1) (10 points) Draw the tunnel diagram for beam displacement deviation prism with the ray path shown. The tunnel diagram must be drawn to the same scale as the prism drawing. It may be helpful to first describe the given prism system in terms of component prisms.
2) (10 points) What is the angular field of view of an optical system consisting of a 38 mm focal length lens with a 2 cm diameter detector? The object is at infinity. Provide the full field of view, and the system is in air.

\[ \tan \frac{\theta}{2} = \frac{10 \text{ mm}}{38 \text{ mm}} = 0.263 \]

\[ \frac{\theta}{2} = 14.7^\circ \quad \theta = 29.4^\circ \]

Since the system is in air, the principal planes and nodal planes are coincident. The ray angle transfers from \( P' \) to \( P \).

Since the object is at infinity, the chief ray is parallel to this object-space ray - independent of pupil position.

Field of View = 29.5 deg
3) (10 points) My favorite building on the campus is the West Wing of the Meinel Building that houses the College of Optical Sciences. The building rises to a height of about 80 feet. You are standing across the mall and taking a picture of this building with a digital camera that has a 1 cm square detector. The mall is about 200 feet wide, and the image of the building should fill the vertical dimension of the sensor. Approximately what focal length lens is required?

$$80 \text{ feet} = 24,000 \text{ mm}$$

$$m = \frac{h'}{h} = -\frac{10 \text{ mm}}{24,000 \text{ mm}} = -\frac{1}{2,400}$$

$$z = -200 \text{ feet} = -61 \text{ m} = -61,000 \text{ mm}$$

$$m = \frac{z'}{z} = \frac{f}{z}$$

$$f = m z$$

$$f \approx 25 \text{ mm}$$

For the image of the building to fill to height of the detector, the optical axis of the camera must be pointed at the middle of the building.

$$f \approx 25 \text{ mm}$$
4) An air-spaced triplet is comprised of three thin lenses in air:

\[
\begin{align*}
&f_1 = 20 \text{ mm} & t_1 = 10 \text{ mm} & \phi_1 = .05/\mu m \\
&f_2 = -20 \text{ mm} & t_2 = 5 \text{ mm} & \phi_2 = -.05/\mu m \\
&f_3 = 20 \text{ mm} & & \phi_3 = .05/\mu m
\end{align*}
\]

Note: No raytrace analysis is permitted for either portion of this problem.

a) (15 points) Use Gaussian methods to determine the system Focal Length and Back Focal Distance.

Reduce the first two lenses:

\[
\phi_{12} = \phi_1 + \phi_2 - \phi_1 \phi_2 t_1,
\]

\[
\phi_{12} = .025/\mu m
\]

\[
d_{12}' = \frac{-\phi_1}{\phi_{12}} t_1 = -20 \mu m
\]

\[
t_{12} = t_2 - d_{12}' = 25 \mu m
\]
b) (15 points) The system stop has a diameter of 10 mm and is located halfway between the first two thin lenses (5 mm behind the first lens). Use Gaussian methods to determine the locations and sizes of the system Entrance and Exit Pupils.

\[ \phi_{1\rightarrow 3} = \phi_{1\rightarrow 2} + \phi_{3\rightarrow 2} - \phi_{1\rightarrow 2} \chi_{1\rightarrow 2} \]

\[ \phi_{1\rightarrow 2} = 0.4375 \text{ /mm} \]

\[ \frac{1}{f} = \frac{1}{\phi_{1\rightarrow 2}} = \frac{22.86 \text{ mm}}{} \]

\[ d_{1\rightarrow 3} = -\frac{\phi_{1\rightarrow 2}}{\phi_{2\rightarrow 2}} \chi_{1\rightarrow 2} = -14.28 \text{ mm} \]

\[ \text{BFD} = f + d_{1\rightarrow 3} = 8.57 \text{ mm} \]

\[ f = 22.86 \text{ mm} \]

\[ \text{BFD} = 8.57 \text{ mm} \]
\[ z_{1a} = 5 \text{ mm} \]
\[ \frac{1}{z'_{EP}} = \frac{1}{z_{1a}} + \frac{1}{f_1} \]
\[ z'_{EP} = \frac{6.67 \text{ mm}}{(R \text{ of } L1)} \]
\[ m_{EP} = \frac{z'_{EP}}{z_{1a}} = \frac{6.67}{5.0} = 1.333 \]
\[ D_{EP} = m_{EP} \cdot D_{STOP} = 13.33 \text{ mm} \]
\[ XP: \text{ Image the stop through } L2 \text{ to obtain an intermediate pupil:} \]
\[ \frac{1}{z'_{2b}} = \frac{1}{z_{2a}} + \frac{1}{f_2} \]
\[ z'_{2b} = -4 \text{ mm} \]
\[ m_2 = \frac{z'_{2b}}{z_{2a}} = \frac{-4}{-5} = 0.8 \]
\[ \text{Now image the intermediate pupil through } L3 \text{ for the } XP: \]
\[ \frac{1}{z'_{XP}} = \frac{1}{z_{3b}} + \frac{1}{f_3} \]
\[ z'_{XP} = \frac{-16.36 \text{ mm}}{(L \text{ of } L3)} \]
\[ m_3 = \frac{z'_{XP}}{z_{3b}} = \frac{-16.36}{-9} = 1.82 \]
\[ D_{XP} = m_2 \cdot m_3 \cdot D_{STOP} = 14.6 \text{ mm} \]

XP can also be determined by reducing lenses 2 + 3.

EP: \( D_{EP} = 13.33 \text{ mm; Located } 6.67 \text{ mm to the } R \text{ of the first lens.} \)

XP: \( D_{XP} = 14.6 \text{ mm; Located } 16.36 \text{ mm to the } L \text{ of the third lens.} \)
Alternate Solution for XP

Reduce elements 2 and 3, then image the stop.

\[ \phi_{23} = \phi_2 + \phi_3 - \phi_2 \phi_3 \frac{1}{f_2} = 0.0125 \text{ mm} \]

\[ f_{23} = 80 \text{ mm} \]

\[ d_{23} = \frac{\phi_3}{f_{23}} \frac{1}{f_2} = 20 \text{ mm} \]

\[ d_{23}' = - \frac{\phi_2}{f_{23}} \frac{1}{f_2} = 20 \text{ mm} \]

\[ s_5 = s_5 - d_{23} = -25 \text{ mm} \]

\[ s_5 = -5 \text{ mm} \quad (L_2 \Rightarrow \text{ stop}) \]

\[ \frac{1}{z'_{XP}} = \frac{1}{s_5} + \frac{1}{f_{23}} \]

\[ z'_{XP} \quad (\text{from } P_{23}) \]

\[ z'_{XP} = -36.36 \text{ mm} \]

\[ s'_{XP} \quad (\text{from } L_3) = z'_{XP} + d_{23}' = -16.36 \text{ mm} \quad (L \text{ of } L_3) \]

\[ m_{XP} = \frac{z'_{XP}}{s_5} = \frac{-36.36 \text{ mm}}{-25.0 \text{ mm}} = 1.45 \]

\[ D_{XP} = m_{XP} D_{STOP} = 14.5 \text{ mm} \]

EP: \[ D_{EP} = \quad \text{mm}; \quad \text{Located} \quad \text{mm to the \quad of the first lens.} \]

XP: \[ D_{XP} = \quad \text{mm}; \quad \text{Located} \quad \text{mm to the \quad of the third lens.} \]
5) An optical system consists of two thick lenses in air with a stop between the lenses as shown. The diameter of the stop is 20 mm.

All of the radii of curvature are 100 mm. Use the diagram to determine the sign for each surface radius of curvature.

The index of the first lens is 1.5, and the index of the second lens is 1.7

This problem is to be worked using raytrace methods only. Gaussian imaging methods may not be used for any portion of this problem.

For grading purposes, the values for your final rays must be entered into the raytrace sheets that follow. Use the raytrace sheets noted for the marginal ray and the chief ray.

Be sure to clearly label your rays, and note which ray is used to determine each answer.

a) (10 points) Determine the locations of the entrance and exit pupils.

Trace a potential chief ray starting at the center of the stop. The EP and XP are located where this ray crosses the axis in object and image space.

\[ A_1: \ EP \rightarrow S_1 = -27.75 \text{ (EP to the R of } S_1) \]
\[ A_2: \ S_4 \rightarrow XP = -59.62 \text{ (XP to the L of } S_4) \]

Entrance Pupil: \(27.75\) mm to the \(R\) of the first surface.

Exit Pupil: \(59.62\) mm to the \(L\) of the last surface.
b) (10 points) Determine the focal length and back focal distance of this optical system.

Trace a potential marginal ray for an axial object at infinity (\(u = 1\), \(u_0 = 0\)).

The rear focal point \(F'\) is located where this ray crosses the axis.

\[ \beta_1 = 194.0 = x_P \rightarrow F' \quad \text{and} \quad \nu_1 = x_P = -59.62 \]

\[ BF0 = 194.0 - 59.62 = 134.4 \text{ mm} \]

\[ \phi = -\frac{\beta_1}{\nu_1} = 0.01156 \]

\[ f = \frac{1}{\phi} = 86.5 \text{ mm} \]

\[ f = 86.5 \text{ mm} \quad \text{BFD} = 134.4 \text{ mm} \]

c) (10 points) Determine the diameters of the Entrance and Exit Pupils.

Scale the potential marginal ray to the stop radius:

\[ D_{\text{stop}} = 20 \text{ mm} \quad a_{\text{stop}} = 10 \text{ mm} \]

Scale factor = \( \frac{a_{\text{stop}}}{D_{\text{stop}}} = \frac{10 \text{ mm}}{20 \text{ mm}} = 0.5 \)

\[ y_{EP} = 7.656 \text{ mm} \quad D_{EP} = 15.31 \text{ mm} \]

\[ y_{XP} = 17.11 \text{ mm} \quad D_{XP} = 34.34 \text{ mm} \]

Entrance Pupil Diameter = 15.31 mm

Exit Pupil Diameter = 34.34 mm
d) (10 points) The system has a Field of View of +/- 10 degrees. What is the image height corresponding to this FOV?

\[ \tilde{\omega}_0 = \tan 10^\circ = 0.1763 \]

Extend the potential chief ray to \( F' \) and scale the ray to this ray angle in object space.

Scale Factor: \[ \frac{\tilde{\omega}_0}{\tilde{\omega}_0} = \frac{0.1763}{0.1306} = 1.350 \]

Image Height at \( F' \) = \( \tilde{g}' = 15.24 \text{ mm} \)

Image Height = +/- 15.24 mm