Name Solutions

Closed book; closed notes. Equation sheets are attached and can be removed.
Spare raytrace forms are also attached.
Use the back sides if required. The time limit is 2 hours.
Calculators are permitted, but do not use any pre-stored information or programs.
If a method of solution is specified in the problem, it must be used.
Note any assumptions you make in solving the problems.
Show your work. Present it in a neat and logical fashion.

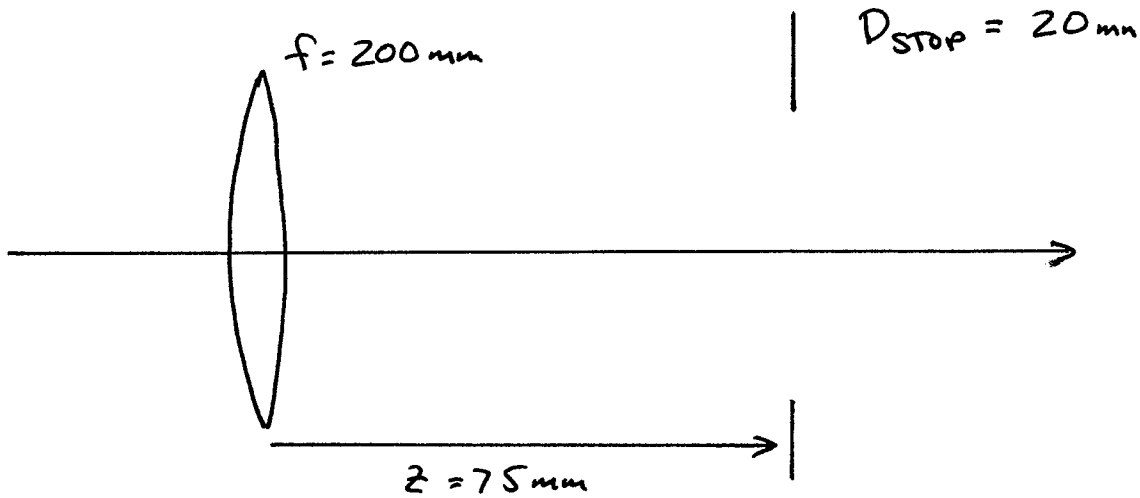
Distance Students: Please return the original exam only; do not FAX an additional copy.

1) (5 points) The letter "A" is imaged by an aberration free thin lens (stop at the lens). A real image is produced. The top half of the lens is now blocked by an opaque card. What happens to the image?

The image gets dimmer.

This is no different than stopping down the lens (or changing its diameter).

2) (10 points) A stop of diameter 20 mm is located 75 mm to the right of a 200 mm focal length thin lens. Use Gaussian methods to determine the location and size of the system entrance pupil.



Light propagates to the left for the real stop $(n = n' = -1)$

$$\frac{-1}{z'} = \frac{-1}{z} + \frac{1}{f} \qquad z' = 120 \text{ mm}$$

The EP is virtual.

$$m_{EP} = \frac{z'/n'}{z/n} = \frac{-120 \text{ mm}}{-75 \text{ mm}} = 1.6$$

$$D_{EP} = m_{EP} D_{STOP} = 32 \text{ mm}$$

EP Location: 120 mm to the right of the lens

EP Diameter = 32 mm

3) (25 points) A system is comprised of three thin lenses. The following partially-completed raytrace of the system is the starting point for this problem, and will be completed during various parts of the problem. Extra lines/rays are provided.

Be sure to properly label your raytraces.

	Object	EP	Lens 1	Stop Lens 2	Lens 3	XP	Image
Surface	0	1	2	3	4	5	6
f			100 mm	-100 mm	100 mm		
$-\phi$			-.01	.01	-.01		
t	∞	-66.67	40 mm	60 mm	-150	197.4	

Potential Chief Ray:

$\tilde{\tilde{y}}$		0	4.0 mm	0.0 mm	-6.0 mm	0	
$\tilde{\tilde{u}}$	-.06	-.06	-0.1	-0.1	-.04	-.04	

Potential Marginal Ray:

$\tilde{y}$	1.0	1.0	1.0	0.6	.36	1.5 (197.4)	0
$\tilde{u}$	0	0	-.01	-.004	-.0076	-.0076	

Marginal Ray - Scale by 16.67

y	16.67	16.67	16.67	10	6.0	25.0	0
u	0	0	-.1667	-.06667	-.1267	-.1267	

Chief Ray - Scale by 2.938

y		0	11.75	0	-17.63	0	-23.2
u	-.1763	-.1763	-.2938	-.2938	-.1175	-.1175	

y							
u							

y							
u							

a) Which element serves as the System Stop (circle one)?

The chief ray goes through the center of the stop

Lens 1 (Lens 2) Lens 3

b) Determine the Entrance and Exit Pupil locations.

Extend the potential chief ray to object and image space. $\tilde{y} = 0$ at EP, XP.

EP: Located 66.67 mm to the right of the first lens.

XP: Located 150 mm to the left of the third lens.

c) Determine the system Focal Length and its Back Focal Distance (BFD).

Trace a potential marginal ray from infinity.

$\tilde{u}' = -.0076$

$\phi = -\tilde{u}'/\tilde{y}_0 = .0076$ $\quad f = 1/\phi = 131.58$ mm

$BFD = (L3 \rightarrow XP) + (XP \rightarrow Image)$

$BFD = -150 + 197.4$

$BFD = 47.4$

f = 131.58 mm

BFD = 47.4 mm

d) The System Stop has a diameter of 20 mm. Determine the diameters of the Entrance Pupil and the Exit Pupil.

Scale the marginal ray to a radius of 10 mm at the stop.

Scale Factor = $\frac{10\text{ mm}}{\tilde{y}_{L2}} = \frac{10\text{mm}}{0.6} = 16.67$

$a_{EP} = 16.67$ mm

$a_{XP} = 25.0$ mm

Entrance Pupil Diameter = 33.33 mm

Exit Pupil Diameter = 50.0 mm

Continues...

e) For distant objects, the system has an unvignetted Field of View of +/- 10 deg. What is the image height in the image plane for this FOV? What are the required Lens Diameters to support this FOV?

$\bar{u}_o = \tan(10°) = .1763$

Scale the chief ray to the object space value

Scale Factor = $\bar{u}_o / \tilde{\bar{u}}_o = .1763/.06 = 2.938$

Extend the chief ray to the image plane to obtain the image height - $\bar{y}' = -23.2$ mm

For Unvignetted $a = |y| + |\bar{y}|$

Lens 1: $y_1 = 16.67$ $\bar{y}_1 = 11.75$ $a_1 = 28.42$ mm

Lens 2: $y_2 = 10.0$ $\bar{y}_2 = 0$ $a_2 = 10.0$ mm

Lens 3: $y_3 = 6.0$ $\bar{y}_3 = -17.63$ $a_3 = 23.63$ mm

Image Height = +/- 23.2 mm

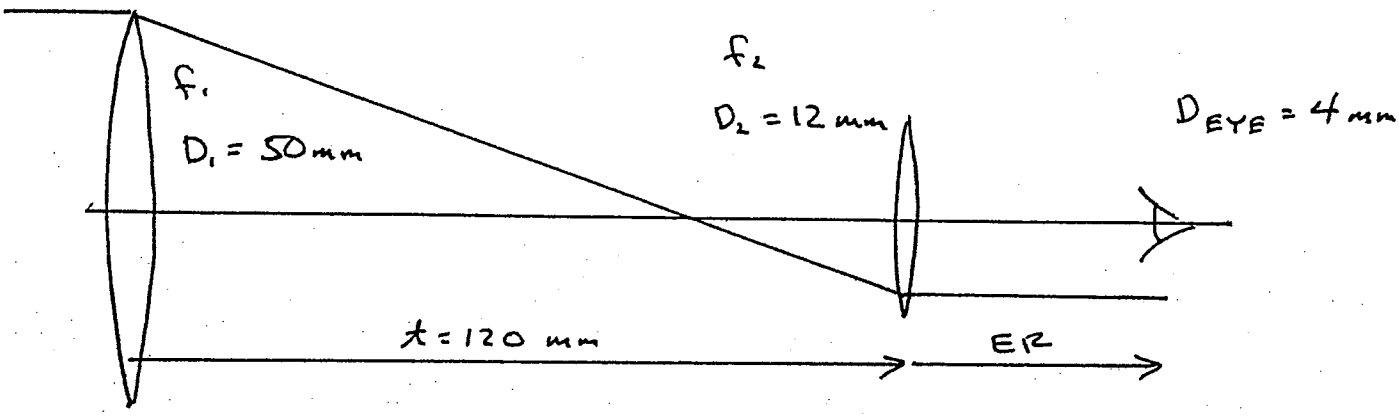
Lens 1 Diameter = 56.84 mm

Lens 2 Diameter = 20.0 mm

Lens 3 Diameter = 47.26 mm

4) (30 points) A 5X Keplerian telescope is comprised of two thin lenses separated by 120 mm. The objective lens is 50 mm in diameter, and the eye lens is 12 mm in diameter. This telescope is to be used with a human eye with a 4 mm diameter pupil. The eye is placed at the exit pupil of the telescope. For distant objects, what is the unvignetted object field of view (in degrees) of this system?

A blank raytrace sheet is on the next page.



First, design the telescope:

$$f_1 + f_2 = t = 120 \text{ mm}$$

$$MP = -5X = -f_1/f_2$$

(The MP is negative because it is a Keplerian)

$$f_1 = 5 f_2$$

$$5f_2 + f_2 = 120 \text{ mm}$$

$$f_2 = 20 \text{ mm}$$

$$f_1 = 100 \text{ mm}$$

Because $D_1 < 5 D_2$, f_1 is the stop of the telescope. The ER is found by imaging the objective through the eye lens (f_2):

$$\frac{1}{ER} = \frac{1}{-t} + \frac{1}{f_2}$$

$t = 120$ mm

$ER = 24.0$ mm

Because the marginal ray is collimated between the eye lens and the eye, the eye must be the stop of the system including the eye. $D_{EYE} < D_1/5$

	Surface 0	f_1 1	f_2 2	Eye 3	4	5	6
f		100	20				
$-\phi$		-.01	-.05				
t	∞	120	ER = 24				
Marginal Ray							
y	10	10	-2	-2			
u	0	-.01	0				
Trial Chief Ray							
$\tilde{\bar{y}}$		0	2.4	0			
$\tilde{\bar{u}}$	.02	.02	-.1 *				
					* arbitrary		
y							
u							
Scaled Chief Ray							
y		0	4.0	0			
u	.03333	.03333	-.1667				

Trace a marginal ray and a trial chief ray from the eye back through the system.

At the eye: $y = -2$ $u = 0$ $\bar{y} = 0$

Continues...

The EP is at f_1 – since vignetting cannot occur at a stop or pupil, the eye lens limits the FOV.

The vignetting condition must be satisfied at f_2 using a scaled chief ray

$$a = |y| + |\bar{y}| = |y| + c|\tilde{\bar{y}}|$$

At f_2: $y_2 = -2$ $\tilde{\bar{y}}_2 = 2.4$

$a_2 = 6$ mm

$$a_2 = 6 \text{ mm} = |y_2| + c\,|\bar{y}_2| = 2\text{ mm} + C \cdot 2.4\text{mm}$$

$$C = 1.667$$

Scale the trial chief ray to get the system chief ray.

In object space —

$$\bar{u}_0 = .03333$$

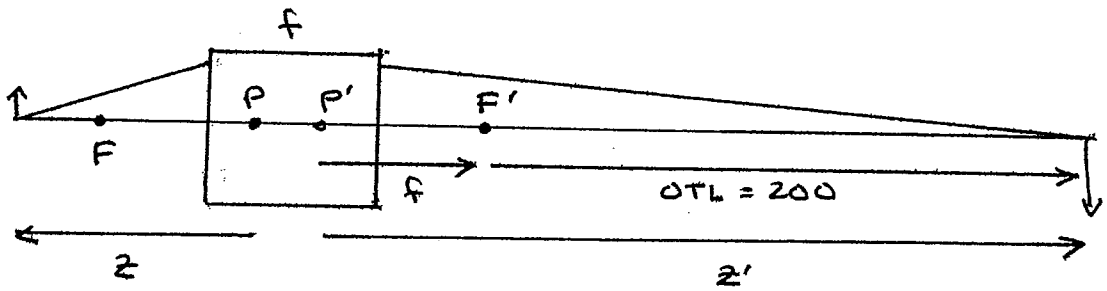
$$HFOV = \tan^{-1}|\bar{u}_0| = 1.91^\circ$$

$$FOV = +/- 1.91^\circ \text{ or } 3.82^\circ$$

Unvignetted FOV = +/- 1.91 degrees

3) (30 points) A ball lens is used as a 100X objective in a microscope with an optical tube length of 200 mm. The optical tube length is the distance from the rear focal point of the objective to the intermediate image presented to the eyepiece of the microscope. The ball lens is a sphere of radius R with an index n = 1.500.

Determine the required radius of curvature for the ball R and the object-side working distance provided by this objective.



Imaging Requirements:

100X: $m_{obj} = -100 = z'/z$ $\qquad z' = f + OTL = f + 200 \text{ mm}$

$z = -z'/100$

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

$$\frac{1}{z'} = \frac{-100}{z'} + \frac{1}{f}$$

$$\frac{101}{z'} = \frac{1}{f}$$

$z' = 101 f = f + 200$

$100 f = 200$

$\underline{f = 2.00 \text{ mm}}$

$z' = 202 \text{ mm}$

$z = -z'/100 = -2.02 \text{ mm}$

Alternate Newtonian Solution:

$m_{obj} = -100 = -\frac{OTL}{f}$

$f = \frac{OTL}{100} = 2.00 \text{ mm}$

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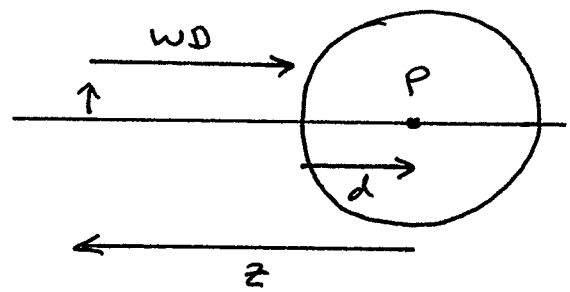
Now the ball ...

$\phi_1 = (n-1)/R = (1.5-1)/R = \frac{1}{2R}$

$\phi_2 = -(1-n)/R = \phi_1$



$t = 2R$

$\phi = \phi_1 + \phi_2 - \phi_1 \phi_2 \frac{t}{n}$

$\phi = \frac{1}{2R} + \frac{1}{2R} - \frac{1}{4R^2} \frac{2R}{n}$

$n = 1.5$

$\phi = \frac{1}{R} - \frac{1}{2nR} = \frac{1}{R} - \frac{1}{3R}$

$\phi = \frac{2}{3R}$ $\quad f = \frac{3R}{2}$

$R = \frac{2f}{3}$ $\quad f = 2.00$ mm

$R = 1.333$ mm

By inspection, both Principal planes are located at the C.C. of the ball lens:

$d = -d' = R$ or $d = \frac{\phi_2}{\phi} \frac{t}{n} = \frac{1/2R}{2/3R} \frac{2R}{1.5} = R$

$WD = |z + d| = |z + R| = |-2.02 + 1.333|$

$WD = 0.68667$ mm

Radius of Curvature R = 1.333 mm

Working Distance = 0.68667 mm

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