F&printed from APPLIED OPTICS, Vol. 13, page 2762, December 1974 Copyright 1974 by the Optical Society of America and reprinted by permission of the copyright owner

## Computer Generated Hologram; Null Lens Test of **Aspheric Wavefronts**

J. C. Wyant and P. K. O'Neill

The authors were with Itek Corp., Lexington, Massachusetts 02173 when this work was done; J. C. Wyant is presently with Optical Sciences Center, University of Arizona, Tucson, Arizona 85721. Received 26 June 1974.

A major obstacle in using aspheric surfaces in optical systems has been the difficulty involved in accurately testing them. A common method of testing an aspheric optical element is to make a second optical system (null lens or null mirror) that converts the wavefront produced by the element under test into either a spherical or plane wavefront.<sup>13</sup>This wavefront is interferometrically compared with a known reference wavefront. In the testing of steep aspherics, the null optics are often expensive to produce accurately.

Recent studies showed that null lenses can be replaced with computer generated holograms (CGH).<sup>4-10</sup> Unfortunately, the CGH required to test steep aspherics are also difficult to produce. It has been suggested that instead of using either a very expensive null lens or a very complicated CGH, often the test can be performed using the combination of a relatively inexpensive null lens and relatively simple CGH.<sup>10</sup> The purpose of this Letter is to illustrate the potential of the combined CGH null lens test by showing the results of a CGH null lens test of the primary mirror of an eccentric Cassegrain system that had a departure



Fig. 1. CGH Maksutov interferometer.

of approximately 455 waves (at 5145 Å) and a maximum slope of approximately 1500 waves/radius. The mirror was a 69-cm diam off-axis segment whose center lies 81 cm from the axis of symmetry of the parent aspheric surface. The null optics was a Maksutov sphere, <sup>11,12</sup> which reduces the departure and slope of the aspheric wavefront from 910 to 45 waves/radius and 3000 to 70 waves/radius, respectively. A hologram was then used to remove the remaining asphericity. These results are compared with the test results obtained testing the primary with a rather expensive three-element null lens.

Figure 1 shows a diagram of the CGH Maksutov setup. When the aspheric wavefront from the mirror under test illuminates the hologram, several wavefronts are produced, one of which would be a perfect plane wave if both the hologram and mirror under test were perfect. That is, the hologram acts as a conventional null lens. The spatial filter (small aperture) shown passes only this plane wavefront and the plane reference wavefront. Thus, if the piece under test were perfect (and also if the CGH were perfect), straight fringes would be obtained in the interferogram plane shown. That is, a null test is being performed. If the surface under test departs from the desired shape, the fringes will depart from straightness in the same manner as for a regular Twyman-Green test.

The interferometer shown in Fig. 1 was ray traced to obtain the wavefront in the image plane of the element under test, which is the plane in which the hologram is placed. It is important that the entire interferometric setup shown be ray traced, since elements such as the diverger may introduce little aberration when used with aspheric wavefronts as in this test. A computer program determines the location of the fringes that would be obtained if the aspheric wavefront in the hologram plane were interfered with a tilted plane wave. From this information the computer calculates the instructions necessary to make the laser beam recorder (LBR) plot the CGH, which has the property that when it is illuminated with the aspheric wavefront emerging from the interferometer it produces a plane wavefront to produce straight line fringes. Figure 2 shows the master CGH produced by the LBR. The 20-cm diam LBR plot (CGH) is photoreduced to approximately 2.5-cm diameter and placed in the interferometer as shown in Fig. 1.

A comment should be made about the selection of the angle of the tilted plane wave used in making the CGH. To properly spatial filter to select out the desired wave-front produced by the hologram, the angle of the tilted

plane wave must be at least as large as the maximum slope of the aspheric wavefront along the intersection of the plane of incidence of the plane wave and the aspheric wavefront. As was shown previously,<sup>s</sup> the peak-to-peak error in the wavefront produced by the hologram is directly proportional to the angle between the tilted plane wave and the aspheric wavefront. Since increasing the slope of the plane reference wavefront decreases the accuracy of the aspheric wavefront produced, advantage was taken of the fact that a smaller reference wavefront tilt could be used in the testing of the nonsymmetric aspheric wavefront if the plane of incidence of the reference wavefront is along the direction of minimum slope of the aspheric wavefront. The slope of the plane reference wavefront was made equal to the maximum slope of the aspheric wavefront, 70 waves/radius.

The test errors resulting from errors in the fabrication and alignment of the CGH can be calculated using the theoretical analysis given previously.<sup>8</sup>The main sources of errors are (1) LBR distortion; (2) positional error of CGH, both translational and rotational; and (3) incorrect CGH size.

The peak-to-peak wavefront error,  $E_{\mu}$ , caused by LBR distortion is given by

$$E_1 = [(2S)/P], \tag{1}$$

where S is the maximum difference between the slope of the aspheric wavefront and the tilted plane wave and P is the number of distortion-free resolution points the LBR gives across a diameter of the plot. P was measured to be about 1400. For our hologram, S was about 70 waves/hologram radius. Thus, the peak-to-peak wavefront error caused by LBR distortion is about 0.1 wave, which corresponds to a surface error of approximately 1/20 wave.

The peak-to-peak wavefront error,  $E_{\nu}$ , caused by translational error in the placement of the hologram is given by

$$E_s = S\Delta x \tag{2}$$

where S is the maximum slope of the aspheric wavefront as measured in the hologram plane and  $\Delta x$  is the positional error of the hologram in the direction of S. To aid in the positioning of the CGH, the LBR drew a vertical and horizontal line that had to be lined up with the edge of the aperture of the mirror. In the vertical direction, S was only a few waves per centimeter, so there was no positioning problem. However, in the horizontal direction, S was about 55 waves/cm. By measuring the repeatability of the test results, it is estimated that  $\Delta x$ , the positional error in the



Fig. 2. Master CGH plotted by the LBR.



(a) Results obtained using Maksutov test without CGH ( $\lambda = 5145$  Å)



(b) Results of CGH Maksutov test ( $\lambda = 5145$  Å)



(c) Results obtained using null lens ( $\lambda = 6328$  Å)

Fig. 3. Test results: (a) results obtained using Maksutov test without CGH ( $\lambda = 5145$  Å); (b) results of CGH Maksutov test ( $\lambda =$ 5145 Å); and (c) results obtained using null lens ( $\lambda = 6328$  Å). horizontal direction, was about 0.0013 cm. Thus, the peak-to-peak wavefront error caused by translational displacement was approximately 0.07 wave, which corresponds to a surface error of approximately 1/30 wave.

The peak-to-peak error caused by rotational error in the hologram is the same as that given by Eq. (2), except now S is the angular slope and  $\Delta x$  is replaced with  $\Delta \theta$ , the angular error. The correct angular position was determined by lining up the vertical line the LBR drew on the CGH with a vertical line drawn on the mirror between two marks put on the mirror in the fabrication process. For our test, S was about 100 wave/rad, and  $\Delta \theta$  was estimated to be approximately 0.5 X 10<sup>-3</sup> rad. Thus, the peak-to-peak wavefront error caused by rotational error was approximately 0.05 wave, which corresponds to a surface error of approximately 1/40 wave.

The peak-to-peak wavefront error, Es, caused by incorrect hologram size, is given by

$$E_3 = S \left| R_c - R_a \right|, \tag{3}$$

where S is again the maximum slope of the aspheric wavefront,  $R_{a}$  is the correct hologram radius, and  $R_{a}$  is the actual hologram radius. For our test, S was about 55 waves/cm and  $|\mathbf{R}_c - \mathbf{R}_a|$  was less than 0.0013 cm. Thus, the peakto-peak wavefront error caused by incorrect hologram size was approximately 0.07 wave. This corresponds to a surface error of approximately 1/30 wave.

In addition to the errors resulting from use of the CGH, there are errors introduced by misalignment of the Maksutov sphere and the interferometer. By ray tracing the system with what was estimated to be reasonable amounts of misalignment, the rms value of these errors was found to be 0.4 wave. The optics in the interferometer and Maksutov sphere had an error less than 0.03 wave.

Figure 3(a) shows an interferogram of the mirror under test as obtained using the Maksutov test without the CGH, and Fig. 3(b) shows the results obtained using the CGH. Figure 3(c) shows the results for performing the same test using a null lens. The CGH was made to test only the re-

gion inside the distorted circle shown in Figure 3(c). Allowing for the fact that the interferogram obtained using the null lens has much more distortion than for the CGH Maksutov test and the difference in sensitivity for the two tests  $\lambda = 6328$  Å for the null lens test, and  $\lambda = 5145$  Å for the CGH Maksutov test, the results for the two tests are seen to be very similar. The hills and valleys on the mirror surface appear the same for both tests, as expected. The peak-to-peak surface error measured using the null lens was 0.46 wave (6328 Å), and for the CGH Maksutov test it was 0.39 wave (5145 Å). The rms surface error was measured to be 0.06 wave (6328 Å) using the null lens and for the CGH Maksutov test it was 0.07 wave (5145 Å). The above results certainly demonstrate that at least in one case, and it is hoped in many cases, expensive null optics can be replaced with relatively inexpensive null optics and a CGH.

The authors wish to thank M. Rimmer, C. King, and M. Kates for the development of the computer programs used in producing the computer generated hologram used in the test, M. Beaulieu for doing much of the experimental work, and C. DeFranzo for doing the ray tracing.

## References

- 1. A. Offner, Appl. Opt. 2,153 (1963).
- 2. R. Hilbert, and M. Rimmer, J. Opt. Soc. Am. 59, 485 (1969).
- 3. R. E. Fischer, J. Opt. Soc. Am. 61, 655 (1971).
- J. Pastor, Appl. Opt. 8, 525 (1969).
  Y. Ichioka, M. Izumi, and T. Suzuki, Appl. Opt. 10, 403 (1971).
- 6. A. J. MacGovern and J. C. Wyant, Appl. Opt. 10, 619 (1971).
- 7. A. F. Fercher and M. Driese, Optik 35, 168 (1972) (German).
- 8. J. C. Wyant and V. P. Bennett, Appl. Opt. 11, 2833 (1972).
- 9. Y. Ichioka and A. W. Lohmann, Appl. Opt. 11, 2597 (1972).
- 10. J. C. Wyant, in Space Optics: Proceedings of Ninth Congress of International Commission for Optics (U.S. National Academy of Sciences, Wash., D.C., 1974), p. 643.
- 11. D. D. Maksutov, Technologie der Astronomischen Optik (VEB Verlag Technik, Berlin, 1954), pp. 202-206.
- 12. R. E. Parks, J. Opt. Soc. Am. 62, 726 (1972).