm}$ $\qquad \phi_{S1} = 0.00442/\text{mm}$

$\phi_{S2} = (1-n_1)/R_1'$ $\qquad \phi_{S1} = (n_1-1)/R_1$

$R_1' = -100$ mm $\qquad R_1 = 135.8$ mm

$R_1 =$ 135.8 mm $\quad R_1' =$ -100 mm

(b) (15 points) For the system comprised of the two thick lenses, determine:
System Focal Length
Location of the Rear Principal Plane of the system relative to the rear vertex of the second lens
Back Focal Distance

$\phi_1 = \phi_2 = 0.01/mm$ $\qquad t = P_1' \rightarrow P_2$

$t = 50 - d_1' + d_2 = 50 + 6.90 + 9.64$

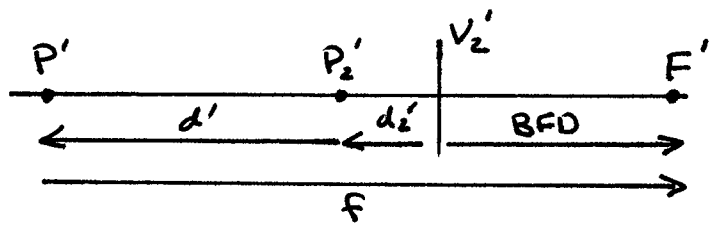
$t = 66.74\ mm$

$n = 1$

$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t = 0.0133/mm$

$f = 1/\phi = \underline{75.0\ mm}$ $\qquad n' = 1$

$\delta' = d' = -\frac{\phi_1}{\phi} t = -50.2\ mm$ (from P_2')



$d_2' = -7.58\ mm$

$\overline{V_2'P'} = d' + d_2' = \underline{-57.8\ mm}$

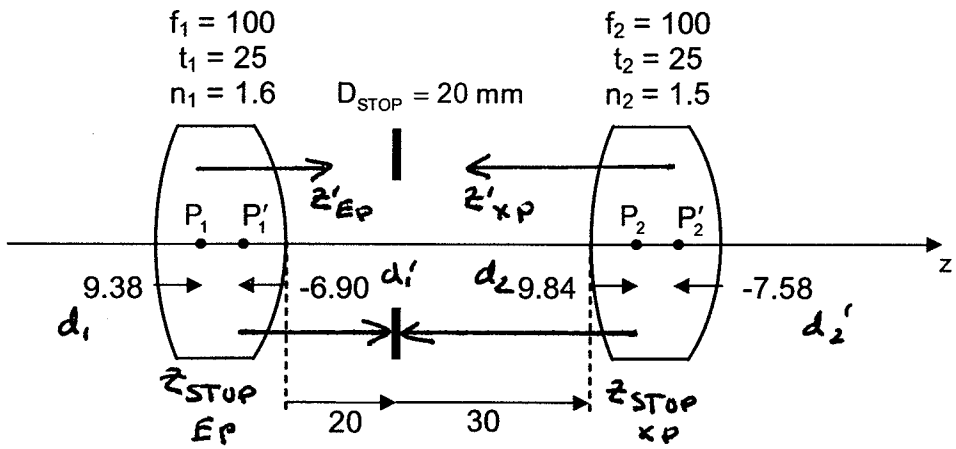
$BFD = f + d' + d_2' = \underline{17.2\ mm}$

System Focal Length = 75.0 mm

System P': Located 57.8 mm to the left of the rear vertex of the second lens.

BFD = 17.2 mm

4) (15 points) A 20 mm diameter stop is now inserted between the two thick lenses (f = 100 mm; t = 25 mm). The stop is 20 mm to the right of the rear vertex of the first lens. The vertex-to-vertex separation of the two lenses is 50 mm. All units are in mm.



Determine the entrance pupil and exit pupil locations and diameters. The entrance pupil is to be located relative to the front vertex of the first lens, and the exit pupil is to be located relative t the rear vertex of the second lens.

NOTE: Only Gaussian methods may be used for this problem.

XP: Image through second lens

$$z_{STOP} = -30 \text{ mm} - d_2 = -39.84 \text{ mm} \quad (\text{from } P_2)$$

$$\frac{1}{z'_{XP}} = \frac{1}{z_{STOP}} + \frac{1}{f_2}$$

$$z'_{XP} = -66.2 \text{ mm} \quad (\text{from } P_2')$$

$$s'_{XP} = z'_{XP} + d_2' = \underline{-73.8 \text{ mm}} \quad (\text{from } V_2')$$

$$m_{XP} = \frac{z'_{XP}}{z_{STOP}} = \frac{-66.2 \text{ mm}}{-39.84 \text{ mm}} = 1.66$$

$$D_{XP} = m_{XP} D_{STOP} = \underline{33.2 \text{ mm}}$$

Continues...

EP: Image stop through first lens

Light R ⇒ L $n = n' = -1$

$z_{STOP} = 20\,mm - d_1' = 26.90\,mm$ (from P_1')

$$\frac{-1}{z'_{EP}} = \frac{-1}{z_{STOP}} + \frac{1}{f_1}$$

$z'_{EP} = 36.8\,mm$ (from P_1)

$S'_{EP} = z'_{EP} + d_1$ $d_1 = 9.38\,mm$

$S'_{EP} = \underline{46.2\,mm}$ (from V_1)

$$m_{EP} = \frac{z'_{EP}/n'}{z_{STOP}/n} = \frac{36.8\,mm}{26.9\,mm} = 1.37$$

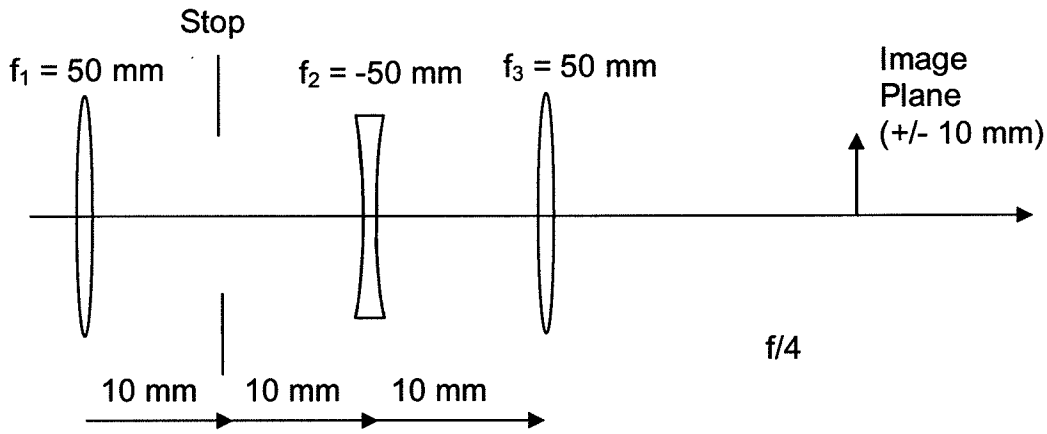
$D_{EP} = m_{EP}\, D_{STOP} = \underline{27.4\,mm}$

EP: D_{EP} = 27.4 mm; Located 46.2 mm to the right of the front vertex of the first lens.

XP: D_{XP} = 33.2 mm; Located 73.8 mm to the left of the rear vertex of the second lens.

5) (30 points) The following diagram shows the design of an objective that is comprised of three thin lenses in air.

The system operates at f/4. The object is at infinity.



The maximum image size is +/- 10 mm.
Determine the following:
- System focal length.
- Back focal distance
- Entrance pupil and exit pupil locations and sizes.
- Stop Diameter.
- Angular field of view (in object space).

NOTE: This problem is to be worked using raytrace methods only. All answers must be determined directly from the rays you trace; for example, the field of view must be determined from the chief ray. Gaussian imaging methods may not be used for any portion of this problem. Be sure to clearly label your rays on the raytrace form.

Your answers must be entered below. Be sure to provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.)

Entrance Pupil: 12.5 mm to the right of the first lens. D_{EP} = 13.6 mm

Exit Pupil: 28.95 mm to the left of the third lens. D_{XP} = 14.3 mm

Stop Diameter = 10.8 mm

System Focal Length = 54.35 mm Back Focal Distance = 28.3 mm

FOV = +/- 10.4 deg in object space

	Obj	EP	L1	Stop	L2	L3	XP	F' Image
Surface	0	1	2	3	4	5	6	7
f			50	-	-50	50		
$-\phi$			-.02	-	.02	-.02		
t		-12.5	10	10	10	-28.95	57.2	

Potential Chief Ray

$\tilde{\bar{y}}$		0	-1.0	0	1.0	2.2	0	
$\tilde{\bar{u}}$		0.08	0.1*	0.1*	0.12	0.076		

Potential Marginal Ray

57.2

$\tilde{y}$	1*	1	1	0.8	0.6	0.52	1.053	0
$\tilde{u}$	0	0	-.02	-.02	-.008	-.0184	-.0184	

Potential Chief Ray-Extended

$\tilde{\bar{y}}$		0	-1.0	0	1.0	2.2	0	4.35
$\tilde{\bar{u}}$		0.08	0.1	0.1	0.12	0.076	0.076	

Marginal Ray - Scale factor = 6.8

y	6.8	6.8	6.8	5.4	4.1	3.5	7.16	0
u	0	0	-0.136	-0.136	-0.054	-0.125	-0.125	

Chief Ray - Scale factor = 2.30

$\bar{y}$		0	-2.3	0	2.3	5.06	0	10.0
$\bar{u}$		0.184	0.23	0.23	0.276	0.175	0.175	

y								
u								

y								
u								

y								
u								

Continues…

*arbitrary

Provide Method of Solution:

<u>EP/XP Location</u>: Trace a potential chief ray starting at the center of the stop. The pupils are located where this ray crosses the axis in object / image space

L1 → EP = 12.5 mm (Right of L1)

L3 → XP = -28.95 mm (Left of L3)

<u>Focal Length</u>: Trace a potential marginal ray parallel to the axis in object space ($\tilde{y}=1$). The rear focal point is located where this ray crosses the axis.

XP → F' = 57.2 mm

BFD = (L3 → XP) + (XP → F') = -28.95 + 57.2

BFD = <u>28.3 mm</u>

$$\phi = -\frac{\tilde{u}'}{\tilde{y}_1} \qquad \tilde{u}' = -.0184 \qquad \tilde{y}_1 = 1.0$$

$\phi = .0184/\text{mm}$

$f = 1/\phi = \underline{54.35\text{ mm}}$

Extend the potential chief ray to F!

Continues...

<u>Entrance Pupil:</u> $f/\# = f/4 = f/D_{EP}$

$$D_{EP} = f/4 = 54.35\,mm/4 = \underline{13.6\,mm}$$

<u>Pupil/Stop Sizes:</u> $r_{EP} = 6.8\,mm$

Scale the potential marginal ray to the proper r_{EP}

Scale factor $= 6.8/1.0 = 6.8$

$r_{STOP} = 5.4\,mm$ $D_{STOP} = \underline{10.8\,mm}$

$r_{XP} = 7.16\,mm$ $D_{XP} = \underline{14.3\,mm}$

<u>FOV:</u> Scale the potential chief ray to the desired image height of 10 mm (from the current value of 4.35 mm),

Scale factor $= 10.0/4.35 = 2.30$

Object Space Chief Ray

$\bar{u}_0 = 0.184$

$HFOV = \tan^{-1}(\bar{u}_0) = 10.4°$

$FOV = 20.8°$ or $+/- 10.4°$

6) (10 points) This diagram shows an optical system consisting of two refracting surfaces and an object. Also shown are the locations and sizes of the stop, the entrance pupil and the exit pupil. Show the paths of the marginal and chief rays through the system along with the location and size of the image. No calculations are required.