Reprinted from APPLIED OPTICS, Vol. 14, page 1619, July 1975

OTF measurements with a white light source: an interferometric technique

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The use of lateral shear interferometers for measuring the optical transfer function of an optical system for a white light source is investigated. It is shown that grating lateral shear interferometers fulfill the requirements necessary to perform measurements of both the optical transfer function and the optical coherence function for a white light source. Several possible grating lateral shear interferometers are described.

Introduction

Since Hopkins showed that a lateral shear interferometer can be used to measure the optical transfer function of an optical system,[1] several types of shearing interferometers for measuring either OTF or MTF have been presented.[2-9] The techniques illustrated previously have been useful only in situations where a quasi-monochromatic light source (or equivalently a white light source and a narrowband spectral filter) could be used, while the technique described in this paper can be used with a white light source, making the interferometric measurement of OTF more useful in the testing of a general optical system.

Theory

To determine the requirements for a lateral shear interferometer to measure the OTF of an optical system for a white light source, the relationship between a shearing interferometer output and OTF must be investigated. The one-dimensional optical transfer function of an optical system $D(\nu_x,\psi)$ is represented by the autocorrelation of the pupil function $f(x,y)$:

$$D(\nu_x,\psi) = \frac{1}{C}\iint_A f\left(x + \frac{1}{2}S, y\right) f^*\left(x - \frac{1}{2}S, y\right) dx dy = |D(\nu_x,\psi)| \exp[i\theta(\nu_x,\psi)], \quad (1)$$

where the azimuth angle ψ is the direction of shear.

S, the shear of the wavefront, is related to the spatial frequency ν_x, by the expression

$$S = D\lambda(f_{no})\nu_x. \quad (2)$$

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Received 10 January 1975.

D is the diameter of the pupil, λ is the wavelength of light, f_{no} is the f number of the system, A is the region of overlap of the apertures, and C is the value of the integral when the shear S is zero.

As shown by Hopkins,[1] one method of obtaining the value of the integral in Eq. (1) is to measure the total flux in the interference pattern obtained using a lateral shear interferometer. If δ is the phase difference between the two sheared interfering wavefronts at the center of the interference area and the interfering wavefronts have equal amplitude, the total flux in the interference pattern is

$$F(\delta,S,\psi) = \iint \left| f\left(x + \frac{1}{2}S, y\right) + f^*\left(x - \frac{1}{2}S, y\right)\exp(-i\delta)\right|^2 dx dy = 2C\{1 + |D(\nu_x,\psi)| \cos[\delta - \theta(\nu_x,\psi)]\}. \quad (3)$$

The term 2C is the average amount of flux in the two interfering wavefronts. Generally δ is made to vary linearly with time, i.e.,

$$\delta = \omega t + \phi_0, \quad (4)$$

so the second term varies sinusoidally with time having an amplitude $2C|D(\nu_x,\psi)|$ and phase $\phi_0 - \theta(\nu_x, y)$. Therefore, by changing the shear S, both the modulus $|D(\nu_x,\psi)|$ and the phase $\theta(\nu_x,\psi)$ of the OTF can be determined.

If a white light source is to be used with a lateral shear interferometer in an OTF measurement, three conditions must be satisfied. The first two conditions are that ω, the frequency shift between the two interfering wavefronts, and ϕ_0, the constant phase shift between the two interfering wavefronts, must be independent of wavelength. The third requirement is that the lateral shear introduced by the interferometer must be proportional to wavelength. This condition follows from Eqs. (2) and (3), which show

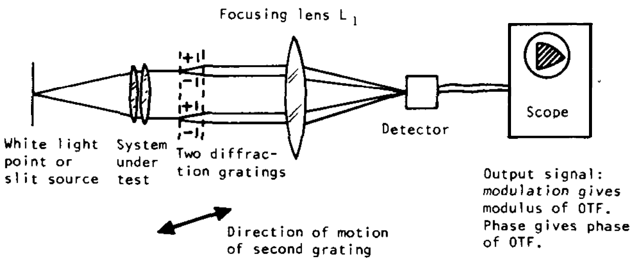

Fig. 1. Grating shearing OTF measuring interferometer for use with white light.

that only if the shear is proportional to wavelength does the flux in the interference pattern give the OTF at a unique spatial frequency ν_x.

Interferometer Details

There exist several grating lateral shear interferometers that possess the three above characteristics. One example is shown in Fig. 1. Collimated light from the optical system under test is incident upon a diffraction grating, which produces +1 and -1 diffraction orders. (The situation where other diffraction orders are produced is discussed below.) A second grating placed some distance from the first grating converts the +l and -1 orders into beams traveling in the same direction as the beam incident upon the first grating. (In general there are beams leaving the second grating at different angles; however, these other beams can be spatially filtered out of the system at the focus of lens L_1.) For small diffraction angles the two beams leaving the second grating are sheared or laterally displaced from one another an amount S given by the equation

$$S = (2\lambda/d)L, \tag{5}$$

where d is the grating period, and L is the distance between the gratings. It should be noted that the shear is proportional to wavelength, a requirement necessary if the OTF measurement is performed using a white light source.

If one grating has a constant velocity v perpendicular to both the grating lines and the direction of propagation of the incident light, one of the first diffraction orders will experience a frequency shift of v/d, and the other first diffraction order will be frequency shifted by an amount $-v/d$. Therefore, Eq. (4) becomes

$$\delta = 2\pi(2v/d)t + \phi_0, \tag{6}$$

where ϕ_0 depends only on the relative position of the two gratings at the time when t = 0. Therefore, δ is independent of wavelength, and all three conditions required to enable the OTF measuring interferometer to work with white light are satisfied. (It should be noted that if unsymmetrical diffraction orders such as 0 and +l are used, a path difference would exist between the two interfering beams, and hence ϕ_0 would depend upon the wavelength.)

By moving one grating with constant velocity in the direction shown in Fig. 1, both the shear and the phase difference between the two sheared beams are made to vary linearly with time. If the light is directed to the detector the AC portion of the signal coming from the detector is proportional to the magnitude of the OTF, and the phase is related to the phase of the OTF. If an oscilloscope is AC coupled to the detector, the oscilloscope traces out a curve whose envelope is the magnitude of the OTF of the system under test. The phase of the OTF can be measured directly if the grating is moving with constant velocity. Otherwise, the phase can be determined by measuring the difference between the phase of the signal from a known high quality beam and phase of the test beam.

The assumption was made above that the gratings produced only plus and minus first orders, i.e., the gratings have sinusoidal amplitude transmission. The modulation frequency produced by interfering diffraction order m_1 with diffraction order m_2 is equal to ($\nu\,|m_2 - m_1|$). Hence, unless the difference between the two diffraction order numbers is equal to 2, the modulation frequency is different from the modulation frequency of interest, and the unwanted signal can be filtered out in the electronics. In a breadboard constructed using holographically produced bleached gratings, no problems with unwanted signals were experienced. However, if required, unwanted orders could be eliminated as shown in Fig. 2, in which an afocal optical system is placed between the two diffraction gratings, and an aperture stop is placed in the focal plane of the first lens to eliminate unwanted orders. As mentioned above, unwanted orders produced by the second diffraction grating are eliminated by placing the appropriate aperture stop at the focus of the lens focusing the light beams onto the detector.

Many other grating shearing interferometers can be used to measure the OTF of an optical system using a white light source. One example is a double frequency grating described previously[10] and illustrated in Fig. 3. If the two grating periods are d_1 and d_2, the angular shear can be approximated as

$$\Delta\alpha = \lambda\left(\frac{1}{d_2} - \frac{1}{d_1}\right). \tag{7}$$

If the difference between the two grating spatial frequencies is made to vary across the grating, the

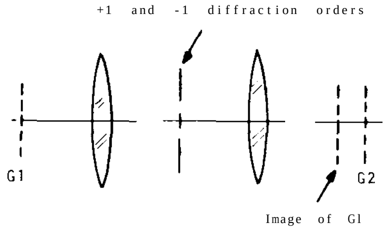

Fig. 2. Afocal lens system to eliminate undesired diffraction orders.

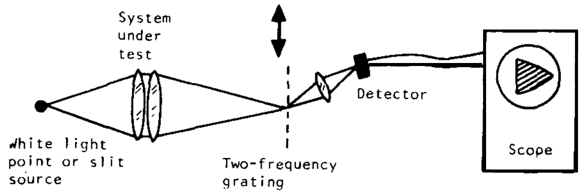

Fig. 3. Two frequency diffraction interferometer having variable difference frequency for measuring OTF.

shear is varied as the grating is translated in a direction perpendicular to the grating lines. At the same time, the phase difference between the interfering beam varies such that the irradiance of the interference pattern is modulated at a frequency

$$\omega = 2\pi v\left(\frac{1}{d_2} - \frac{1}{d_1}\right). \tag{8}$$

Hence, all the conditions for obtaining white light operation are satisfied. As long as the grating spatial frequencies are sufficiently high that the various diffraction orders do not overlap, additional diffraction orders pose no problems. The biggest drawback of the technique is that while the modulation frequency is independent of wavelength, the modulation frequency is a function of the grating spatial frequency difference and thus varies as the OTF spatial frequency is varied. This makes measuring the phase of the OTF more difficult, but certainly not impossible.

A third grating shearing interferometer for measuring the OTF is described in Refs. 11 and 12 for making wavefront measurements. The experimental setup is the same as shown in Fig. 3, except the two-frequency grating is replaced with two identical single frequency gratings placed in close contact. In this situation the shear is produced by rotating the two diffraction gratings with respect to one another. If d is the grating line spacing, f_{no} is the f number of the beam converging upon the gratings, and α is the angle one grating is rotated with respect to the second grating, the percentage shear, i.e., the ratio of the shear distance to the beam radius can, for small diffraction angles, be approximated as

$$\text{percentage shear} = [4f_{no}\sin(\alpha/2)]\lambda/d. \tag{9}$$

The principal advantage of the rotating diffraction grating OTF measuring interferometer over the two interferometers described above is that since the variable shear and modulation are achieved using rotational mechanical motion, instead of translational motion, data can be taken at higher rates.

Discussion

Equation (3) was written for the situation where the light source is either a point or a narrow slit source. For the general situation of an incoherent extended source, Eq. (3) becomes

$$F(\delta, S, \psi) = 2C\{1 + |\gamma(\nu_x, \psi)|\,|D(\nu_x, \psi)| \times \cos[\delta - \beta(\nu_x, \psi) - \theta(\nu_x, \psi)]\}, \tag{10}$$

where $|\gamma(\nu_x, \psi)|$ is the magnitude of the optical coherence function, and $\beta(\nu_x, \psi)$ is its phase. It should be noted that for the same reasons a grating shearing interferometer measures the OTF for a white light source, the grating shearing interferometer can also be used to measure the optical spatial coherence function for a white light source.[13,14]

It is interesting, but not surprising, to note the similarity between the autocorrelation method of measuring the OTF of an optical system as described in this paper and scanning methods for measuring the OTF (see, for example, Ref. 15). In the scanning method for OTF measurement either a sinusoidal grating is used as a test object and the image of the grating is scanned by a slit or point detector, or, equivalently, a slit or point source is used as a test object and the image of the slit is scanned by a sinusoidal grating. Square wave gratings can be used instead of a sinusoidal grating if the fundamental harmonic is selected electronically. Thus, a close relationship is seen between the scanning method of OTF measurement and grating shearing interferometer OTF measurement, especially for the last two grating shearing interferometers described in this paper. The two-frequency diffraction grating lateral shear interferometer is equivalent to using a variable frequency sinusoidal grating in a scanning method measurement. (The shearing interferometer could actually have been a variable frequency sinusoidal diffraction grating; however, it is believed it is easier to make the two-frequency grating than a sinusoidal grating.) Likewise the rotating diffraction grating lateral shear interferometer OTF measurement technique is equivalent to using Moiré fringes in a scanning mode, as described in Ref. 15.

Major portions of this work were performed while the author was at the Itek Corporation, Lexington, Massachusetts 02173. The work was partially supported by the USAF Space and Missile Systems Organization under contract F04695-67-C-0197.

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