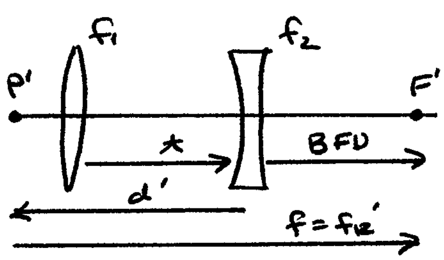
Name Solutions

Closed book; closed notes. Time limit: 2 hours.
An equation sheet is attached and can be removed. A spare raytrace sheet is also attached.
Use the back sides if required.
Assume thin lenses in air if not specified.
As usual, only the magnitude of a magnification or magnifying power may be given.
If a method of solution is specified in the problem, that method must be used.
You must show your work and/or method of solution in order to receive
credit or partial credit for your answer.
Only a basic scientific calculator may be used. This calculator must not have
programming or graphing capabilities. An acceptable example is the TI-30
calculator. Each student is responsible for obtaining their own calculator.

Distance Students: Please return the original exam only; do not scan/FAX/email an additional copy. Your proctor should keep a copy of the completed exam.

1) (10 points) A 200 mm focal length telephoto objective is constructed out of two thin lenses in air. The first lens has a focal length of 150 mm. The system Back Focal Distance is 100 mm. Determine the focal length of the second lens and the separation between the two lenses. **Use Gaussian methods.**



$\phi = \frac{1}{f} = \frac{1}{200} = 0.005/mm$

$\phi_1 = \frac{1}{f_1} = \frac{1}{150} = 0.00667/mm$

$BFD = d' + f_R' = d' + 200 = 100mm$

$d' = -100mm = -\frac{\phi_1}{\phi} t$

$-100mm = -\frac{0.00667}{0.005} t$

$t = 75\ mm$

$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 t$

$\phi_2 (1 - \phi_1 t) = \phi - \phi_1$

$\phi_2 (1 - 0.00667(75)) = 0.005 - 0.00667$

$\phi_2 = -0.00333/mm$

$f_2 = -300\ mm$

Separation = 75 mm f_2 = -300 mm

2) (15 points) The design of a thin lens achromatic doublet is given by the following two equations:

$$\frac{\phi_1}{\phi} = \frac{v_1}{v_1 - v_2} \qquad \frac{\phi_2}{\phi} = -\frac{v_2}{v_1 - v_2}$$

Derive one of these two equations.

$$\phi = \phi_1 + \phi_2 \qquad \delta\phi = \frac{\phi_d}{v} = \frac{\phi}{v}$$

$$\delta\phi_{FC} = \delta\phi_1 + \delta\phi_2$$

$$\delta\phi_{FC} = \frac{\phi_1}{v_1} + \frac{\phi_2}{v_2}$$

For Achromat: $\delta\phi_{FC} = 0$

$$\frac{\phi_1}{v_1} = -\frac{\phi_2}{v_2}$$

$$\phi_2 = -\frac{v_2}{v_1}\phi_1$$

But $\phi = \phi_1 + \phi_2$

$$\phi = \phi_1 - \frac{v_2}{v_1}\phi_1$$

$$\phi = \phi_1\left(\frac{v_1 - v_2}{v_1}\right)$$

$$\underline{\frac{\phi_1}{\phi} = \frac{v_1}{v_1 - v_2}}$$

For ϕ_2:

$$\phi = \phi_1 + \phi_2$$

$$\phi = \phi\left(\frac{v_1}{v_1 - v_2}\right) + \phi_2$$

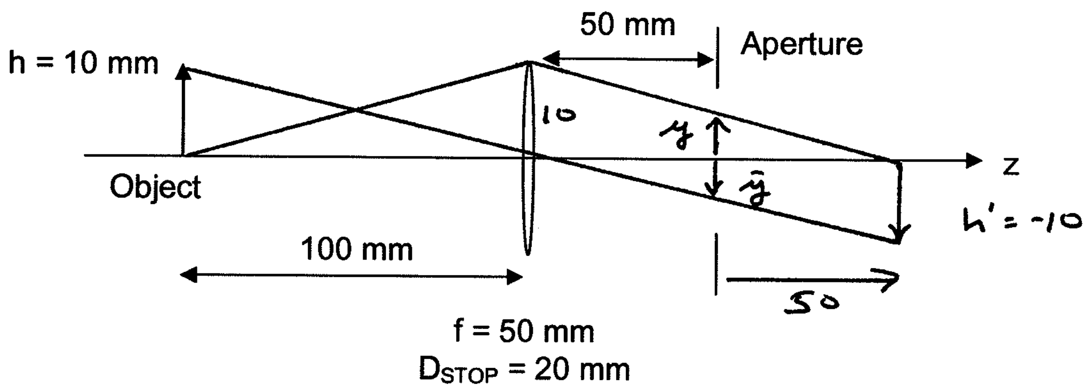
$$\phi_2 = \phi\left(1 - \frac{v_1}{v_1 - v_2}\right)$$

$$\phi_2 = \phi\left(\frac{-v_2}{v_1 - v_2}\right)$$

$$\underline{\frac{\phi_2}{\phi} = -\frac{v_2}{v_1 - v_2}}$$

3) (20 points) An object is located 100 mm to the left of a 50 mm focal length thin lens. The object has a height of 10 mm above the optical axis of the lens. The lens diameter is 20 mm, and the lens serves as the system stop. An additional aperture is placed 50 mm to the right of the lens. What is the required diameter of this aperture so that the system operates without vignetting?

The method of solution is not specified.



Imaging: $z = -100$ mm $f = 50$ mm $\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$

$z' = 100$ mm

System is working at 1:1 conjugates

$h' = -10$ mm

Draw the Marginal + Chief Rays - the ray heights at the aperture plane can be done by inspection:

$y = 5$ mm

$\bar{y} = -5$ mm

For no vignetting:

$a_{APERTURE} \geq |y| + |\bar{y}| = 10$ mm

$D_{APERTURE} \geq 20$ mm

Continues...

The problem can also be solved directly by ray trace:

Raytrace sheet, if needed:

			Obj		Lens		Aper.		Image				
Surface	0		1		2		3		4		5		6
f					50		-						
−φ					-.02		-						
t				100		50		50					
y			0		10		5		0				
u				0.1		-0.1		-0.1					
$\bar{y}$			10		0		-5		-10				
$\bar{u}$				-0.1		-0.1		-0.1					
y													
u													
y													
u													

At Aperture $y = 5$ $\bar{y} = -5$

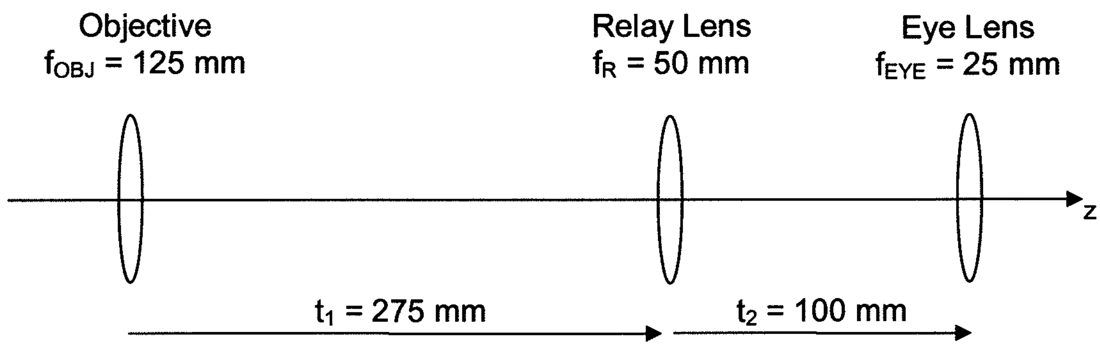
No Vignetting: $a_{APERTURE} \geq |y| + |\bar{y}| = 10\,mm$

$D_{APERTURE} \geq 20\,mm$

$D_{APERTURE} \geq$ 20 mm

4) (30 points) A relayed Keplarian telescope is constructed with three thin lenses in air.

All three lenses have the same diameter of 24 mm.



Determine: The Magnifying Power of the telescope.
The half-vignetted Field of View in degrees in object space.
The locations of the Entrance and Exit Pupils.
The diameters of the Entrance and Exit Pupils.

NOTE: This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem. The field of view must be determined from the chief ray.

Be sure to clearly label your rays on the raytrace form. Your answers must be entered below. You must provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.).

Magnifying Power = 2.5 X FOV = +/- 3.0 deg in object space

Entrance Pupil: 229 mm to the L of the objective lens. D_{EP} = 20 mm

Exit Pupil: 33.3 mm to the R of the eye lens. D_{XP} = 8 mm

			EP		Obj		Relay		Eye		XP		
Surface	0		1		2		3		4		5		6
f					125		50		25				
$-\phi$					-.008		-.02		-.04				
t						275		100					

Potential Marginal Ray

	0		1		2		3		4		5		6
$\tilde{y}$			1		1		-1.2		0.4		0.4		
$\tilde{u}$				0		-.008		.0160		0			

Marginal Ray: Scale Factor = $\lvert 12/-1.2 \rvert = 10$

	0		1		2		3		4		5		6
y			10		10		-12		4		4		
u				0		-.08		.16		0			

Potential Chief Ray

	0		1		2		3		4		5		6
$\tilde{\bar{y}}$					-27.5		0		10				
$\tilde{\bar{u}}$				-0.12		0.1		0.1		-0.3			

Chief Ray: Scale = $12/27.5 = 0.4364$

	0		1		2		3		4		5		6
y					-12		0		4.364				
u				-.05236		.04364		.04364		-.1309			

EP/XP 229.2 33.3

	0		1		2		3		4		5		6
y			0		-12		0		4.364		0		
u				-.05236		.04364		.04364		-.1309			

	0		1		2		3		4		5		6
y													
u													

	0		1		2		3		4		5		6
y													
u													

Continues…

Provide Method of Solution:

- Trace a Potential Marginal Ray

The system magnification can be determined from the ray heights in object and image space:

$$\tilde{y} = 1.0 \qquad \tilde{y}' = 0.4 \qquad m = \frac{\tilde{y}'}{\tilde{y}} = 0.4$$

$$MP = \frac{1}{m} = \underline{2.5\,X} \qquad \text{(erect image)}$$

The base Keplerian telescope operates at a $MP_K = -5\,X$; the magnification of the relay is $-1/2$.

- The aperture stop can be determined by comparing the ray height at each element. All the lens radii are 12 mm ($a_{obj} = a_{relay} = a_{eye} = 12$ mm)

$$\left|\frac{a_{OBJ}}{\tilde{y}_{obj}}\right| = 12 \qquad \left|\frac{a_{relay}}{\tilde{y}_{relay}}\right| = 10 \qquad \left|\frac{a_{eye}}{\tilde{y}_{eye}}\right| = 30$$

The stop is located where this ratio is a minimum – the relay lens is the stop.

- The marginal ray is found by scaling:

$$\text{Scale factor} = \left|\frac{a_{relay}}{\tilde{y}_{relay}}\right| = 10$$

- Since the marginal ray is parallel to the axis in object and image space, the marginal ray height gives the EP + XP radii.

Continues…

Provide Method of Solution:

$a_{EP} = 10 \qquad \underline{D_{EP} = 20\,mm} \qquad a_{XP} = 4 \qquad \underline{D_{XP} = 8\,mm}$

- Launch a potential chief ray through the center of the stop (relay lens).

- For Half-Vignetting $a = |\bar{y}|$

  The objective lens clearly limits the chief ray.

  The chief ray is scaled to the radius of the objective:

$$Scale = \left| \frac{a_{obj}}{\bar{y}_{obj}} \right| = \left| \frac{12}{-27.5} \right| = 0.4364$$

- The chief ray angle (slope) in object space provides the FOV

$$\bar{u} = -0.05236 \qquad HFOV = \tan^{-1}(\bar{u})$$

$$HFOV = -3.0\ deg$$

  Note that the ratio of $\bar{u}'$ to $\bar{u}$ is the MP

$$\frac{\bar{u}'}{\bar{u}} = \frac{-.1309}{-0.05236} = 2.5\,X$$

- The EP and XP locations are found by extending the chief ray until it crosses the axis.

  EP: 229.2 mm to the left of the objective

  XP: 33.3 mm to the right of the eye lens

Both pupils are real.

5) (10 points) The image in the eye is formed in an index of refraction of 1.336. The rear focal length of the eye is 22.4 mm. An object is 1 meter in front of the eye (in air) and has a height of 20 mm. What is the height of the image formed in the eye (on the retina)? **Use Gaussian methods.**

Assume that the eye changes length to keep the image in focus.

$f_R' = 22.4$ mm $\qquad n' = 1.336$

$f = f_R'/n' = 16.77$ mm

$\phi = 0.0596$ /mm

Imaging: $\quad z = -1$ m $= -1000$ mm $\qquad n = 1.0$

$$\frac{n'}{z'} = \frac{n}{z} + \phi = -.001 + .0596$$

$$\frac{n'}{z'} = .0586 \qquad z' = 22.8 \text{ mm}$$

$$m = \frac{z'/n'}{z/n} = \frac{22.8/1.336}{-1000} = -.0171$$

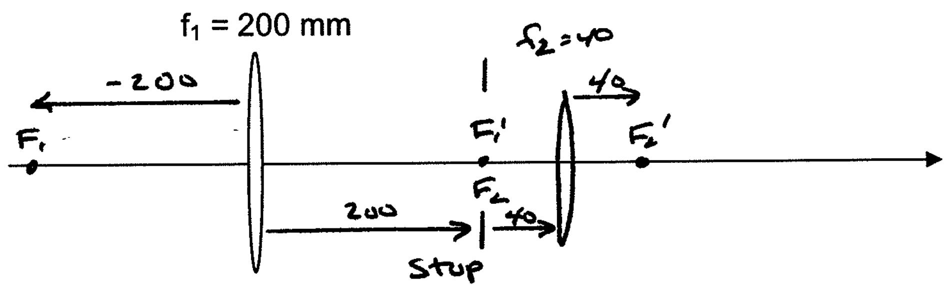
$h' = m h \qquad h = 20$ mm

$h' = -0.341$ mm (Inverted)

Image Height = -.341 mm

6) (15 points) A double telecentric system is constructed out of two thin lenses in air. It has a magnification of 1/5. The focal length of the first lens of the system is 200 mm. Provide a layout of the system showing the second lens, spacings and the stop.

Two objects are located 400 mm and 100 mm to the left of the first lens in the system. Where are the respective image planes (located relative to the second lens element)?



Design: $m = -1/5 = -f_2/f_1 = -f_2/200$

$f_2 = 40$ mm

$L = f_1 + f_2 = 240$ mm

Stop at the common focal point.

Imaging: The reference conjugates are at F_1 ; F_2'

Use $\bar{m} = m^2 = 1/25$

Object at 400 mm: $\Delta z = -200$ mm (from F_1)

$\Delta z' = \bar{m}\,\Delta z = -8$ mm (from F_2')

$z' =$ 32 mm to the right of L2

Object at 100 mm: $\Delta z = 100$ mm (from F_1)

$\Delta z' = \bar{m}\,\Delta z = 4$ mm (from F_2')

$z' =$ 44 mm to the right of L2

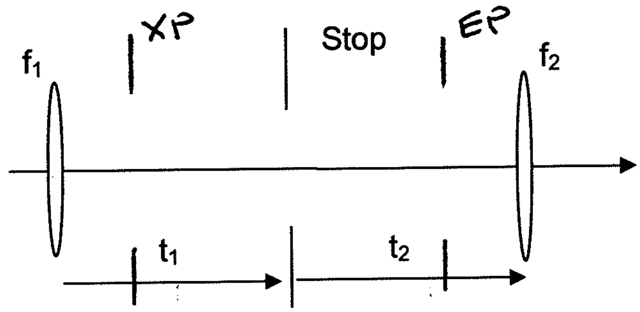
Focal length of the second lens = 40 mm

Object at 400 mm: Image is 32 mm to the R of the second lens

Object at 100 mm: Image is 44 mm to the R of the second lens

Bonus/Extra Credit (5 points) We all know that in Gaussian reduction, a system of lenses is replaced with a pair of principal planes and a system focal length or power. One thing we did not talk about is what happens to the stop!

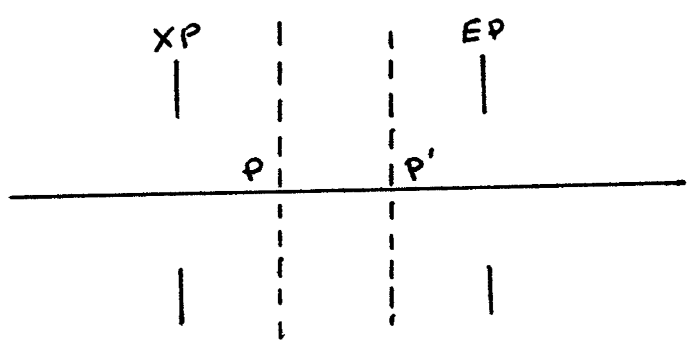
Consider the following system which has a stop located between two thin lenses:



Discuss how you would include the optical properties of the stop in the reduced system.

Following reduction, the intermediate optical space is no longer present and the stop will not be present in the reduced representation. However, the EP and the XP of the original system are in object space and image space and are present in the reduced representation.

As a result, the reduced model will contain a pair of separated principal planes along with the EP and the XP. The separation of the principal planes preserves the physical location of the pupils.



Both pupils are shown as being virtual.