1) (5 points) The letter "A" is imaged by an aberration free thin lens (stop at the lens). A real image is produced. The top half of the lens is now blocked by an opaque card. What happens to the image?

The image gets dimmer.

This is no different than stopping down the lens (or changing its diameter).
2) (10 points) A stop of diameter 20 mm is located 75 mm to the right of a 200 mm focal length thin lens. Use Gaussian methods to determine the location and size of the system entrance pupil.

\[ f = 200 \text{ mm} \quad D_{\text{Stop}} = 20 \text{ mm} \]

\[ z = 75 \text{ mm} \]

Light propagates to the left for the real stop \((u = u' = -1)\)

\[ \frac{-1}{z'} = \frac{-1}{2} + \frac{1}{f} \quad z' = 120 \text{ mm} \]

The EP is virtual.

\[ m_{\text{EP}} = \frac{z'}{z} = \frac{-120 \text{ mm}}{-75 \text{ mm}} = 1.6 \]

\[ D_{\text{EP}} = m_{\text{EP}} D_{\text{Stop}} = 32 \text{ mm} \]

EP Location: 120 mm to the right of the lens

EP Diameter = 32 mm
3) (25 points) A system is comprised of three thin lenses. The following partially-completed raytrace of the system is the starting point for this problem, and will be completed during various parts of the problem. Extra lines/rays are provided.

Be sure to properly label your raytraces.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Object</th>
<th>EP</th>
<th>Lens 1</th>
<th>Lens 2</th>
<th>Lens 3</th>
<th>XP</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

\[ f = \begin{array}{llll}
100\ mm & -100\ mm & 100\ mm \\
\end{array} \]

\[ \Phi = \begin{array}{llll}
-0.1 & 0.1 & -0.1 \\
\end{array} \]

\[ t = \begin{array}{llll}
-60.67 & 40\ mm & 60\ mm & \text{(-150)} & 1197.4 \\
\end{array} \]

Potential Chief Ray:

\[ \begin{array}{ccccccc}
\bar{y} & 0 & 4.0\ mm & 0.0\ mm & -6.0\ mm & 0 \\
\bar{u} & -0.06 & -0.06 & -0.1 & -0.1 & -0.04 & -0.04 \\
\end{array} \]

Marginal Ray - Scale by 14.67

\[ \begin{array}{ccccccc}
y & 14.67 & 14.67 & 14.67 & 10 & 6.0 & 25.0 & 0 \\
u & 0 & 0 & -0.1 & -0.004 & -0.0076 & -0.0076 \\
\end{array} \]

Chief Ray - Scale by 2.938

\[ \begin{array}{ccccccc}
y & 0 & 11.75 & 0 & -17.63 & 0 & -23.2 \\
u & -1763 & -1763 & -2938 & -2938 & -1175 & -1175 \\
\end{array} \]
a) Which element serves as the System Stop (circle one)?

The chief ray goes through the center of the stop

Lens 1 [Diagram]  Circle  Lens 2  Lens 3

b) Determine the Entrance and Exit Pupil locations.

Extend the potential chief ray to object and image space. \( \frac{y}{y_o} = 0 \) at EP, XP.

EP: Located 66.67 mm to the right of the first lens.

XP: Located 150 mm to the left of the third lens.

c) Determine the system Focal Length and its Back Focal Distance (BFD).

Trace a potential marginal ray from infinity.

\( \theta' = 0.0076 \)

\( \phi = -\theta' / y_o = 0.0076 \)

\( f = \frac{1}{\phi} = 131.58 \text{ mm} \)

\( f = 131.58 \text{ mm} \)

BFD = \( L_3 \rightarrow \text{XP} \) + (XP \rightarrow \text{Image})

BFD = -150 + 197.4

BFD = 47.4

BFD = 47.4 mm

D) The System Stop has a diameter of 20 mm. Determine the diameters of the Entrance Pupil and the Exit Pupil.

Scale the marginal ray to a radius of 10 mm at the stop.

Scale Factor: \( \frac{10 \text{ mm}}{0.6} = \frac{10 \text{ mm}}{0.6} = 16.67 \)

Entrance Pupil Diameter = \( \frac{33.33 \text{ mm}}{16.67} \)

Exit Pupil Diameter = \( \frac{50.0 \text{ mm}}{16.67} \)

Continues...
e) For distant objects, the system has an unvignetted Field of View of +/- 10 deg. What is the image height in the image plane for this FOV? What are the required Lens Diameters to support this FOV?

\[ \bar{u}_0 = \tan(10^\circ) = 0.1763 \]

Scale the chief ray to the object space value.

Scale Factor: \( \frac{\bar{u}_0}{u_0} = \frac{0.1763}{0.06} = 2.938 \)

Extend the chief ray to the image plane to obtain the image height: \( \bar{y}' = -23.2 \text{ mm} \)

For unvignetted \( a = 1 + y + 1 \bar{y}' \)

Lens 1: \( y_1 = 16.67 \quad \bar{y}_1 = 11.75 \quad a_1 = 28.42 \text{ mm} \)

Lens 2: \( y_2 = 10.0 \quad \bar{y}_2 = 0 \quad a_2 = 10.0 \text{ mm} \)

Lens 3: \( y_3 = 6.0 \quad \bar{y}_3 = -17.43 \quad a_3 = 23.63 \text{ mm} \)

Image Height = \(-23.2 \text{ mm}\)

Lens 1 Diameter = \(56.84 \text{ mm}\)

Lens 2 Diameter = \(20.0 \text{ mm}\)

Lens 3 Diameter = \(47.26 \text{ mm}\)
4) (30 points) A 5X Keplerian telescope is comprised of two thin lenses separated by 120 mm. The objective lens is 50 mm in diameter, and the eye lens is 12 mm in diameter. This telescope is to be used with a human eye with a 4 mm diameter pupil. The eye is placed at the exit pupil of the telescope. For distant objects, what is the unvignetted object field of view (in degrees) of this system?

A blank raytrace sheet is on the next page.

First, design the telescope:

\[ f_1 + f_2 = x = 120\text{ mm} \]

\[ MP = -5x = -\frac{f_1}{f_2} \]

(The MP is negative because it is a Keplerian)

\[ f_1 = 5f_2 \]

\[ 5f_2 + f_2 = 120\text{ mm} \]

\[ f_2 = 20\text{ mm} \]

\[ f_1 = 100\text{ mm} \]

Because \( D_1 < 5D_2 \), \( f_1 \) is the stop of the telescope.

The ER is found by imaging the objective through the eye lens (\( f_2 \)):

\[ \frac{1}{ER} = \frac{1}{-x} + \frac{1}{f_2} \]

\[ \frac{1}{ER} = \frac{1}{120}\text{ mm} + \frac{1}{20}\text{ mm} \]

\[ ER = 24.0\text{ mm} \]
Because the marginal ray is collimated between the eye lens and the eye, the eye must be the stop of the system including the eye.  \( D_{\text{Eye}} < D_{1/5} \)

<table>
<thead>
<tr>
<th>Surface</th>
<th>0</th>
<th>1</th>
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<th>3</th>
<th>4</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>100</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( -\phi )</td>
<td>-0.01</td>
<td>-0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t )</td>
<td>20</td>
<td>120</td>
<td>BE2 = 2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Marginal Ray**

| \( y \) | 10 | 10 | -2 | -2 |   |   |   |
| \( u \)  | 0  | -0.01 | 0 |   |   |   |   |

**Trial Chief Ray**

| \( \bar{y} \) | 0 | 2.4 | 0 |   |   |   |   |
| \( \bar{u} \)  | 0.02 | 0.02 | -1 \# |   |   |   |   |

\# arbitrary

**Scaled Chief Ray**

| \( y \) | 0 | 40 | 0 |   |   |   |   |
| \( u \)  | 0.0333 | 0.0333 | -1667 |   |   |   |   |

Trace a marginal ray and a trial chief ray from the eye back through the system.

At the eye: \( y = -2 \)  \( u = 0 \)  \( \bar{y} = 0 \)
The EP is at $f_1$ - since vignetting cannot occur at a stop or pupil, the eye lens limits the FOV.

The vignetting condition must be satisfied at $f_2$ using a scaled chief ray

$$a = |g_1 + c\frac{g_2}{g_1}| = |g_1 + c\frac{2.4}{g_1}|$$

At $f_1$:

$$g_2 = 2 \quad \frac{g_2}{g_1} = 2.4$$

$$a_2 = 6 \text{ mm}$$

$$a_2 = 6 \text{ mm} = |g_{2}| + c |\frac{g_2}{g_1}| = 2 \text{ mm} + c \cdot 2.4 \text{ mm}$$

$$c = 1.667$$

Scale the trial chief ray to get the system chief ray.

In object space —

$$\tilde{w}_0 = 0.0333$$

$$HFov = \tan^{-1} |\tilde{w}_0| = 1.91^\circ$$

$$FOV = \pm 1.91^\circ \text{ or } 3.82^\circ$$

Unvignetted FOV $= \pm/\ - 1.91 \text{ degrees}$
3) (30 points) A ball lens is used as a 100X objective in a microscope with an optical tube length of 200 mm. The optical tube length is the distance from the rear focal point of the objective to the intermediate image presented to the eyepiece of the microscope. The ball lens is a sphere of radius $R$ with an index $n = 1.500$.

Determine the required radius of curvature for the ball $R$ and the object-side working distance provided by this objective.

Imaging Requirements:

100X: $m_{obj} = -100 = \frac{z'}{z}$

$z = -\frac{z'}{100}$

$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$

$\frac{1}{z'} = \frac{-100}{z'} + \frac{1}{f}$

$101 = \frac{1}{f}$

$z' = 101 \cdot f = f + 200$

$100f = 200$

$f = 2.00 \text{ mm}$

$z' = 202 \text{ mm}$

$z = -\frac{z'}{100} = -2.02 \text{ mm}$

Alternate Newtonian Solution:

$m_{obj} = -100 = -\frac{OTL}{f}$

$f = \frac{OTL}{100} = 2.00 \text{ mm}$

$z' = 202 \text{ mm}$

$z = -\frac{z'}{100} = -2.02 \text{ mm}$

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Now the bell...

\[ \phi_1 = \frac{(n-1)}{R} = \frac{(1.5-1)}{R} = \frac{1}{2R} \]

\[ \phi_2 = -(1-n)R = \phi_1 \]

\[ A = 2R \]

\[ \phi = \phi_1 + \phi_2 - \phi_2 \frac{A^2}{n} \]

\[ \phi = \frac{1}{2R} + \frac{1}{2R} - \frac{1}{4R^2} \frac{2R}{n} \]

\[ \phi = \frac{n}{2R} - \frac{1}{2nR} = \frac{1}{2R} - \frac{1}{3R} \]

\[ \phi = \frac{2}{3R} \]

\[ f = \frac{3R}{2} \]

\[ R = \frac{2f}{3} \]

\[ f = 200 \text{ mm} \]

\[ R = 1.333 \text{ mm} \]

By inspection, both principal planes are located at the C.C. of the bell lens:

\[ d = -d' = R \]

\[ WD = |z + d| = |z + R| = |-2.02 + 1.333| \]

\[ WD = 0.68667 \text{ mm} \]

Radius of Curvature \( R = 1.333 \text{ mm} \)

Working Distance = 0.68667 mm

\[ \text{Blank page follows...} \]