1) (10 points) A 200 mm focal length telephoto objective is constructed out of two thin lenses in air. The first lens has a focal length of 150 mm. The system Back Focal Distance is 100 mm. Determine the focal length of the second lens and the separation between the two lenses. Use Gaussian methods.

\[ \phi = \frac{1}{f_1} = \frac{1}{150} = 0.00667/m\]

\[ d' = -100 = -\frac{\phi_1}{\phi} x \]

\[ x = 7.5\ mm \]

\[ f_2 = -300\ mm \]

\[ \text{Separation} = 7.5\ mm \]

\[ f_2 = -300\ mm \]
2) (15 points) The design of a thin lens achromatic doublet is given by the following two equations:

\[
\phi_1 = \frac{\nu_1}{\nu_1 - \nu_2} \quad \phi_2 = -\frac{\nu_2}{\nu_1 - \nu_2}
\]

Derive one of these two equations.

\[
\begin{align*}
\phi &= \phi_1 + \phi_2 \\
\delta \phi &= \delta \phi_1 + \delta \phi_2 \\
\delta \phi_{RL} &= \frac{\phi_1}{\nu_1} + \frac{\phi_2}{\nu_2}
\end{align*}
\]

For Achromatic: \( \delta \phi_{RL} = 0 \)

\[
\begin{align*}
\frac{\phi_1}{\nu_1} &= -\frac{\phi_2}{\nu_2} \\
\phi_2 &= -\frac{\nu_2}{\nu_1} \phi_1 \\
\text{But} \quad \phi &= \phi_1 + \phi_2 \\
\phi &= \phi_1 - \frac{\nu_2}{\nu_1} \phi_1 \\
\phi &= \phi_1 \left( \frac{\nu_1 - \nu_2}{\nu_1} \right) \\
\frac{\phi_1}{\phi} &= \frac{\nu_1}{\nu_1 - \nu_2}
\end{align*}
\]

For \( \phi_2 \):

\[
\begin{align*}
\phi &= \phi_1 + \phi_2 \\
\phi &= \phi \left( \frac{\nu_1}{\nu_1 - \nu_2} \right) + \phi_2 \\
\phi_2 &= \phi \left( 1 - \frac{\nu_2}{\nu_1 - \nu_2} \right) \\
\phi_2 &= \phi \left( \frac{-\nu_2}{\nu_1 - \nu_2} \right) \\
\frac{\phi_2}{\phi} &= -\frac{\nu_2}{\nu_1 - \nu_2}
\end{align*}
\]
3) (20 points) An object is located 100 mm to the left of a 50 mm focal length thin lens. The object has a height of 10 mm above the optical axis of the lens. The lens diameter is 20 mm, and the lens serves as the system stop. An additional aperture is placed 50 mm to the right of the lens. What is the required diameter of this aperture so that the system operates without vignetting?

The method of solution is not specified.

Imaging: \( \frac{1}{z} = \frac{1}{f} + \frac{1}{z'} \)

\( f = 50 \text{ mm} \)
\( z = -100 \text{ mm} \)
\( z' = 100 \text{ mm} \)

System is working at 1:1 conjugate;
\( h' = -10 \text{ mm} \)

Draw the Marginal and Chief Rays - the ray heights at the aperture plane can be done by inspection:
\( y = 5 \text{ mm} \)
\( y = -5 \text{ mm} \)

For no vignetting:
\( d_{\text{APERTURE}} \geq |y| + |y| = 10 \text{ mm} \)
\( D_{\text{APERTURE}} \geq 20 \text{ mm} \)

Continues...
The problem can also be solved directly by ray trace.

Raytrace sheet, if needed:

<table>
<thead>
<tr>
<th>Surface</th>
<th>Obj</th>
<th>Lens</th>
<th>Aper.</th>
<th>Im.</th>
<th>n</th>
</tr>
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<tbody>
<tr>
<td>f</td>
<td>50</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-φ</td>
<td>-0.2</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| y    | 0   | 10  | 5   | 0   |   |
| u    | 0.1 | -0.1| -0.1|     |   |

| y    | 10  | 0   | -5  | -10 |   |
| u    | -0.1| -0.1| -0.1|     |   |

At Aperture: \( y = 5 \) \( \overline{y} = -5 \)

No vignetting: \( |y| + |\overline{y}| = 10 \text{mm} \)

\( D_{\text{Aperture}} \geq 20 \text{ mm} \)
4) (30 points) A relayed Keplarian telescope is constructed with three thin lenses in air. All three lenses have the same diameter of 24 mm.

The objective, relay lens, and eye lens are labeled in the diagram.

Objective

\( f_{\text{OBJ}} = 125 \text{ mm} \)

Relay Lens

\( f_{R} = 50 \text{ mm} \)

Eye Lens

\( f_{\text{EYE}} = 25 \text{ mm} \)

\( t_1 = 275 \text{ mm} \)

\( t_2 = 100 \text{ mm} \)

Determine:

- The Magnifying Power of the telescope.
- The half-vignetted Field of View in degrees in object space.
- The locations of the Entrance and Exit Pupils.
- The diameters of the Entrance and Exit Pupils.

NOTE: This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem. The field of view must be determined from the chief ray.

Be sure to clearly label your rays on the raytrace form. Your answers must be entered below. You must provide details on the pages that follow to indicate your method of solution (how did you get your answer: which ray was used, analysis of ray data, etc.).

Magnifying Power = \( 2.5 \times \)

FOV = +/- 3.0 deg in object space

Entrance Pupil: 229 mm to the \( \text{L} \) of the objective lens. \( D_{EP} = 20 \) mm

Exit Pupil: 33.3 mm to the \( \text{R} \) of the eye lens. \( D_{XP} = 8 \) mm
<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
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<th>5</th>
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<td></td>
<td>12.5</td>
<td>50</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-φ</td>
<td></td>
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<td>-.02</td>
<td>-.04</td>
<td></td>
<td></td>
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<tr>
<td>t</td>
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**Potential Marginal Ray**

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<tr>
<th>y</th>
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<td>.0160</td>
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**Marginal Ray: Scale Factor = \(|12/1.2| = 10\)**

<table>
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<th>-12</th>
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<th>4</th>
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<th></th>
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<tbody>
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</table>

**Potential Chief Ray**

<table>
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<th>y</th>
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<th>-27.5</th>
<th>0</th>
<th>10</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>-.012</td>
<td>0.1</td>
<td>0.1</td>
<td>-0.3</td>
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</table>

**Chief Ray: Scale = 12/27.5 = 0.4364**

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<tbody>
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<td>.04364</td>
<td>-.1309</td>
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</tbody>
</table>

**EP/XP**

<table>
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<th>0</th>
<th>4.364</th>
<th>0</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td></td>
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<td>.04364</td>
<td>.04364</td>
<td>-.1309</td>
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<td></td>
</tr>
</tbody>
</table>
Provide Method of Solution:

- Trace a Potential Marginal Ray

The system magnification can be determined from the ray heights in object and image space:

\[ \frac{h_o}{h'_{o}} = 1.0, \quad \frac{h_i}{h'_{i}} = 0.4, \quad m = \frac{h'_{o}}{h_{o}} = 0.4 \]

\[ MP = \frac{1}{m} = 2.5 \times \text{(erect image)} \]

The base Keplerian telescope operates at a

\[ MP_{k} = -5 \times \]; the magnification of the relay is \(-\frac{1}{2}\).

- The aperture stop can be determined by comparing the ray height at each element. All the lens radii are 12 mm (\(a_{obj} = a_{relay} = a_{eye} = 12 \text{ mm}\))

\[ \left| \frac{a_{obj}}{h_{obj}} \right| = 12, \quad \left| \frac{a_{relay}}{h_{relay}} \right| = 10, \quad \left| \frac{a_{eye}}{h_{eye}} \right| = 30 \]

The stop is located where this ratio is a minimum – the relay lens is the stop.

- The marginal ray is found by scaling:

Scale factor = \[ \left| \frac{a_{relay}}{h_{relay}} \right| = 10 \]

- Since the marginal ray is parallel to the axis in object and image space, the marginal ray height gives the EP + XP radii.

Continues...
Provide Method of Solution:

\[ a_{EP} = 10 \quad D_{EP} = 20 \text{ mm} \quad a_{XP} = 4 \quad D_{XP} = 8 \text{ mm} \]

- Launch a potential chief ray through the center of the stop (relay lens).

- For half Vignetting \( a = \frac{1}{2} \)

The objective lens clearly limits the chief ray.

The chief ray is scaled to the radius of the objective:

\[ \text{scale} = \left| \frac{a_{obj}}{a_{obj}} \right| = \left| \frac{12}{-27.5} \right| = 0.436 \text{ V} \]

- The chief ray angle (slope) in object space provides the FOV

\[ \bar{u} = -0.05236 \quad \text{HFOV} = \tan^{-1}(\bar{u}) \]

\[ \text{HFOV} = -3.0 \text{ deg} \]

Note that the ratio of \( \bar{u}' \) to \( \bar{u} \) is the HP

\[ \frac{\bar{u}'}{\bar{u}} = \frac{-1.1309}{-0.05236} = 21.5 \]

- The EP and XP locations are found by extending the chief ray until it crosses the axis.

EP: 229.2 mm to the left of the objective

XP: 33.3 mm to the right of the eye lens

Both pupils are real.
5) (10 points) The image in the eye is formed in an index of refraction of 1.336. The rear focal length of the eye is 22.4 mm. An object is 1 meter in front of the eye (in air) and has a height of 20 mm. What is the height of the image formed in the eye (on the retina)? Use Gaussian methods.

Assume that the eye changes length to keep the image in focus.

\[ f' = 22.4 \text{ mm} \quad n' = 1.336 \]

\[ f = \frac{f'}{n'} = 16.77 \text{ mm} \]

\[ \phi = 0.0596 / \text{mm} \]

Imaging:

\[ \frac{h'}{z'} = \frac{h}{z} + \phi = -0.001 + 0.0596 \]

\[ \frac{h'}{z'} = 0.0586 \quad z' = 22.8 \text{ mm} \]

\[ m = \frac{z'/n'}{z/n} = \frac{22.8/1.336}{-1000} = -0.0171 \]

\[ h' = m \cdot h \quad h = 20 \text{ mm} \]

\[ h' = -0.341 \text{ mm} \quad (\text{Inverted}) \]

Image Height = -0.341 mm
6) (15 points) A double telecentric system is constructed out of two thin lenses in air. It has a magnification of 1/5. The focal length of the first lens of the system is 200 mm. Provide a layout of the system showing the second lens, spacings and the stop.

Two objects are located 400 mm and 100 mm to the left of the first lens in the system. Where are the respective image planes (located relative to the second lens element)?

\[
\begin{align*}
\text{Object at 400 mm:} & \quad \omega z = -200 \text{ mm (from } F_1) \\
& \quad \omega z' = \bar{m} \omega z = -8 \text{ mm (from } F_2') \\
& \quad z' = 32 \text{ mm to the right of } L_2 \\
\text{Object at 100 mm:} & \quad \omega z = 100 \text{ mm (from } F_1) \\
& \quad \omega z' = \bar{m} \omega z = 4 \text{ mm (from } F_2') \\
& \quad z' = 44 \text{ mm to the right of } L_2
\end{align*}
\]

Focal length of the second lens = \_40\_ mm

Object at 400 mm: Image is \_32\_ mm to the \_R\_ of the second lens

Object at 100 mm: Image is \_44\_ mm to the \_R\_ of the second lens
Bonus/Extra Credit (5 points) We all know that in Gaussian reduction, a system of lenses is replaced with a pair of principal planes and a system focal length or power. One thing we did not talk about is what happens to the stop!

Consider the following system which has a stop located between two thin lenses:

Discuss how you would include the optical properties of the stop in the reduced system.

Following reduction, the intermediate optical space is no longer present and the stop will not be present in the reduced representation. However, the EP and the XP of the original system are in object space and image space and are present in the reduced representation.

As a result, the reduced model will contain a pair of separated principal planes along with the EP and the XP. The separation of the principal planes preserves the physical location of the pupils.

Both pupils are shown as being virtual.