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Testing Aspherics Using Two-Wavelength Holography

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It is shown that both single exposure and double exposure two-wavelength holography provide a good method of using visible light to obtain an interferogram identical to what would be obtained if a longer nonvisible wavelength were used. Both techniques provide for the real-time adjustment of defocus and tilt in the final interferogram. When both hologram exposures are made simultaneously, the sensitivity to air turbulence is essentially the same as if the longer nonvisible wavelength were used. Results are shown for testing both lenses and mirrors at equivalent wavelengths at 6.45 μ, 9.47 μ, 14.20 μ, 20.22 μ, and 28.50 μ obtained by using an argon laser for the visible light source.

An aspheric optical element is generally tested using either a mechanical or an optical probe or interferometry. Interferometry is generally the preferred method, because the complete surface is covered in a single measurement, unlike a probe, which measures the contour along only one diameter at a time. The major problem in using interferometry for testing a largely deformed aspheric is that the resulting interferogram contains too many fringes to analyze. Null lenses are often used in the interferometer to reduce the number of fringes in the final interferogram, but making a null lens is frequently very expensive, and it also must be tested some way.

Another method of reducing the number of fringes in the interferogram would be to use a longer wavelength light source in the interferometer. For example, if a CO_2 laser operating at 10.6 μ were used for the light source, rather than the commonly used He-Ne laser operating at 0.6328 μ, the interferogram would contain only about 1/17 (the ratio of the wavelengths) as many fringes. Increasing the wavelength, of course, decreases the sensitivity of the interferometric test, but for many cases, in particular in fabrication stage testing, a 10-μ wavelength yields adequate sensitivity.

There are three main disadvantages in using a longer wavelength, nonvisible light source in the interferometer: (1) ordinary refractive elements cannot be tested this way, (2) film cannot be used to record the interferogram directly, and (3) not being able to see the radiation causes added experimental difficulty. These three problems can be solved by using two-wavelength holography (TWH).[1-3] TWH provides a means of using only visible light to obtain an interferogram identical to the one that would be obtained if a longer wavelength were used. Due to the large number of different wavelengths that can be obtained from commercially available lasers, a wide range of equivalent wavelengths can be obtained using TWH.

There are two methods of TWH for testing optical elements. The first method consists of photographing the fringe pattern obtained by testing an optical element using a wavelength λ_1 in an interferometer such as the Mach-Zehnder type shown in Fig. 1. This photographic recording of the fringe pattern (hologram) is then developed and replaced in the interferometer in the exact position it occupied during exposure, and it is illuminated with the fringe pattern obtained by testing the optical element using a different wavelength λ_2. As will be shown in Appendix II, the moiré pattern obtained is identical to the interferogram that would have been obtained if the optical element were tested using a wavelength λ_{eq} where

$$\lambda_{eq} = \lambda_1\lambda_2/|\lambda_1 - \lambda_2|. \qquad (1)$$

See Table I for various values of λ_{eq} that can be obtained using various pairs of wavelengths from an argon and He-Ne laser.

This moire pattern will not have high contrast if the two fringe patterns giving the moire pattern do not have high contrast. If desired, the contrast of the final interferogram can be increased by spatial filtering. If this filtering is to be effective, the angle between the two interfering beams in the interferometer should be such that only the object beam, and not the reference beam, passes through aperture A1 (spatial filter) shown in Fig. 1. The spatially filtered moire pattern, which is imaged in plane B in Fig. 1, is a result of the interfer-

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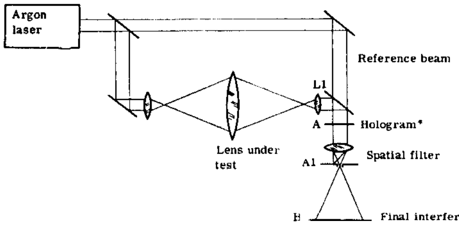

Fig. 1. Experimental setup for using TWH for testing lenses, (* image of exit pupil of lens under test; † image of hologram).

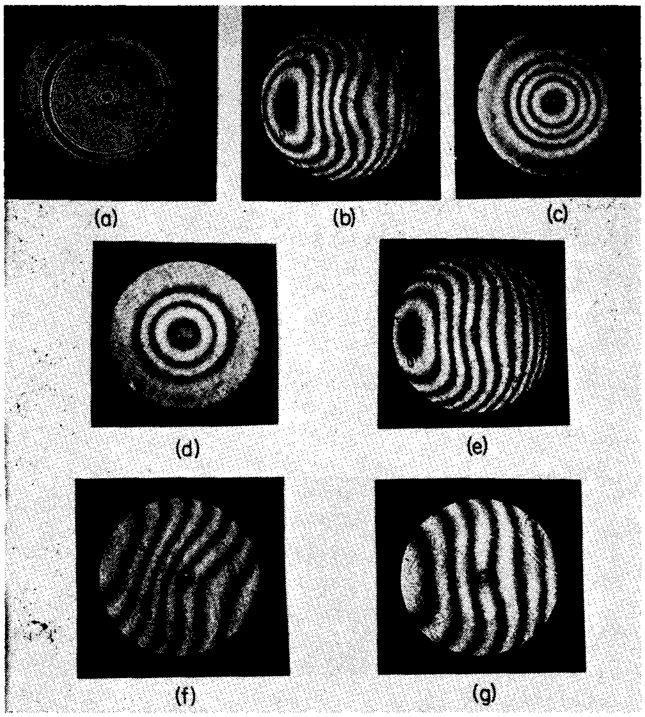

Fig. 2. Interferograms of a single lens. (a) λ = 0.4880 μ, (b) λ_{eq} = 6.45 μ, (c) λ_{eq} = 6.45 μ, (d) λ_{eq}, 9.47 μ, (e) λ_{eq} = 9.47 μ, (f) λ_{eq} = 20.22 μ, (g) λ_{eq} = 28.5 μ.

ence between the reconstruction of the hologram recorded using wavelength λ_1 and the wavefront obtained from the optical element using wavelength λ_2.

It is important that the fringe pattern (hologram) is recorded in the image plane of the exit pupil of the optical element under test, since the interferogram obtained using TWH correctly gives the difference between the two interfering beams only in the plane of the hologram. The final photograph of the interferogram should be recorded in the image plane of the hologram, i.e., in the image plane of the exit pupil of the optical element under test.

Figure 2(a) shows a Mach-Zehnder interferogram of a lens tested using a wavelength of 0.4580 μ. The other interferograms shown in the figure were obtained using TWH and the Mach-Zehnder interferometer shown in

Table I. Possible Equivalent Wavelengths, λ_{eq} Obtainable Using an Argon and a He-Ne Laser

	λ_1, μ					
λ_2, μ	0.4765	0.4880	0.4965	0.5017	0.5145	0.6328
0.4765	-	20.22	11.83	9.49	6.45	1.93
0.4880	20.22	-	28.5	17.87	9.47	2.13
0.4965	11.83	28.5	-	47.9	14.19	2.30
0.5017	9.49	17.87	47.9	-	20.16	2.42
0.5145	6.45	9.47	14.19	20.16	-	2.75
0.6328	1.93	2.13	2.30	2.42	2.75	-

Fig. 1. The interferograms shown in Fig. 2, (b), (c), (d), and (e) were obtained by first recording an interferogram (hologram) using a wavelength of 0.5145 μ and then illuminating the recording with a fringe pattern obtained using a wavelength of 0.4765 μ for Fig. 2, (b) and (c) and 0.4880 μ for Fig. 2, (d) and (e). The interferograms were spatially filtered as shown in Fig. 1. The amount of tilt shown in the interferograms was adjusted in real time by changing the angle at which the reference wavefront was incident upon the hologram during the reconstruction. The amount of defocus shown in the interferograms was also adjusted in real time by moving lens Ll in Fig. 1.

The interferograms shown in Fig. 2, (f) and (g) were obtained by first recording an interferogram using a wavelength of 0.4880 μ and then illuminating this recording with a fringe pattern obtained using a wavelength of 0.4765 μ and 0.4965 μ, respectively. As mentioned above, one of the real advantages of using TWH for testing aspheric optical elements is the wide

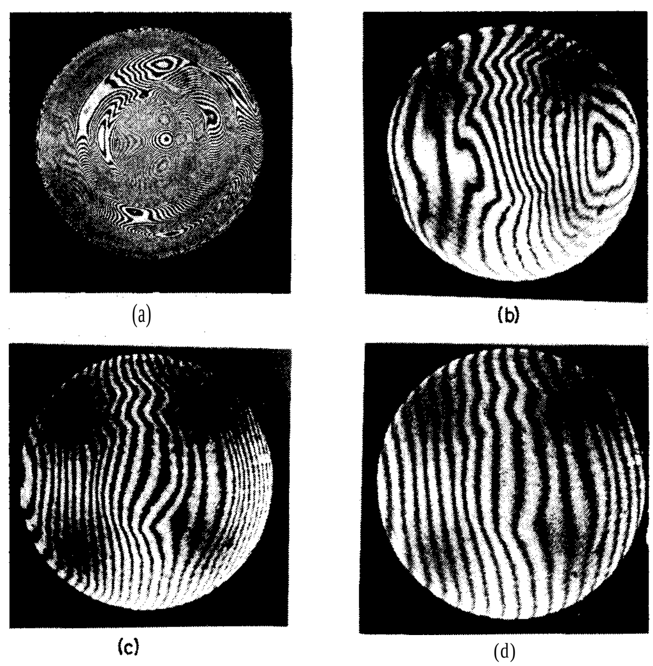

Fig. 3. Interferograms of an aspheric lens. (a) λ = 0.5145 μ, (b) λ_{eq} = 6.45 μ, (c) λ_{eq} = 9.47 μ, (d) λ_{eq} = 14.2 μ.

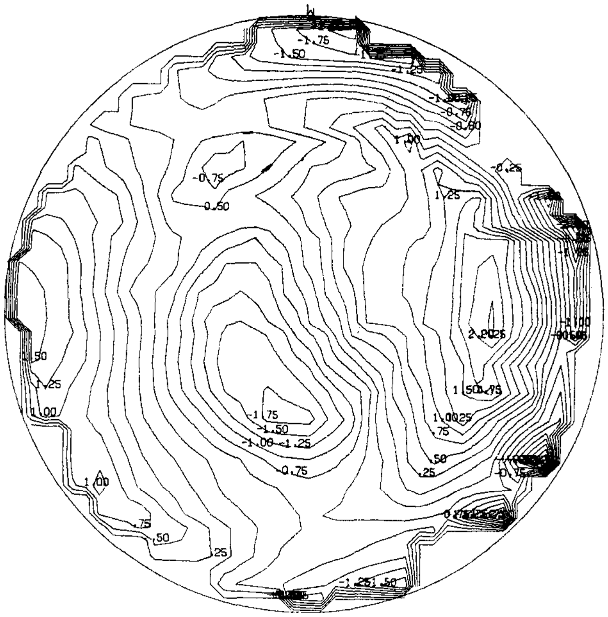

Fig. 4. Contour map obtained from interferogram shown in Fig. 3(b) (rms error = 1.00 λ, peak-to-peak error = 4.982 λ, λ = 6.45 μ).

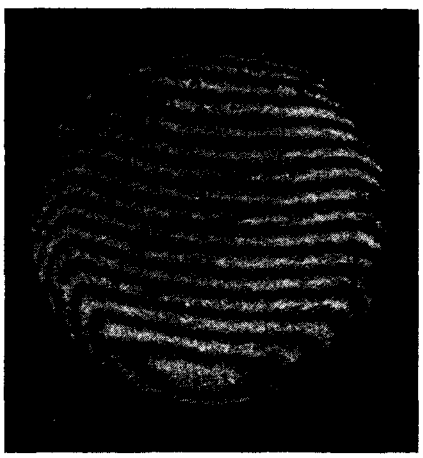

Fig. 3. Interferogram of waxed ground glass mirror (λ_{eq} = 9.47 μ).

range of equivalent, wavelengths that can be used for the test.

Figure 3(a) shows a Mach-Zehnder interferogram of a more complex aspheric lens tested using a wavelength of 0.5145 μ. The interferograms shown in Fig. 3, (b), (c), and (d) were obtained using TWH and the Mach-Zehnder interferometer shown in Fig. 1. Although the interferogram made using a wavelength of 0.5145 μ contains all the information on the aberrations in the element, it is too complicated to analyze to obtain a contour map of the wavefront produced by the lens. The interferograms made at the longer equivalent wavelengths can be analyzed to obtain a contour map as shown in Fig. 4.

It must be remembered that, any chromatic aberration in the optics in the interferometer or in a refractive element being tested will produce false results, since we are actually finding the difference between two interferograms obtained using two different wavelengths. Since the wavelength difference is small, and since the largest' part of chromatic aberration results in defocus (which can be adjusted in real time in our interferometer), chromatic aberration has not, yet caused us any trouble. If chromatic aberration were to introduce a sizable error in the results, it could of course be calculated and subtracted from the test, results.

Mirrors can be tested using TWH in a Twyman-Green interferometer modified in the same manner as the Mach-Zehnder described above. Figure 5 shows an interferogram of a waxed ground glass mirror[4] obtained this way.

In the method of TWH described above we are finding the difference between a fringe pattern recorded at one instant of time and a fringe pattern existing at some later instant of time. If the two fringe patterns are different for reasons other than wavelength change, e.g., air turbulence, incorrect, results are obtained. For example, if air turbulence causes one fringe change between the fringe pattern obtained using $\lambda_1 = 0.4880\ \mu$ and the fringe pattern obtained using $\lambda_2 = 0.5145\ \mu$, the moiré interferogram will contain one fringe error, which as Table I indicates, corresponds to an error of 9.47 μ.

The effect of air turbulence can be reduced by recording the two interferograms resulting from the two wavelengths simultaneously. When this interferogram (hologram) is illuminated with a plane wave, spatially filtered, and reimaged in the same manner as shown in Fig. 1, one obtains an interferogram identical to that obtained using the first method of TWH described above. Since both fringe patterns are recorded simultaneously, and air dispersion is small ($n_{0.4880\ \mu} - n_{0.5485} \cong 10^{-6}$), the sensitivity of the interferometer to

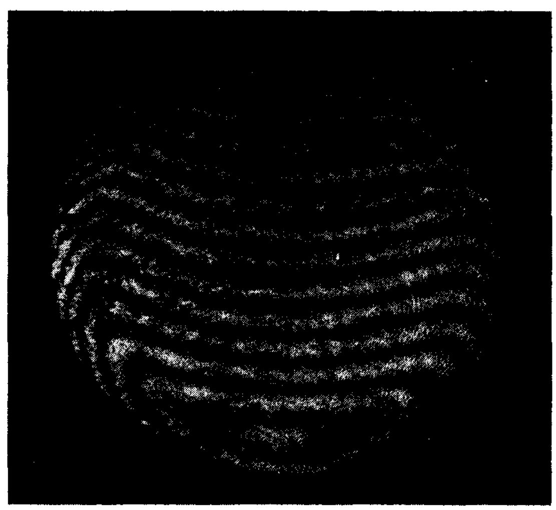

Fig. 6. Double exposure TWH interferogram of a waxed ground glass mirror taken with turbulence present.

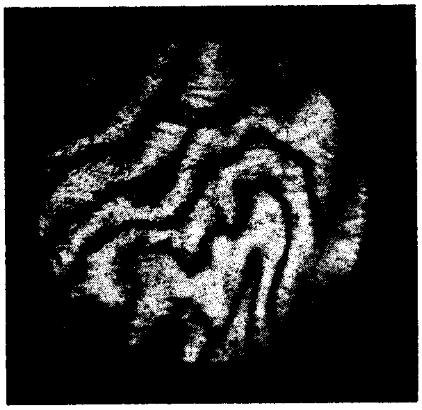

Fig. 7. Double exposure holographic interferogram of forced turbulence present in interferometer ($\lambda = 0.5145\ \mu$).

air turbulence is essentially the same as if a long wavelength light source were used in the interferometer.

One problem in using double exposure TWH, as just described, is that the amount of tilt and defocus in the final interferogram cannot be adjusted after the hologram is recorded. When desired, this problem can be solved by using the procedure described in Appendix I.

Double exposure TWH was used to test a waxed ground glass mirror. To demonstrate that reasonable amounts of turbulence caused no problems, a pan of hot water was placed in front of the mirror during the test to produce a large amount of turbulence. The resulting interferogram is shown in Fig. 6. The amount of tilt in this interferogram was adjusted in real time as described in Appendix II so it could be compared with the interferogram of the same mirror shown in Fig. 5 obtained using single exposure TWH with no turbulence present. As can be seen, the turbulence produced essentially no change in the interferogram.

The amount of turbulence introduced was measured using the conventional method of double exposure, single wavelength, hologram interferometry.[5] That is, a hologram was made of the mirror under test without the turbulence present, then a second exposure (using the same wavelength) was made with the turbulence present. The resulting interferogram, which gives a measure of the turbulence present, is shown in Fig. 7. The turbulence amounts to a fringe or two when a wavelength of 0.5145 μ is used. When we desensitize our interferogram, using TWH, to an equivalent wavelength of 9.47 μ, the turbulence of course produces a very small effect on the results.

It must be remarked that if the hologram exposure time is so long that the hologram fringes wash out over certain regions of the hologram, the corresponding regions of the final interferogram will be dark. However, even with a large amount of forced turbulence, this has not been a problem. For our work we have been using a 200-mW argon laser and Agfa l0E-56 photographic plates, and the hologram exposure time for testing waxed ground glass mirrors has been on the order of 1/60 sec.

Just as 10.6 μ from a CO_2 laser can be used to obtain interferograms of ground glass surfaces,[6] so can TWH. Figure 8 shows two TWH interferograms of approximately one half of an f/12, 7.5-cm diam ground glass mirror. As can be seen, the fringes have amazingly good contrast. However, there are two problems in using TWH for testing ground glass surfaces. First, since the hologram is made using visible light, the ground glass surface scatters the light so much that very little light gets back through the imaging lens onto the hologram. Thus, long exposures are required. The second problem is the difficulty involved in setting up an interferometer when the piece under test does not give a specular reflection.

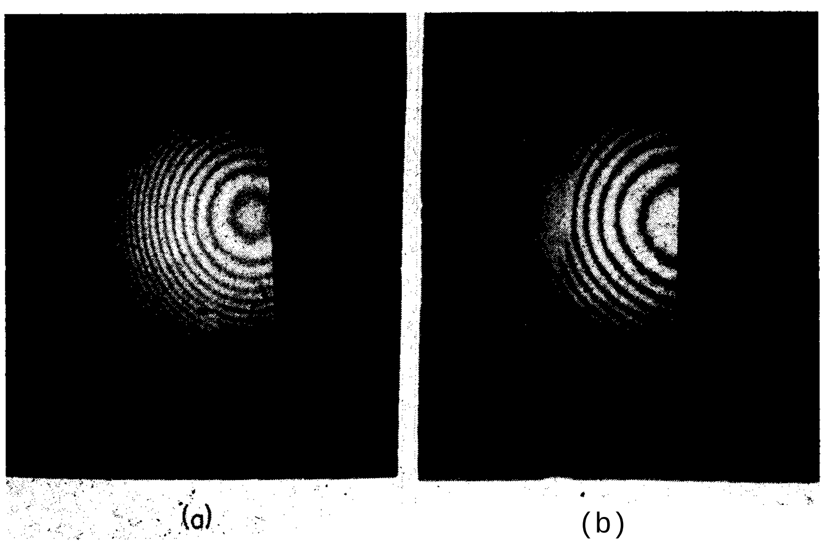

Fig. 8. Interferograms of a portion of a ground glass mirror. (a) $\lambda_{eq} = 9.47\ \mu$, (b) $\lambda_{eq} = 14.19\ \mu$.

Conclusion

It has been shown that both single exposure and double exposure TWH provide a good method of using visible light to obtain an interferogram identical to the one that would be obtained if a longer wavelength were used. A wide range of equivalent wavelengths can be obtained using commercially available lasers. Both techniques provide for the real-time adjustment of defocus and tilt in the final interferogram. When both hologram exposures are made simultaneously, the sensitivity to air turbulence is essentially the same, as if the longer nonvisible wavelength were used. TWH should prove to be very useful for the fabrication stage testing of both aspheric lenses and mirrors.

Appendix I

If the following procedure is used, the amount of tilt and defocus in the final interferogram obtained using double exposure TWH can be adjusted after the hologram is recorded.

The hologram should be recorded such that it is possible to spatially filter the hologram reconstruction so as to select the reconstruction due to only one of the original wavelengths. This requirement is fulfilled if the two fringe patterns making up the hologram are recorded with a sufficiently large angle between the plane reference wavefront and the object wavefront. Alternatively, a small angle between reference and object wavefronts can be used if the angle is sufficiently different for the two wavelengths.

If the above requirement is fulfilled, when the hologram is illuminated with two plane waves (both having a wavelength λ_3), the angle between the two plane waves can be selected such that the spatial filter passes only the reconstruction of the hologram recorded using λ_1 and reconstructed using plane wave 1, and the reconstruction of the hologram recorded using λ_2 and reconstructed using plane wave 2. Thus, in the image plane of the hologram we have the desired interferogram. The tilt in the interferogram can be adjusted by changing the angles between the two plane waves illuminating the hologram. A small amount of defocus can be introduced into the final interferogram by making one of the beams illuminating the hologram either slightly convergent or divergent. The amount of defocus added this way should be kept to a minimum to reduce the possibility of introducing added aberration into the reconstructed wavefronts.

Appendix II

It will be shown that both methods of TWH described in this paper give a final interferogram that is identical to the interferogram that would have been obtained if a wavelength λ_{eq}, as given in Eq. (l), were used. We will first look at the single exposure case.

Let both the reference wavefront, which is a plane wave tilted at an angle θ with respect to the normal to the hologram plane, and the object wavefront, which has a phase distribution $[(2\pi/\lambda_1)\phi(x,y)]$ as measured at the hologram plane, have unit amplitude. Then the amplitude of the light at the hologram plane is

$$\exp\,[i(2\pi/\lambda_1)\phi(x,y)] + \exp\,(i(2\pi/\lambda_1)x\sin\theta_1), \qquad \text{(A1)}$$

and the normalized intensity of the light at the hologram plane is

$$2 + (\exp\{i(2\pi/\lambda_1)[\phi(x,y) - x\sin\theta_1]\} + \text{C.C.}), \qquad \text{(A2)}$$

where C.C. means complex conjugate.

We will make the usual assumption that after exposure and development the amplitude transmission of the hologram is proportional to the exposure intensity. Thus, if the hologram is illuminated with the same amplitude distribution as was used during the exposure, except that now the reference wavefront is incident on the hologram at an angle θ_2 and the wavelength is changed to λ_2, the amplitude distribution transmitted through the hologram is equal to the product of Eq. (A2) times an equation just like Eq. (Al), except that λ_1 and θ_1 are replaced with λ_2 and θ_2, respectively. If this multiplication is carried out, one finds that two of the terms present in this product are

$$2\exp\left[2\pi i\,\frac{\phi(x,y)}{\lambda_2}\right] + \exp\left\{2\pi i\left[\frac{\phi(x,y)}{\lambda_1} + x\left(\frac{\sin\theta_2}{\lambda_2} - \frac{\sin\theta_1}{\lambda_1}\right)\right]\right\}. \qquad \text{(A3)}$$

The first term in Eq. (A3) is proportional to the new object beam (the wavefront from the optical element under test), and the second term is the hologram reconstruction of the original object beam. If θ_1 and θ_2 are correctly chosen such that the new object beam and the hologram reconstruction of the original object beam can be separated from the other wavefronts leaving the hologram, the intensity distribution in the image of the hologram is given by Eq. (A3) times its complex conjugate, which gives

$$I(x,y) = 5 + 4\cos 2\pi\{\phi(x,y)[(1/\lambda_1) - (1/\lambda_2)] + x[(\sin\theta_2/\lambda_2) - (\sin\theta_1/\lambda_1)]\}. \qquad \text{(A4)}$$

Thus, in the image of the hologram plane we obtain an interferogram that is identical to the interferogram that would be obtained if we were to interfere the wavefront from the optical element under test with a tilted plane wave if we used a wavelength λ_{eq} such that $1/\lambda_{eq} = |(1/\lambda_1) - (1/\lambda_2)|$. The amount of tilt is adjusted by changing θ_2. A small amount of defocus can be introduced into the object wave used in the reconstruction to adjust the amount of defocus in the interference pattern. Thus, we have real-time adjustment of both tilt and defocus.

Before spatially filtering, the intensity distribution in the hologram plane is the moiré pattern between the interferogram recorded using λ_1 and the interferogram obtained using λ_2. If the correct photographic process is chosen, the intensity distribution of this moire pattern is given by the product of Eq. (A2) times a similar equation with λ_1 and θ_1 changed to λ_2 and θ_2, respectively. The resulting normalized equation is

$$I(x,y) = 2 + \cos 2\pi \left[\phi(x,y) \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) + x \left(\frac{\sin\theta_2}{\lambda_2} - \frac{\sin\theta_1}{\lambda_1} \right) \right]$$

$$+ 2 \left\{ \cos \frac{2\pi}{\lambda_1} [\phi(x,y) - x \sin\theta_1] + \cos \frac{2\pi}{\lambda_2} [\phi(x,y) - x \sin\theta_2] \right\}$$

$$+ \cos 2\pi \left[\phi(x,y) \left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) - x \left(\frac{\sin\theta_2}{\lambda_2} + \frac{\sin\theta_1}{\lambda_1} \right) \right]. \quad \text{(A5)}$$

The first two terms of Eq. (A5) show that even without spatial filtering we still have an interferogram identical to what we would have obtained if we had used a longer wavelength, λ_{eq}, such that $1/\lambda_{eq} = |(1/\lambda_1) - (1/\lambda_2)|$. However, the contrast of the desired interferogram is reduced, because we also have some higher frequency fringe patterns present.

An alternative technique for obtaining similar results would be to use double exposure holography. The two exposures could be either simultaneous or sequential. The intensity of one exposure is given by Eq. (A2), and that of the second exposure would be identical to that of Eq. (A2), except that λ_1 and θ_1 would be replaced with λ_2 and θ_2, respectively. If we can again make the assumption that the amplitude transmission of the hologram is proportional to the exposure intensity, the normalized amplitude transmission is given by

$$4 + \exp\{i(2\pi/\lambda_1)[\phi(x,y) - x \sin\theta_1]\}$$
$$+ \exp\{i(2\pi/\lambda_2)[\phi(x,y) - x \sin\theta_2]\} + \text{C.C.} \quad \text{(A6)}$$

If the hologram is illuminated with a plane wave and if θ_1 and θ_2 were chosen wisely so that a spatial filter can be used to select the second and third term of Eq. (A6), the normalized intensity distribution in the image of the hologram is

$$1 + \cos 2\pi \{\phi(x,y)[(1/\lambda_1) - (1/\lambda_2)]$$
$$+ x[(\sin\theta_2/\lambda_2) - (\sin\theta_1/\lambda_1)]\}. \quad \text{(A7)}$$

It is noted that this expression is essentially the same as Eq. (A4).

If the hologram is illuminated with two plane waves having a wavelength λ_3, one incident at an angle θ_3 and the other at an angle θ_4, it follows from Eq. (A6) that the amplitude distribution transmitted through the hologram contains the two terms

$$\exp \left\{ 2\pi i \left[\frac{\phi(x,y)}{\lambda_1} + x \left(\frac{\sin\theta_3}{\lambda_3} - \frac{\sin\theta_1}{\lambda_1} \right) \right] \right\}$$

$$+ \exp \left\{ 2\pi i \left[\frac{\phi(x,y)}{\lambda_2} + x \left(\frac{\sin\theta_4}{\lambda_3} - \frac{\sin\theta_2}{\lambda_2} \right) \right] \right\}. \quad \text{(A8)}$$

If the angles θ_1, θ_2, θ_3, and θ_4 are selected correctly, it is possible to use spatial filtering to select these two terms in the amplitude distribution from the other terms. Thus, the normalized intensity distribution in the image of the hologram is

$$1 + \cos 2\pi \left[\phi(x,y) \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) + x \left(\frac{\sin\theta_2}{\lambda_2} - \frac{\sin\theta_1}{\lambda_1} + \frac{\sin\theta_3 - \sin\theta_4}{\lambda_3} \right) \right]. \quad \text{(A9)}$$

Thus, it is seen that the amount of tilt in the resulting interferogram can be adjusted by changing θ_3 and θ_4. Likewise, if a small amount of defocus is desired, one of the beams used in the reconstruction process should be made either slightly convergent or divergent. The amount of defocus added this way should be kept to a minimum to reduce the possibility of introducing added aberration into the reconstructed wavefronts.

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