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Effects of Photographic Gamma on Hologram Reconstructions*†

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An expression derived for hologram exposures made along the straight-line portion of an H-D curve of a photographic plate gives the first-order transmittance of a hologram made of several object points exposed simultaneously (conventional holograms). This expression is compared with a similar expression derived previously for holograms made of several object points exposed sequentially (synthetic holograms). Theory and experiments show the effect of the nonlinearity of the photographic process on the contrast of the reconstruction of conventional holograms. Synthetic and conventional holograms are studied theoretically and experimentally to determine how the total amount of light in the reconstruction image depends upon the number of object points when the total amount of light in the object is constant. It is shown that the reconstructed image formed by a conventional hologram contains more light than the image formed by a synthetic hologram of the same number of object points. Both synthetic and conventional holograms are also studied to determine the ratio of reference-beam illuminance to object-beam illuminance that maximizes the amount of light in the reconstructed image.

INDEX HEADINGS: Holography; Photography; Image formation.

In an earlier paper,[1] we showed that, for holograms made of several object points exposed sequentially (synthetic holograms), the relationship between the luminance of the reconstructed hologram image and the luminance of the original object depends on the value and sign of the gamma of the photographic process. In particular, for synthetic holograms, the nonlinearity of the photographic process tends to decrease the luminances of bright reconstructed object points relative to the luminances of dim reconstructed object points.

In this paper, the work given in the earlier paper' will be extended to include holograms made of several object points exposed simultaneously (conventional holograms). The results obtained for conventional holograms will be compared with results obtained previously for synthetic holograms.

Another purpose of this paper is to help solve the question of the optimum ratio of reference- to object-beam illuminances. It is generally agreed that, to reduce the effects of the interference between different object points, the illuminance of the reference beam should be greater than the illuminance of the object beam.[2] It will be shown that to maximize the amount of light in the reconstructed image, the optimum ratio of reference-beam illuminance to object-beam illuminance depends upon the photographic process and the total hologram exposure.

Synthetic and conventional holograms will also be studied to determine if the same total amount of light is found in the reconstruction from a hologram of one object point that produces exposure E as is found in the reconstruction from a hologram of N object points, each of which produces exposure E/N.

Kozma[3] has previously considered the problem of nonlinearity of the emulsion, using a different approach.

THEORY

For hologram exposures made along the straight-line portion of the H-D curve of the photographic plate, the amplitude modulus of the transmittance function of

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[1] J. C. Wyant and M. P. Givens, J. Opt. Soc. Am. 58, 357 (1968).
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[3] A. Kozma, J. Opt. Soc. Am. 56, 428 (1966).

sides of the plate. However, we also obtain the complex-conjugate wave field, which appears only on one side of the plate because of the refractive-index conditions. This field will exist only if the angle between the object and reference waves is smaller than the critical angle, as is always the case in the configurations shown in Figs. 4 and 5. It would also exist in the cases of Figs. 2 and 3 if the angle between o and r were smaller than α_c. Reversing the direction of the evanescent wave used for reconstruction results in changing the reconstructed true wave field o into a complex-conjugate wave field o* and vice versa [Figs. 4(c) and 5(c)].

No pronounced difference was found between holograms recorded with the light polarized parallel or perpendicular to the plane of incidence. As in the former case, perpendicular polarized light is preferable for reconstruction.

IV. RECONSTRUCTION IN WHITE LIGHT FROM THIN HOLOGRAMS

Another feature of evanescent-wave holography is that white light can be used for reconstruction. In all of the cases shown in Figs. 2-5, it is also possible to reconstruct in white light. Reconstructions in different colors occur in different directions, but the one in the recording wavelength is especially strong and of good quality. The evanescent waves travel along the thin hologram and the effect is equivalent to the Bragg scattering conditions in a volume hologram. Because the hologram is confined to the very surface of the emulsion, shrinkage in depth after development does not influence the reconstruction. Furthermore, multiple scattering in the hologram is limited to a minimum.

It is possible to change the wavelength of the evanescent wave λ_e, by changing the angle of incidence α, even though the frequency of the light remains constant. Thus, when using white light, we may change the angle of incidence so that a frequency other than that used in recording will match the hologram spatial frequency and form a reconstruction in the direction used in the recording. This reconstruction is in a different color from that used in recording; however, it is also distorted.

V. DISCUSSION AND COMMENTS

Holography with evanescent waves offers a possibility for storing the information carried by these waves. It is, thus, a further step toward recording the whole optical wave field. The potentialities of this kind of holography seem to open up new developments and possibilities in the application of holographic techniques-the most apparent one seems to be in formation of high-aperture and high-resolution optical images. It also appears to be a powerful tool for obtaining deeper insight into the properties of evanescent waves.

Some unique features were found characteristic for this type of holography : (1) Thin holograms can be formed. The thickness is not determined by the emulsion thickness, but by the penetration depth of the evanescent waves which, when they are formed by internal reflection, is determined by the wavelength, polarization, and the angle of incidence of the light. Usually, the hologram does not exceed a depth of 1μ; it can be made thinner by choosing proper values of the mentioned quantities. (2) The fact that an evanescent reconstructing wave travels along the hologram gives this kind of holography some peculiar features. Two similar wave fields (either true or complex conjugate, depending on the direction of the evanescent wave) are reconstructed and, furthermore, white light may be used for the reconstruction. (3) The wavelength of the evanescent wave can be chosen arbitrarily by selection of a proper value of the angle of incidence of the internally reflected light.

For evanescent-wave holography, a high-resolution recording material is necessary in some cases. Furthermore, the refractive index of the emulsion has to be less than that of its surrounding on the incidence side. One way to meet these requirements would be to coat the emulsion on a high-refractive-index glass base.[3] Another way, which was chosen here, is to immerse commercially available high-resolution photographic plates in a highly refractive liquid. The latter way offers some flexibility of choice of a proper refractive index; a different index can be chosen for reconstruction. On the other hand, the immersion method is inferior for the production of bleached holograms because of surface roughness. The hologram frequencies, however, are generally too high to produce a relief image, so the density variations in the hologram are only transformed into refractive-index variations.

The wave fields reconstructed with evanescent-wave holograms have the same properties as those reconstructed from conventional holograms, i.e., the image reconstructed by the true-object wave field o is orthoscopic and that reconstructed by the complex-conjugate wave field o* is pseudoscopic. Reconstruction by an evanescent wave creates the same wave field on each side of the hologram, the only difference being a mirror image.

A hologram formed by two interfering evanescent waves propagating in the same direction can be reconstructed in two ways; by use of either an evanescent or an ordinary illuminating wave. In the latter case, the distortions can be compensated to obtain a satisfactory reconstruction. We also made holograms in which the two evanescent waves had an angular separation. In this case, it was also possible to reconstruct with an ordinary wave, provided that the angle between the evanescent waves was not too large.

The diffraction efficiency of the evanescent-wave holograms is about the same as in conventional thin absorption-type holograms. For the case using the Air the hologram is given by

$$T_A = (E/E')^{-\gamma/2}, \qquad (1)$$

where E is the exposure of the photographic plate, E' is the inertia of the plate, and γ is the slope of the straight-line portion of the H-D curve of the plate.[1]

In making a conventional hologram, the amplitude, A, at a point (X,Y) on the photographic plate is

$$A = R + \sum_{n=1}^{N} A_n, \qquad (2)$$

where R is the complex amplitude of the reference beam at the point (X,Y) on the plate, A_n is the complex amplitude at the point (X,Y) due to the nth object point, and N is the number of object points. A_n and R depend on position on the plate, but not on time. That is, the periodic time factor has been omitted.

If t is the exposure time, the exposure, E_H, at a point (X, Y) on the plate is

$$E_H = t(AA^*) = t(R + \sum_{n=1} A_n)(R + \sum_{n=1} A_n)^*$$
$$= t(I_R + I + R^* \sum_{n=1} A_n + R \sum_{n=1} A_n^* + \sum_{n=1} \sum_{\substack{m=1 \\ m \neq n}} A_n A_m^*), \qquad (3)$$

where * denotes complex conjugate. $I_R = |R|^2$ is equal to the illuminance of the reference beam, and $I = \sum_{n=1}^{N} |A_n|^2$ is the sum of the illuminances contributed by each of the object points individually.

E, the total exposure, is equal to the sum of the exposure given by Eq. (3) and any uniform initial "exposure, E_0, used to bring the total exposure to the straight-line portion of the H-D curve.

Thus, the amplitude modulus of the transmittance function of the hologram can be written as

$$|T_A| = (E'C)^{\gamma/2}[1 + C(a+b+d)]^{-\gamma/2}, \qquad (4)$$

where

$$C = [E_0 + t(I_R + I)]^{-1}, \quad a = tR^* \sum_{n=1} A_n, \quad b = tR \sum_{n=1} A_n^*,$$

and

$$d = t \sum_{i=1} \sum_{\substack{j=1 \\ j \neq i}} A_i A_j^*.$$

If the complex amplitude of the reconstructing beam is D, the total amplitude of all of the light transmitted by the photographic plate is given by $D|T_A|$. We assume that the same beam is used for reconstruction as was used for the reference beam, with a possible change of the illuminance of the beam. We also assume that T_A is real, i.e., we neglect phase variation introduced by nonuniform shrinkage of the hologram emulsion.

If we expand Eq. (4) by use of the binomial theorem, and select the terms that give rise to I_K', the luminance of the reconstruction of the kth object point, we obtain

$$I_K' = |D|^2 (E'C)^{\gamma} (\tfrac{1}{2}\gamma)^2 C^2 t^2 I_R I_K (1 - U + V - S + \text{higher-order terms})^2, \qquad (5)$$

where U, V, and S are given by

$$U = (\tfrac{1}{2}\gamma + 1)Ct \sum_{\substack{n=1 \\ n \neq k}}^{N} I_n \quad \text{and} \quad I_n = |A_n|^2, \qquad (6)$$

$$V = (\tfrac{1}{2}\gamma+1)(\tfrac{1}{2}\gamma+2)C^2t^2[I_R(\tfrac{1}{2}I_K + \sum_{\substack{n=1 \\ n \neq k}} I_n) + I_K \sum_{\substack{n=1 \\ n \neq k}} I_n + \tfrac{3}{2} \sum_{\substack{n=1 \\ n \neq k}}^{N} \sum_{\substack{j=1 \\ j \neq k \\ j \neq n}}^{N} I_n I_j], \qquad (7)$$

$$S = (\tfrac{1}{2}\gamma+1)(\tfrac{1}{2}\gamma+2)(\tfrac{1}{2}\gamma+3)C^3(\tfrac{1}{6})t^3(I_R(3 \sum_{n=1}^{N} \sum_{\substack{i=1 \\ i \neq n}}^{N} I_n I_i + 6I_K \sum_{\substack{j=1 \\ j \neq k}}^{N} I_j + 9 \sum_{\substack{n=1 \\ n \neq k}}^{N} \sum_{\substack{i=1 \\ i \neq n \\ i \neq k}}^{N} I_n I_i) + \sum_{n=1}^{N} \sum_{\substack{i=1 \\ i \neq n}}^{N} \sum_{\substack{j=1 \\ j \neq k}}^{N} I_n I_i I_j$$
$$+ 2 \sum_{n=1}^{N} \sum_{\substack{i=1 \\ i \neq n}}^{N} \sum_{\substack{j=1 \\ j \neq i \\ j \neq n}}^{N} I_n I_i I_j + 3 \sum_{\substack{i=1 \\ i \neq k}}^{N} \sum_{\substack{j=1 \\ j \neq k \\ j \neq i}}^{N} \sum_{\substack{n=1 \\ n \neq j \\ n \neq k}}^{N} I_i I_j I_n$$
$$+ \sum_{n=1}^{N} \sum_{\substack{i=1 \\ i \neq n \\ i \neq k}}^{N} \sum_{\substack{j=1 \\ j \neq k \\ j \neq n}}^{N} I_n I_i I_j + \sum_{i=1}^{N} \sum_{\substack{n=1 \\ n \neq i \\ n \neq k}}^{N} \sum_{\substack{j=1 \\ j \neq n \\ j \neq i \\ j \neq k}}^{N} I_i I_n I_j$$
$$+ \sum_{\substack{n=1 \\ n \neq k}}^{N} \sum_{\substack{i=1 \\ i \neq n}}^{N} \sum_{\substack{j=1 \\ j \neq i \\ j \neq n}}^{N} I_n I_i I_j + 2 \sum_{\substack{n=1 \\ n \neq k}}^{N} \sum_{\substack{i=1 \\ i \neq n \\ i \neq k}}^{N} \sum_{\substack{j=1 \\ j \neq k \\ j \neq i \\ j \neq n}}^{N} I_n I_i I_j), \qquad (8)$$

where U, V, and S are functions of K, the reconstructed point.

Equation (5) shows that there are two different situations in which the contrast of the reconstructed image is identical to the contrast of the original object. The first occurs when U, V, S, and the higher-order terms are small enough compared to unity that only the first term in Eq. (5) has to be considered. The second situation occurs when even though U, V, S, and the high-order terms are not small, they are essentially the same for all values of K.

There are three cases in which it is sufficient to consider only the first term in Eq. (5). The first is when gamma is equal to - 2 ; then all of the terms, with the exception of the first, are identically zero, because each contains the factor $(1/2\gamma+1)$.

The other two cases are similar. If either E_0, the exposure to a uniform illuminance distribution required to make the hologram exposure be along the straight-line portion of the H-D curve, is much greater than the hologram exposure, or if the illuminance of the object beam is much less than the illuminance of the reference beam, all of the terms, with the exception of the first, can be disregarded. The last two conditions are said to be similar because they both represent the condition of low-contrast fringes in the hologram. Practical holograms do not satisfy either of these conditions.

Equations (6)-(8) show that U, V, and S will be approximately the same for all values of K, if no one object point contains a major portion of the total amount of light in the object beam.

Reference 1 showed that, for synthetic holograms, the luminance of the reconstruction of the Kth object point is given by a similar equation. The difference between the transmittance of conventional holograms and the transmittance of synthetic holograms results because term d appears in the transmittance function for conventional holograms given in Eq. (4), but not in the transmittance function for synthetic holograms. This term arises from interference between different object points; if only one object point is exposed at a time, as for synthetic holograms, there is no interference between different object points.

EXPERIMENTAL SETUP AND PROCEDURE

All of the holograms were made using Kodak 649-F photographic plates. Before each hologram was made, a portion of the 649-F plate was preflashed to a General Electric AG-1 flashbulb placed 2.5 m from the plate. Only a small part of the preflashed portion of the plate was used for hologram exposures. On the remaining portion of the preflashed part of the plate, several different exposures to ,a uniform illuminance were made, to determine the H-D curve of the photographic process. These exposures were made using the same laser as was used to make the holograms. Several more exposures to the same uniform illuminance distribution were also made on the part of the plate that was not pre-exposed to the flashbulb. From the H-D curve of the part of the plate that was not preflashed, E_0, the exposure due to the laser light needed to produce the same density as was produced by the AG-1 flashbulb, could be determined.

All of the photographic plates were developed in D-19 at 68°F for 5 min. The density of the different exposures was measured using an Ansco-Sweet densitometer.

Figure 1 shows the experimental setup used for making a conventional hologram of two object points. A He-Ne laser operating at 6328 Å was used as the coherent light source. A 20X microscope objective was used to produce a diverging beam, which was collimated with a 75-cm focal-length objective. Two 3-diopter negative lenses were placed in part of the collimated beam to produce two spherical waves. The hologram was made using the spherical waves as the object beam and the collimated wave as the reference beam.

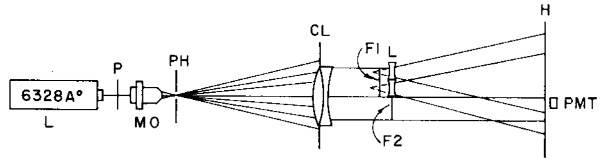

FIG. 1. Apparatus for making the holograms. L, He-Ne laser; MO, 20X microscope objective; PH, 15-µ pinhole; PMT, photomultiplier tube; CL, collimating lens; L, two -3 diopter negative lenses; P, Polaroid film; F1 and F2, neutral-density filters; and H, hologram plane.

For making some conventional holograms, we placed as many as twelve negative lenses in part of the collimated beam to produce the object beam. The lenses were always put in a plane parallel to the hologram and all of the lenses had the same focal length, so that all of the object points were in one plane, parallel to the hologram. Neutral-density filters were placed over some of the negative lenses, so that not all of the object points had the same illuminance.

When making synthetic holograms, we placed only one negative lens in the collimated beam. Either the negative lens or the photographic plate was moved approximately 5 mm in a direction parallel to the hologram plane between each pair of superimposed hologram exposures.

The Polaroid film, P, placed between the laser and the microscope objective could be rotated to change the illuminance of the light source, so that the desired exposure could be obtained with a convenient exposure time. In Fig. 1, Fl and F2 are neutral-density filters used to provide the desired beam-balance ratio, i.e., reference-beam illuminance divided by object-beam illuminance.

A 931-A photomultiplier tube with a red filter and a piece of ground glass in front of it was used to measure the illuminance of the reference and object beams.

The same type of wavefront was used for the reconstructing beam as was used for the reference beam in making the holograms. Since each hologram is effectively the hologram of one or more point sources (diverging wavefronts), the first-order real-image reconstruction is of one or more points. A 931-A photomultiplier tube with a piece of ground glass in front of it was used to measure the luminance of these reconstructed points. All of the ground glass in front of the photomultiplier tube was masked off with black tape, except for a region approximately 1.5 mm square. One at a time, the reconstructed object points were focused on this bare portion of ground glass. In the experiments, the carrier frequency was approximately 50 lines/mm.

EXPERIMENTAL MEASUREMENTS AND RESULTS

The first experiment shows the effect of the nonlinearity of the photographic process on the contrast of the reconstruction of conventional holograms of a few object points.

For all of the holograms, gamma was measured to be about 5.5 and the pre-exposed density of the plates was approximately 0.2. Holograms were made of two object points of unequal luminance.

Table I shows the results of the measured and calculated luminance ratios of the reconstructed images, as well as what the ratios would have been if gamma were -2. The beam-balance ratio shown in Table I is equal to the reference-beam illuminance divided by the total object-beam illuminance. The densities given are the average densities of the holograms. For a given density, the closer the beam-balance is to one, the greater the hologram fringe contrast. For a given beam-balance ratio, the greater the density, the greater the hologram fringe contrast, owing to the high gamma.

Table I shows that there is good agreement between the measured and calculated luminance ratios. The discrepancy between the observed and calculated luminance ratios is within the experimental uncertainty he measurements.

Reference 1 showed that, for synthetic holograms, the nonlinearity of the photographic process tends to decrease the luminances of the bright points relative to the dim points. This is just the opposite of the effect of the nonlinearity of the photographic process on the reconstruction of conventional holograms.

These results show that, for reasonable hologram fringe contrasts, the gamma of the photographic process affects the relative luminances of different reconstructed object points, if the original object consists of a few points. We will now investigate what happens if the object consists of many object points.

In the case of synthetic holograms, the term V is the dominant correction, and V contains no terms involving the products I_nI_j. If many points are present and if each represents a small fraction of the total hologram exposure, then V is approximately the same for all reconstructed object points (all values of K). Under these conditions, the ratio of the luminances of any two reconstructed points is equal to the ratio of the product of the illuminance of the original reference beam, the illuminance of the original object point, and the square of the exposure time for the two points. This is true for all values of gamma, under the condition that V is independent of *K*.

TABLE I. Ratio of reconstructed-image luminances of conventional holograms of two object points.

Object illuminance ratio	Reconstructed-image luminance ratio: Measured	Reconstructed-image luminance ratio: Calculated using measured value of gamma	Reconstructed-image luminance ratio: Calculated if $\gamma=-2$	Beam-balance ratio	Average hologram density
1.5	1.6	1.6	1.5	4.0	0.88
3.3	3.7	3.6	3.3	6.0	0.90
1200	1440	1440	1200	6.5	1.24
10 000	11 300	12 200	10 000	5.3	1.10
17.5	17	17.6	17.5	120	1.10
175	175	176	175	120	1.10
88	100	99	88	8.0	0.92
8.8	9.3	9.8	8.8	7.5	0.95

TABLE II. Ratio of reconstructed-image luminance for different numbers of superimposed holograms.

Hologram exposure ratio	Number of object points for hologram	Reconstructed luminance ratio: Measured	Reconstructed luminance ratio: Calculated using measured value of gamma	Reconstructed luminance ratio: Calculated if $\gamma = -2$
Plate 1				
		3.6	3.73	4
2	2	3.9	3.98	4
2	11	4.0	3.99	4
Plate 2				
			8.16	9
3	2	2	8.97	9
3	11	8.8	8.99	9
Plate 3				
3	2	7.7	7.80	9
3	6	9.0	8.94	9
3	11	9.0	8.99	9
Plate 4				
3	2	8.1	8.02	9
3	6	8.9	8.96	9
3	11	8.9	8.99	9
Plate 5				
2	2	3.6	3.60	4
2	2	3.9	3.97	4
2	11	3.9	3.99	4

The conclusions were confirmed by the following experiment. Three different holograms were made on a photographic plate. The beam-balance ratio and total hologram exposure were the same for all three of the holograms. The only difference between the three holograms was the number of superimposed exposures. One hologram consisted of one large and one small hologram exposure. Another hologram consisted of five large and one small hologram exposures, and the third hologram consisted of ten large and one small hologram exposures. The ratio of a large hologram exposure to the small hologram exposure was the same for all the holograms. So, the effect of gamma on the relative luminances of the reconstructed object points should be greatest for the hologram of two object points, least for the hologram of eleven object points. The results are shown in Table II.

We see that the measured and calculated luminance ratios agree to within the experimental uncertainty of the measurements. We also see that the effect of gamma on the relative luminances of the reconstructed object points of a synthetic hologram decreases when the exposure due to any one hologram exposure is a smaller fraction of the total hologram exposure. This does not mean that gamma does not affect the luminance of a given reconstructed object point; it simply means that gamma affects equally the luminances of all reconstructed object points, regardless of the exposure of the hologram that produces the given reconstructed point.

Equation (5) shows that for conventional holograms, the contrast of the reconstructed image is the same as the original object if no one object point contains a sizable portion of the total amount of light in the object beam.

Therefore, an experiment similar to the one described above for synthetic holograms was performed for conventional holograms. Holograms were made of 2, 6, and 11 object points. For all of the holograms, one of the object points was dimmer than the other object points, which were nearly equally bright. The ratio of the illuminance of one of the bright object points to that of the dim object point was exactly the same for all of the holograms. Also, the beam-balance ratio and the total hologram exposure was the same for all of the conventional holograms. The results of this experiment are shown in Table III.

TABLE III. Ratio of reconstructed-image luminances of conventional holograms of different number of object points.

Object illuminance ratio	Number of object points for hologram	Reconstructed luminance ratio: Measured	Calculated using measured value of gamma	Calculated if $\gamma=-2$
Plate 1				
9.5	2	11.2	10.9	9.5
9.5	6	10.5	9.9	9.5
9.5	11	10.3	9.7	9.5
Plate 2				
74	2	86	86.5	74
74	6	77	76.9	74
74	11	74	75.4	74
Plate 3				
13.2	2	14.4	14.8	13.2
13.2	6	13.8	13.6	13.2
13.2	11	13.2	13.4	13.2
Plate 4				
97	2	112	108.8	97
97	6	97	99.7	97
97	11	93	98.3	97
Plate 5				
10	2	12.0	12.3	10
10	6	10.4	10.6	10
10	11	10.0	10.3	10
Plate 6				
49	2	56	60.7	49
49	6	51	52.0	49
49	11	50	50.5	49

The measured and calculated luminance ratios agree to within the experimental uncertainty of the measurements. Also, as the fraction of the total object-beam illuminance contained in any one object point is decreased, the effect of the gamma of the photographic process on the contrast of the reconstructed image decreases.

Thus, we have the result that for both conventional and synthetic holograms, the gamma of the photographic process does affect the luminance of the reconstructed object points. However, if the portion of the hologram exposure responsible for the reconstruction of each object point is a small fraction of the total, hologram exposure, the gamma of the photographic process affects nearly equally the luminances of all of the reconstructed object points.

The next experiment was performed to determine if the same total amount of light is contained in the reconstruction of a hologram of one object point producing an exposure E, as is found in the reconstruction of synthetic and conventional holograms of N object points, each producing an exposure E/N. The results for synthetic holograms will be given first.

TABLE IV. Ratio of total amount of light in reconstruction of hologram of N superimposed exposures, each of exposure E/N, to amount of light reconstruction of hologram of single exposure E.

N, number of exposures	Total-light ratio: Measured	Calculated using measured value of gamma	Calculated if $\gamma=-2$	Average density of hologram
2	1/1.94	1/2	1/2	0.26
5	1/5.1	1/5	1/5	0.26
8	1/8.7	1/8.7	1/8	0.7
4	1/4.95	1/5.95	1/4	0.7
8	1/8.33	1/8.1	1/8	0.5

For all of the holograms on any one plate, the illuminance of the plate by the object and reference beams was held constant. The total exposure time of all of the holograms was the same. This means that each exposure of the hologram that contained N superimposed exposures was l/N the exposure of the singly exposed hologram.

Table IV shows the measured and calculated ratios of the total amount of light in the reconstruction of the hologram of N superimposed exposures to the amount of light in the reconstruction of the singly exposed hologram. The densities are average densities of the holograms. Because, for all of the holograms, the beam-balance ratio was approximately 4, the greater the density, the greater the hologram fringe contrast.

The measured and calculated ratios shown in Table IV agree to within the experimental uncertainty of the measurements. For synthetic holograms, breaking the total exposure into many smaller exposures greatly reduces the total amount of light in the reconstruction, regardless of the value of gamma. In fact, if a larger exposure is broken up into N equal smaller exposures, the total amount of light in the reconstruction is reduced by a factor of approximately l/N.

The explanation for the above results can be seen from Eq. (5), as simplified for synthetic holograms. If we break a hologram exposure, *E*, into N equal smaller exposures, the luminance of each reconstructed object point will be reduced by a factor of approximately $1/N^2$; because we have N such reconstructed points, the total amount of light in the reconstruction will be reduced by a factor of approximately l/N.

The above experiment was also performed for conventional holograms. That is, on one photographic plate, a hologram was made of one object point and another hologram was made of N equally bright object points. The exposure time, reference-beam illuminance, and total object-beam illuminance was the same for both holograms.

TABLE V. Ratio of total amount of light in reconstruction of holograms of N object points, each of illuminance I/N, to amount of light in reconstruction of hologram of one object point of illuminance *I*.

Number of object points	Total-light ratio: Measured	Total-light ratio: Calculated using measured value of gamma	Total-light ratio: Calculated if $\gamma = -2$	Beam-balance ratio	Average density of hologram
2	0.91	0.88	1	4.5	0.8
2	1.0	0.99	1	31.0	0.8
3	0.92	0.90	1	6.0	0.7
5	0.86	0.89	1	8.0	0.9
5	0.75	0.76	1	4.5	0.9
4	0.80	0.77	1	4.0	0.75
8	0.71	0.71	1	4.0	0.75
12	0.70	0.69	1	4.0	0.75

Table V shows the measured and calculated ratios of the total amount of light in the reconstruction of the hologram of N object points, each of illuminance I/N, to the amount of light in the reconstruction of the hologram of the single object point of illuminance I.

The measured and calculated total-light ratios shown in Table V agree to within the experimental uncertainty of the measurements. For conventional holograms, if gamma is approximately 5 instead of -2, there is less light in the reconstructed image of N object points, each of illuminance I/N, than in the reconstruction of a hologram of one object point of illuminance I. It is also seen that as the contrast of the fringes is increased, the total luminance of the reconstructed image is more dependent upon the number of object points.

Figure 2 is a graph of the ratio of the total amount of light in the reconstruction of a hologram of N object points, each of luminance *I/N*, to the amount of light in the reconstruction of a hologram of one object point of luminance I. The graph is drawn for the gamma, pre-exposure, and total hologram exposure corresponding to the last three data points in Table V.

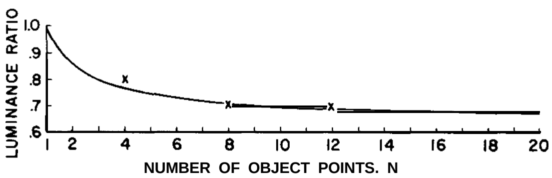

FIG. 2. Ratio of total amount of light in the hologram reconstruction of N object points, each of illuminance Z/N, to amount of light in hologram reconstruction of one object point of illuminance I, as function of number of object points, N. For N equal ∞, the luminance ratio equals 0.643. x are experimentally measured data points. Gamma=4.7, pre-exposed density=0.18, and average hologram density is approximately 0.75.

Figure 2 shows that the rate of change of the total amount of light in the reconstructed image as a function of the number of object points decreases as the number of object points increases. For a sufficiently large number of object points, the change of the total amount of light in the reconstructed image caused by an increase of the number of object points is negligible. This result can be seen by examining Eq. (5).

The total amount of light in the reconstruction of a hologram of N equally bright object points is equal to N times Eq. (5). For a given object-beam and reference-beam illuminance, N times Eq. (5) will depend upon N only because *U*, *V*, and *S* depend upon N. Thus, for a given hologram exposure, the total amount of light in the reconstructed image is less for several object points than for one object point because $(1-U+V-S)^2$ is smaller for N> 1 than for N= 1. However, the rate of change of *U*, *V*, and *S* with a change of the number of object points decreases as the number of object points increases. Finally, when the number of object points becomes so large that the amount of light from any one object point is negligible compared to *I*, the total amount of light from the object, *U*, *V*, and *S* become constant.

The above experiment shows that for a given total hologram exposure, a conventional hologram of N equally bright object points will put approximately N times as much light in the reconstruction as a synthetic hologram of N object points.

The next experiment shows that, for synthetic as well as conventional holograms, a beam-balance ratio of 1 does not necessarily give the maximum amount of light in the reconstruction.

Two sets of hologram exposures, plus exposures for an H-D curve, were made on each photographic plate. The two sets of holograms were exposed to a different average density, in order to study the influence of the average hologram density on the beam-balance ratio that gives the maximum amount of light in the reconstructed image.

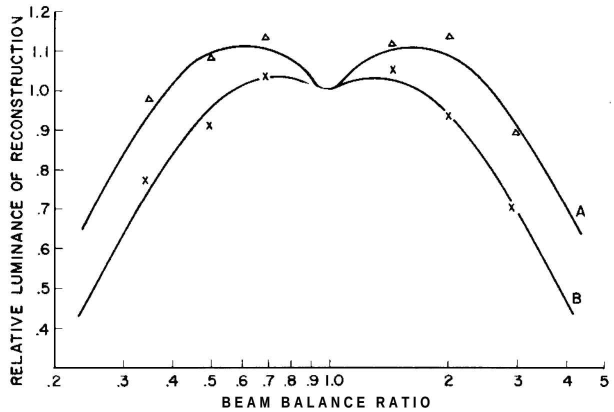

FIG. 3. Relative luminance of hologram reconstruction of single object point as a function of beam-balance ratio. Experimentally measured data points are given by A for Curve A and x for Curve B. Gamma=5.8. For beam-balance ratio= 1, average hologram density equals 0.73 for Curve A and 0.86 for Curve B.

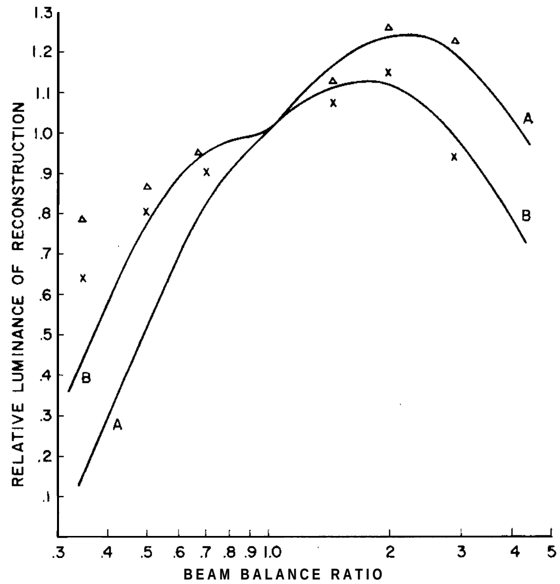

Fig. 5. Relative luminance of conventional hologram reconstruction of three object points as a function of beam-balance ratio. Experimentally measured data points are given by A for Curve A and x for Curve B. Gamma=3.85. For beam balance ratio= 1, average hologram density equals 0.75 for Curve A and 0.9 for Curve B.

Each set of hologram exposures was made with the following beam-balance ratios: 1, 1.45, l/1.45, 2, l/2, 2.9. and l/2.9. For each set. the exposure due to the object beam was the same for all exposures that had beam-balance ratios greater than or equal to 1. The exposure due to the reference beam was the same for all exposures that had beam-balance ratios less than or equal to 1.

The luminance of the hologram reconstructions was normalized to one for a beam-balance ratio of 1. Figures 3-5 show the theoretical and experimentally measured results.

Figures 3 and 4 show that the experimental data points for the relative luminances of the reconstruction of a hologram of one object point and a synthetic hologram of three object points as a function of the beam-balance ratio agree with the experimental curves to within the experimental uncertainty involved in the experiments.

Figure 5 shows that for a beam-balance ratio greater than 1, the measured values of the relative luminances of the conventional hologram reconstructions of three object points agree with the theoretical values to within the experimental uncertainty of the measurements. However, for a beam-balance ratio less than 1, there is not good agreement between theory and experimental results. This discrepancy between the theoretical and measured ratios for a beam-balance ratio less than 1 is not surprising because the limited binomial expansion of Eq. (4) is not accurate under these conditions.

The last experiment shows that, for a given exposure to the object beam, a brighter image may result for conventional holograms if the reference beam has greater illuminance than the object beam. For synthetic holograms, a brighter image may result if the object and reference beams have unequal illuminances, but

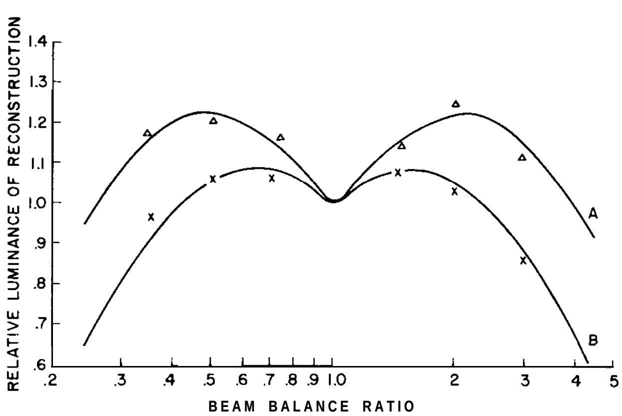

FIG. 4. Relative luminance of synthetic hologram reconstruction of three object points as a function of beam-balance ratio. Experimentally measured data points are given by A for Curve A and x for Curve B. Gamma=4.8. For beam-balance ratio= 1, average hologram density equals 0.72 for Curve A and 0.92 for Curve B.

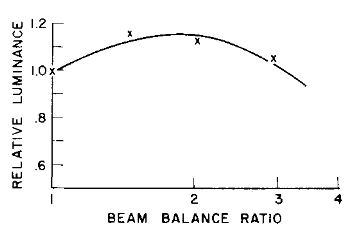

FIG. 6. Relative luminance of hologram reconstruction of one object point as a function of beam-balance ratio. x are experimentally measured data points.

it does not make any difference which beam has the greater.

For both synthetic and conventional holograms, the percentage increase of the luminance in the reconstructed image that can be gained by having the beam-balance ratio different from 1 is larger, the less dense the hologram. Also, the less dense the hologram, the larger the difference should be between the illuminance of the object beam and the illuminance of the reference beam, to obtain the maximum luminance in the reconstructed image.

The above experiment shows the influence of the average hologram density on the beam-balance ratio that gives the maximum luminance in the reconstructed image for holograms of a few object points. We will now investigate what happens if instead of having a conventional hologram of a few object points, we have a conventional hologram of many object points.

The only reason why the curves shown in Fig. 5 depend upon the number of object points is that the terms U, V, and S in Eq. (5) depend upon the number of object points. U, V, and S change very rapidly as functions of the number of object points for a small number of object points; but when the number of object points reaches eleven or twelve, U, V, and S have nearly reached the values they have for an infinite number of object points.

Figures 6-9 show graphs of the theoretical and experimentally measured values of the relative luminances of the hologram reconstructions as functions of beam-balance ratio for holograms of 1, 4, 8, and 12 object points. Figure 9 also shows the theoretical curve for the case of an infinite number of object points. For the four figures, gamma is equal to 5.0. The average hologram density is approximately 0.72 for a beam-balance ratio of 1.0.

Figures 6-9 show that for a given pre-exposure, hologram exposure time, and object-beam illuminance, there is a small change between the shape of the curve giving the relative luminance of the hologram reconstruction vs beam-balance ratio for the hologram of a single point and that for the conventional hologram of four object points. However, there is very little change between the curves for four, eight, and twelve object points. Furthermore, there is little change between the theoretical curve for an object consisting of twelve object points and an object containing the same total amount of light, but consisting of an infinite number of object points, i.e., an extended object.

Thus, the conclusion can be reached that the beam-balance ratio that gives the maximum amount of light in the reconstructed image depends little upon the number of object points.

The following experiment was performed to show that the above theory and experimental results agree to within 1% or 2%. On each photographic plate, eight hologram exposures were made using a beam-balance ratio of 1, and eight hologram exposures were made using a beam-balance ratio of something different than 1. All of the hologram exposures were made with the same exposure time and the same object-beam illuminance.

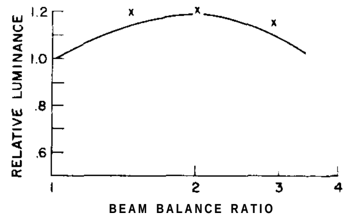

FIG. 7. Relative luminance of conventional hologram reconstruction of four object points as a function of beam-balance ratio. x are experimentally measured data points.

Table VI gives the results of the experiment. For each value of the relative luminance of a hologram reconstruction shown, there are five calculated values of the relative luminance. The first value was calculated using the value of gamma and pre-exposure that best fits the data for the H-D curve. The second and third values were calculated from the measured value of pre-exposure and the smallest and largest values of gamma that can reasonably be obtained from the data points making up the H-D curve. The last two values were calculated from the smallest and largest value of pre-exposure within the experimental uncertainty of the measured value of pre-exposure, and the values of gamma obtained by using these values of pre-exposure. Thus, the correct calculated relative luminance for

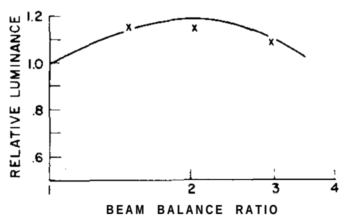

FIG. 8. Relative luminance of conventional hologram reconstruction of eight object points as a function of beam-balance ratio. x are experimentally measured data points.

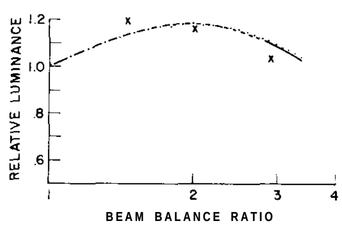

FIG. 9. Relative luminance of conventional hologram reconstruction of twelve object points and an infinite number of object points, as a function of beam-balance ratio. x are experimentally measured data points.

TABLE VI. Relative luminance of hologram reconstruction as a function of beam-balance ratio. Note: Relative luminance of reconstruction equals 1 for a beam-balance ratio of 1.

Number of object points	Beam-balance ratio	Average measured relative luminance of hologram reconstruction	% standard deviation of average measured relative luminance	Calculated relative luminance for given values of E_0 and γ: Luminance	E_o	γ	% deviation from measurement
1	2.9	0.921	2.1%	0.9335	10	4.80	1.36%
				0.9193	10	4.90	-0.185
				0.9553	10	4.65	3.58
				0.9347	11	5.30	1.60
				0.9090	9	4.45	-1.30
4	1.45	1.2028	1.38	1.2031	11.5	3.25	0.26
				1.2010	11.5	3.30	0.083
				1.2072	11.5	3.15	0.60
				1.2048	12.5	3.55	0.40
				1.1944	10.5	3.10	-0.30
8	2.0	1.1096	1.77	1.1161	11	5.35	-0.55
				1.1074	11	5.45	-0.30
				1.1248	11	5.25	1.33
				1.0704	10	5.0	-3.60
				1.0859	9	5.7	-2.20
12	1.45	1.1503	0.94	1.1556	10	3.75	0.46
				1.1444	10	4.00	-0.55
				1.1647	10	3.55	1.25
				1.1636	11	3.95	1.16
				1.1452	9	3.60	-0.434

each set of hologram exposures should be within the range of the five different calculated relative luminances.

The experimental and theoretical results shown are seen to agree to within the experimental uncertainties. By repeating each hologram exposure eight times, the experimental uncertainty of the relative luminance of the hologram reconstruction is reduced by a factor of $\sqrt{7}$ from the uncertainty obtained if only one hologram exposure were made.[4]

Thus, the conclusion is reached that for a given exposure to the object beam, a brighter reconstructed image may result if the reference beam is brighter than the object beam. The percentage increase of the lumiance nance of the reconstruction that can be gained by having the beam-balance ratio greater than 1 is larger the less dense the hologram. Also, the less dense the hologram, the larger the beam-balance ratio should be, to obtain the maximum amount of light in the reconstructed image.

Thus, if the hologram object is relatively dim, and the hologram exposure must be relatively short, the amount of light in the reconstructed image can be increased by having the reference beam considerably brighter than the object beam.

[4] A. G. Worthing and J. Geffner, *Treatment of Experimental Data* (John Wiley & Sons, Inc., New York, 1960), p. 167.