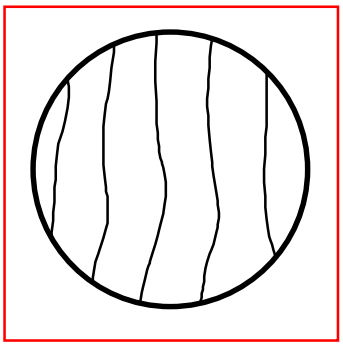
Part 9
Phase Shifting Interferometry

- Classical Interferogram Analysis
- Phase Shifting Advantages
- Phase Shifters
- Algorithms
- Removing Phase Ambiguities
- Error Sources

Classical Analysis of Interferograms

Surface Error =
$(\lambda/2)\ (\Delta/S)$



- **Classical Analysis**
  - **Measure positions of fringe centers.**
  - **Deviations from straightness and equal spacing gives aberration.**

Computer Analysis of Interferograms

Largest Problem

Getting interferogram data into computer

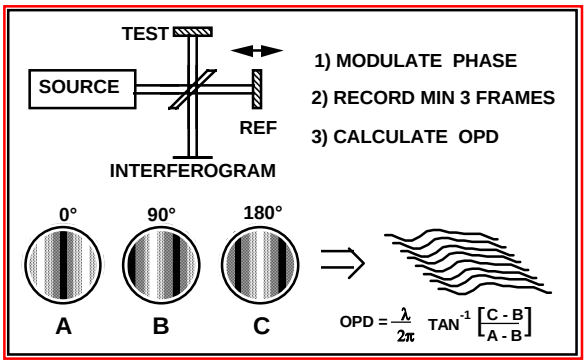
Solutions

- Graphics Tablet
- Scanner
- CCD Camera
- Phase-Shifting Interferometry

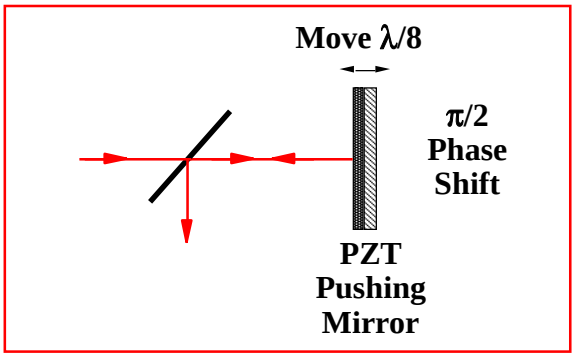
Advantages of Phase-Shifting Interferometry

- High measurement accuracy (>1/1000 fringe, fringe following only 1/10 fringe)
- Rapid measurement
- Good results with low contrast fringes
- Results independent of intensity variations across pupil
- Phase obtained at fixed grid of points
- Easy to use with large solid-state detector arrays

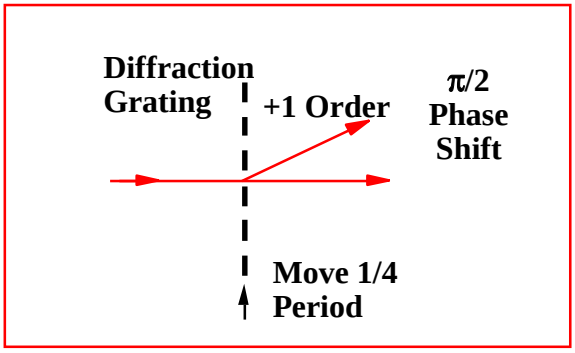
Phase-Shifting Interferometry



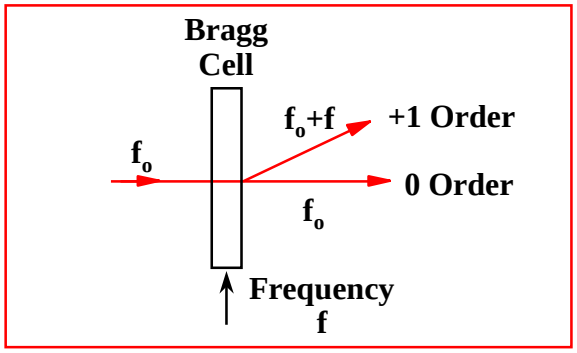
Phase Shifting - Moving Mirror



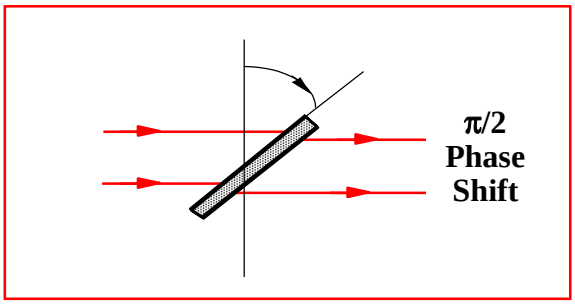
Phase Shifting - Diffraction Grating



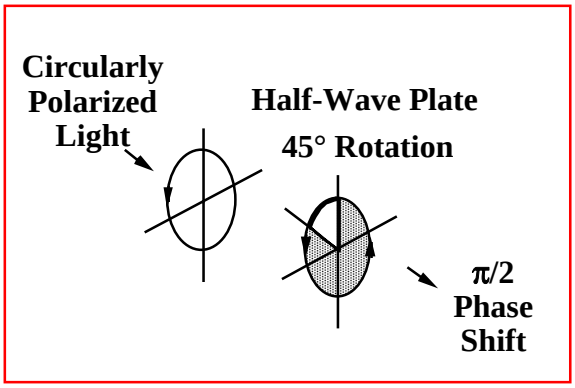
Phase Shifting - Bragg Cell



Phase Shifting - Tilted Glass Plate



Phase Shifting - Rotating Half-Wave Plate



Four Step Method

$$I(x,y) = I_0 + I' \cos[\phi(x,y) + \phi(t)]$$

$I_1(x,y) = I_0 + I' \cos[\phi(x,y)]$ $\quad \phi(t) = 0$

$I_2(x,y) = I_0 - I' \sin[\phi(x,y)]$ $\quad \phi(t) = \pi/2$

$I_3(x,y) = I_0 - I' \cos[\phi(x,y)]$ $\quad \phi(t) = \pi$

$I_4(x,y) = I_0 + I' \sin[\phi(x,y)]$ $\quad \phi(t) = 3\pi/2$

$$\mathrm{Tan}[\phi(x,y)] = \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}$$

Relationship between Phase and Height

$$\phi(x,y) = Tan^{-1}\left[\frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}\right]$$

$$\text{Height Error}(x,y) = \frac{\lambda}{4\pi}\phi(x,y)$$

Phase-Measurement Algorithms

Three Measurements $\phi = \tan^{-1}\left[\frac{I_3 - I_2}{I_1 - I_2}\right]$

Four Measurements $\phi = \tan^{-1}\left[\frac{I_4 - I_2}{I_1 - I_3}\right]$

Hariharan Five Measurements $\phi = \tan^{-1}\left[\frac{2(I_2 - I_4)}{2I_3 - I_5 - I_1}\right]$

Carré Equation

$$\phi = \tan^{-1}\left[\frac{\sqrt{[3(I_2 - I_3) - (I_1 - I_4)][(I_2 - I_3) - (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)}\right]$$

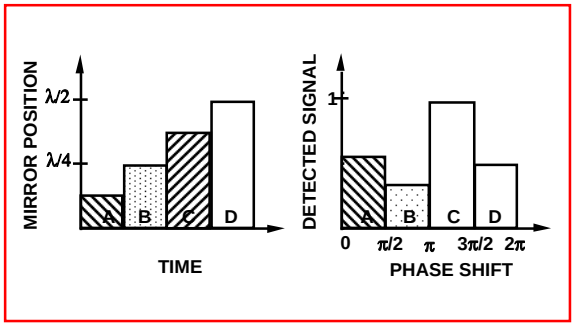
Phase-Measurement Algorithm for N Intensity Measurements

N Measurements $\phi = -\tan^{-1}\left[\frac{\sum_{i=1}^{N} I_i \sin\alpha_i}{\sum_{i=1}^{N} I_i \cos\alpha_i}\right]$

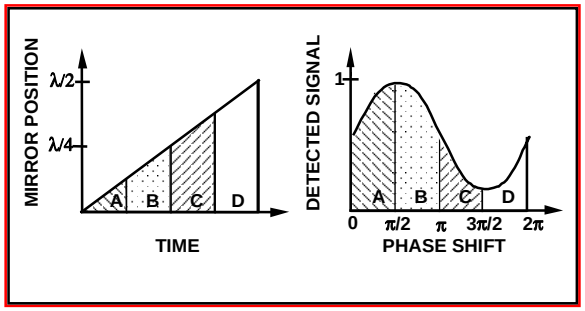
$$\alpha_i = \frac{2\pi i}{N} \quad \text{for } i = 1, \ldots, N$$

Technique is also known as synchronous detection

Phase-Stepping Phase Measurement



Integrated-Bucket Phase Measurement



Integrating-Bucket and Phase-Stepping Interferometry

Measured irradiance given by

$$I_i = \frac{1}{\Delta}\int_{\alpha_i-\Delta/2}^{\alpha_i+\Delta/2} I_o\{1+\gamma_o\cos[\phi+\alpha_i(t)]\}d\alpha(t)$$

$$= I_o\left\{1+\gamma_o \mathrm{sinc}\left[\frac{\Delta}{2}\right]\cos[\phi+\alpha_i]\right\}$$

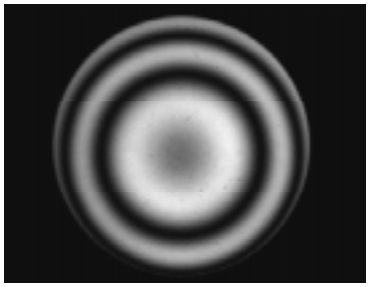
Integrating-Bucket $\Delta=\alpha$

Phase-Stepping $\Delta=0$

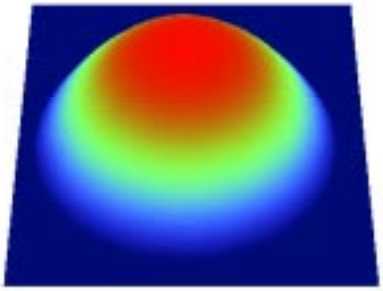
Phase Ambiguities

- **If we know sign of Sin and sign of Cosine the Arc Tangent is calculated modulo 2π.**
- **Must correct for 2π ambiguities.**

Typical Fringes For Spherical Surfaces



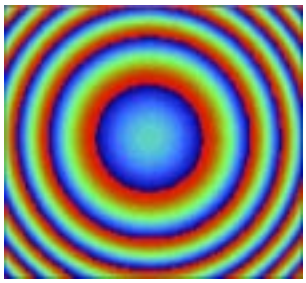
Fringes

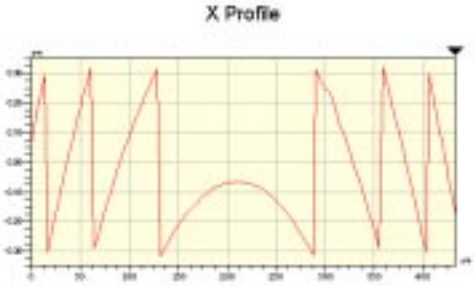


Phase map

Phase Ambiguities
-Before Integration

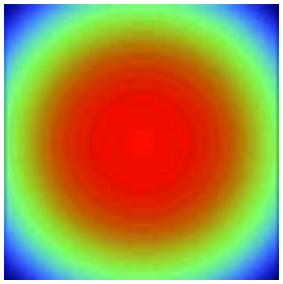
2 π Phase Steps





Phase Ambiguities
- After Integration

Phase Steps Removed



Removing Phase Ambiguities

- **Arctan Mod 2π (Mod 1 wave)**
- **Require adjacent pixels less than π difference**

  (1/2 wave OPD)

- **Trace path**
- **When phase jumps by > π**

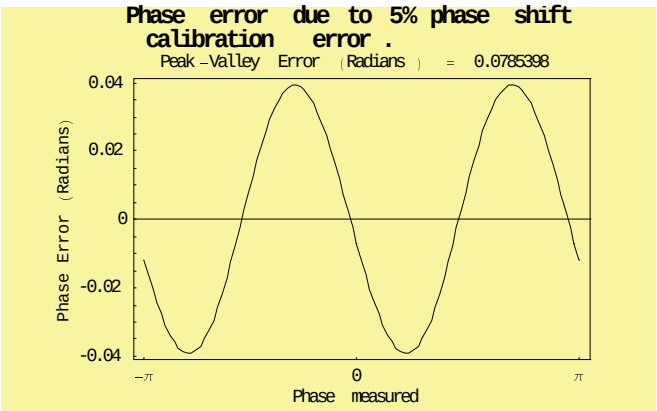
  Add or subtract N2π

  Adjust so < π

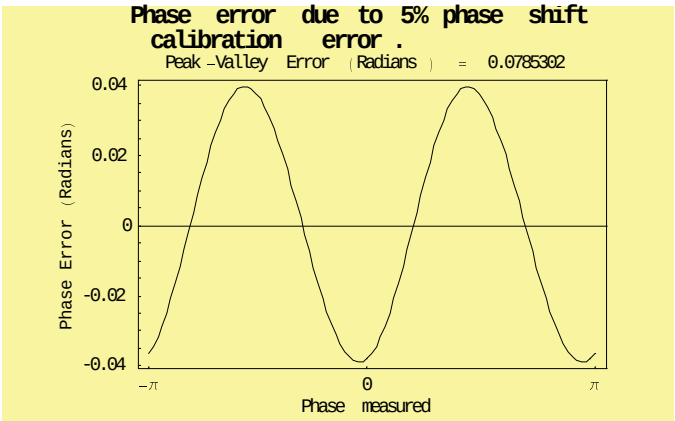
Error Sources

- Incorrect phase shift between data frames
- Vibrations
- Detector non-linearity
- Stray reflections
- Quantization errors
- Frequency stability
- Intensity fluctuations

Three π/2 Steps



Four π/2 Steps



Phase Error Compensating Techniques

Two data sets with π/2 phase shift.

- **Calculate a phase for each set from algorithm, and then average phases.**
- **Average Numerator and Denominator, and then calculate phase.**

Example of Algorithm Derivation

Averaging Technique

4-FRAME (offset = 0)

frames # 1,2,3,4

$$\frac{I_4 - I_2}{I_1 - I_3} = \frac{N_1}{D_1}$$

4-FRAME (offset = $\pi/2$)

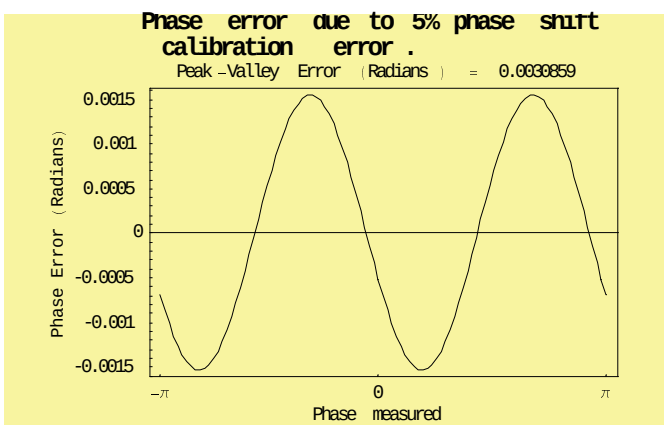
frames# 2,3,4,5

$$\frac{I_4 - I_2}{I_5 - I_3} = \frac{N_2}{D_2}$$

5-FRAME

$$\tan\varphi = \frac{N_1 + N_2}{D_1 + D_2} = \frac{2(I_4 - I_2)}{I_1 + I_5 - 2I_3}$$

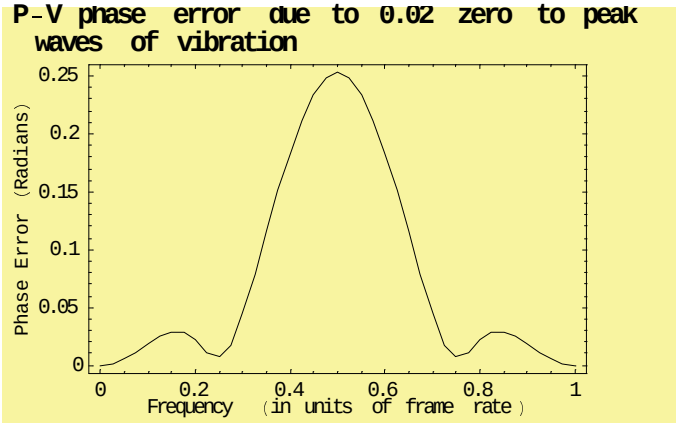
Schwider-Hariharan Five $\pi/2$ Step Algorithm



Error due to vibration

- **Probably the most serious impediment to wider use of PSI is its sensitivity to external vibrations.**
- **Vibrations cause incorrect phase shifts between data frames.**
- **Error depends upon frequency of vibration present as well as phase of vibration relative to the phase shifting.**

Error due to vibration for Schwider-Hariharan algorithm



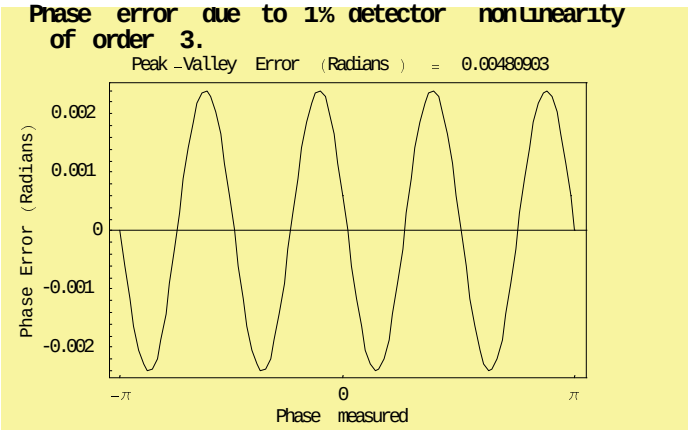
Best way to fix vibration problem

- Reduce vibration
- Take data fast
- Take all frames at once
- Measure vibration and introduce vibration 180 degrees out of phase to cancel vibration

Error due to detector nonlinearity

- Generally CCD's have extremely linear response to irradiance
- Sometimes electronics between detector and digitizing electronics introduce nonlinearity
- Detector nonlinearity not problem in well designed system.
- Schwider-Hariharan algorithm has no error due to 2nd order nonlinearity. Small error due to 3rd order.

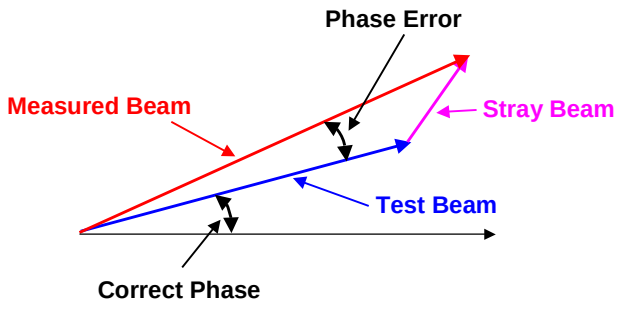
Schwider-Hariharan



Error due to Stray Reflections

- **Stray reflections in laser source interferometers introduce extraneous interference fringes.**
- **Stray reflections add to test beam to give a new beam of some amplitude and phase.**
- **Difference between this resulting phase and phase of test beam gives the phase error.**

Error – Stray Reflections



Phase-Shifting Interferometer