Transfer function characterization of laser Fizeau interferometer for high spatial frequency phase measurements

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ABSTRACT

Large, high power laser systems such as that being constructed by Lawrence Livermore National Laboratories for the National Ignition Facility (NIF) require accurate measurement of spatial frequencies of up to 2.5 lines/mm over a 100mm field of view. In order to ensure accurate measurements of the parts, the test apparatus must be well characterized. The system transfer function (STF) of the interferometer under development to perform these measurements was calculated by comparing the power spectra of measurements of known phase objects to their theoretical power spectra.

Several potential problem areas were identified and studied. Of primary concern was the effect on the STF of the rotating diffuser and incoherent relay system employed in most commercial laser Fizeau interferometers. It was determined that such an arrangement degraded the transfer function beyond acceptability. The other major concern was possible inability to measure certain frequencies due to propagation between the test piece and alignment of the system optics. Use of strictly coherent imaging and small propagation distances between the test piece and return flat, the system transfer function could be kept at acceptable levels within the range of interest.

Keywords: Laser Fizeau Interferometer, Power spectrum, Transfer Function, Interferometry

1. INTRODUCTION

Commercial interferometers typically employ a rotating diffuser in an intermediate image plane. The image formed on this diffuser is relayed to the detector, often with a zoom lens to allow parts of varying sizes to be easily measured. The system is therefore a combination of a coherent and an incoherent system. The system transfer function is the product of the coherent transfer function of the interferometer, the incoherent transfer function associated with the ground glass and zoom system, and the detector.

Typically, laser Fizeau interferometric measurements are concerned with low frequency figure errors. The optics in high powered laser systems, however, must be free from periodic structures of up to 8 lines/mm. Such structures may cause small ripple amplification and lead to poor performance and system damage.[1] Thus, accurate measurements of higher spatial frequencies than are common are desired. The interferometer used to measure the parts for NIF project is required to have greater than a 60% system transfer function at half the Nyquist frequency. Over the 100mm sub-aperture used for the NIF optics measurements, this translates to measurement of frequencies up to 2.5 lines/mm.

The first part of this paper will analyze the transfer function of a WYKO 6000 laser Fizeau interferometer both with and without the rotating diffuser and zoom system. After this, the effects of misalignment of the system optics are studied to determine how critical this is to accurate measurements of the parts. Finally, the effects of defocus of the test part and varying distances between the test piece and return flat are studied.

2. SYSTEM CHARACTERIZATION

The Power Spectral Density Function, or PSD, is a measure of the spatial frequency content of the wavefront being analyzed. By comparing the PSD of an actual measurement with the theoretical PSD of the test part, one may easily determine the STF of the instrument across any desired spatial frequency range. The system transfer function is defined here as the ratio of the measured Fourier amplitude to the actual Fourier amplitude of the test piece over all spatial frequencies. Since the PSD gives the square of the Fourier amplitude coefficients, the STF may be calculated as follows:

$$STF(\nu) = \sqrt{\frac{PSD_{meas}(\nu)}{PSD_{ideal}(\nu)}} \tag{1}$$

In order to determine the STF, one must therefore take measurements on samples with known power spectral density characteristics. Ideally, one would measure a delta function object, which contains all spatial frequencies of equal amplitude. For such a case, PSD_{ideal} would be a constant for all frequencies and the system transfer function calculation is trivial. Unfortunately, no such object exists, so other objects with known Fourier spectra must be used.

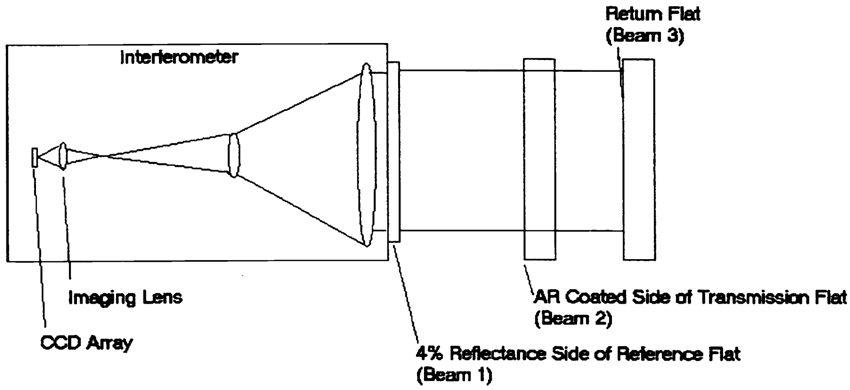

Figure 1: Configuration for creation of simulated sinusoidal phase grating from 3 beam interference. Varying the tilt between beams 2 and 3 allows the frequency and orientation of the effective phase grating to be varied.

One method which may be employed to determine the system transfer function uses three beam interference to create an effective sinusoidal phase grating. Figure 1 illustrates the setup employed to produce such a grating. In the setup, a small angle is introduced between the AR coated side of the third transmission flat and the second transmission flat. Considering the last two flats, standard two beam interference equations give the phase of the combination as[2]:

$$\Phi = \arctan\left[\frac{A_1 * \sin(\frac{2\pi}{\lambda}\sin(\theta_1) * x) + A_2 * \sin(\frac{2\pi}{\lambda}\sin(\theta_2) * x)}{A_1 * cos(\frac{2\pi}{\lambda}\sin(\theta_1) * x) + A_2 * \cos(\frac{2\pi}{\lambda}\sin(\theta_2) * x)}\right] \tag{2}$$

In the above equation, A_1 and A_2 are the amplitude reflectances of the two beams, λ is the wavelength, and θ_1 and θ_2 are the angles the two beams make with respect to the coordinate system. If the coordinates are defined such that $\theta_1=0$ and $\theta_2=\theta$ is the difference in angles between the two beams, one gets:

$$\Phi = \arctan\left[\frac{A_2 * \sin(\frac{2\pi}{\lambda}\sin(\theta) * x)}{A_1 + A_2 * \cos(\frac{2\pi}{\lambda}\sin(\theta) * x)}\right] \tag{3}$$

Provided the difference between A_1 and A_2 is significant, about a factor of two or greater, Φ is approximately sinusoidal. For the setup shown in Figure 1, A_1 was .20 and A_2 was .05, so this condition is met. Thus, an effective sinusoidal grating is presented to the interferometer for measurement. Typical phase shifting techniques are used to measure this effective grating. By varying the tilt of the AR coated flat, the period of the phase grating may be modified over a broad range while the amplitude remains constant.

Theoretically, the PSD of such a sine wave should show a single spike located at the spatial frequency of the grating.[3] Noise, imperfections in the flats, turbulence effects, and FFT effects broaden the spike and superimpose a finite background around the spike. Thus, for STF calculations, the area underneath the spike was used rather than just the peak height. In addition, the transfer function was normalized to one at the lowest spatial frequency measured rather than normalized strictly to the theoretical PSD of a sine wave.

For these measurements and the others described in this paper, a digital CCD camera with 740 pixels in the direction of interest was used for data acquisition. The field of view of the instrument was 100mm, giving a Nyquist frequency of 3.7 lines/mm. Grating measurements were taken over a frequency range of .1 to 2 lines/mm. Results of measurements taken on the same interferometer both with and without the incoherent relay and zoom system are presented in Figure 2.

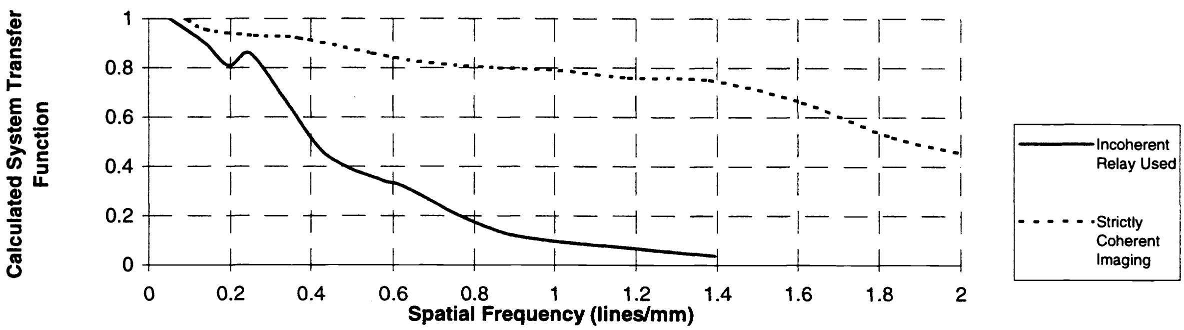

Figure 2: Calculated system transfer function from simulated sinusoidal phase gratings. The stricIty coherent system has much better performance than that with the incoherent relay system.

The plot clearly shows a severe degradation of the STF when the incoherent relay and zoom system are present. When strictly coherent imaging is used, the STF remains above 60% out to 1.7 lines/mm. The cutoff frequency with the incoherent relay system in place is only about .36 lines/mm. It should be noted that the coherent STF is not a rectangle function as predicted by theory primarily due to the camera employed by the system. When fringes are projected directly onto the camera and the modulation of the fringes measured, the falloff with spatial frequency is nearly identical to the coherent results presented above.

Another technique which may be used to calculate the system transfer function is to measure a high quality phase step. The PSD of a step of height H, measured with N data points over distance L is approximately[4]:

$$PSD(f_m) = \frac{2}{\pi^2 * L} * W * H^2 * \frac{1}{f_m^2} \tag{4}$$

A single step thus contains all spatial frequencies. Comparison of the PSD of a measured step to the theoretical PSD will allow the STF to be calculated across all frequencies with a single measurement. The power spectra of measurements of an actual fused silica step were compared with those from a mathematically generated ideal step and the transfer function calculated as described previously. Again measurements were taken both with and without the incoherent relay system .

Results are presented in Figure 3. For this data, no arbitrary normalization to low spatial frequencies was required due to the high quality of the phase step whose height of 131nm was sufficient to make noise sources relatively insignificant. Spatial frequencies below .17 lines/mm are not presented because low frequency figure errors in the step and FFT effects cause large differences in amplitude coefficients between the theoretical and measured data.

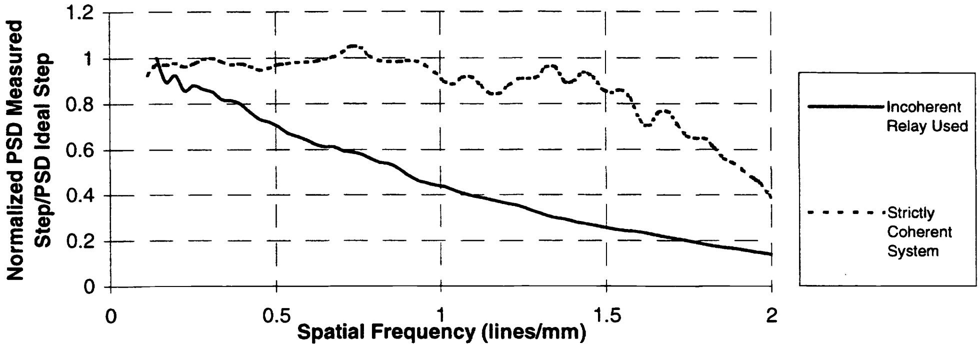

Figure 3: System transfer function as calculated from measuring a high quality phase step in reflection. As with the sinusoidal grating measurements, the transfer function is much higher when the incoherent relay and zoom system are removed.

As before, the elimination of the incoherent relay system significantly improves the system transfer function. The cutoff frequency with strictly coherent imaging is 1.8 lines/mm, similar to that measured with the simulated sinusoidal phase gratings. The cutoff frequency with the incoherent relay is .6 lines/mm, however, higher than that calculated by the other technique. The difference in the STF calculation for the incoherent relay is most likely due to the fact the phase grating technique effectively calculates the STF over the entire field. The step technique, however, only calculates the transfer function along the line of the step. Thus, off-axis aberrations introduced by the zoom system will have less of an effect on the calculated STF for the step calculation, causing it to fall off less drastically with this calculation method.

The same conclusions may be drawn from either calculation procedure for the STF. In order to achieve a significant transfer function out to half the Nyquist frequency, the rotating diffuser and incoherent relay system must be eliminated. Strictly coherent imaging requires that care be taken that nearly plane parallel surfaces, dust, and stray light not introduce artifacts into the measurement which would be eliminated by the incoherent imaging system. However, such precautions are necessary to ensure satisfactory measurements out to the required spatial frequency.

3. DEFOCUS AND PROPAGATION EFFECTS

Some of the optics for the NIF system are designed for use at Brewster's angle and therefore should also be tested at this angle. Due to the large size of these optics (400x900mm) there will be a significant propagation distance between certain parts of the test piece and return flat. Other portions of the beam will travel only a short distance between the test piece and return flat. Any variations in the measurement due both to the defocus of portions of the test part and Fresnel propagation effects between the part and return flat must be characterized. Even if the system transfer function is high under ideal circumstances, if high spatial frequencies cannot be measured due to the test configuration, the measurements may be inaccurate.

Since all functions may be broken down into sums of appropriately scaled sine waves, a consideration of the propagation of a sinusoidal phase disturbance is in order. A uniform, normalized beam passing through a sinusoidal phase grating may be represented by the following formula:[5]

$$u_i(x) = \exp\left[j\frac{m}{2}\sin(2\pi f_o x)\right] \tag{5}$$

In the above equation, m represents the peak to valley extent of the phase delay and f_o is the spatial frequency of the grating.

Provided m is sufficiently small (less than 0.1), we may approximate equation (5) as:

$$u_i(x) \approx J_o(m/2) + J_1(m/2)\exp(j2\pi f_o x) + J_{-1}(m/2)\exp(-j2\pi f_o x) \tag{6}$$

The Fourier transform of this is:

$$U_i(f) = J_o(m/2)\delta(f) + J_1(m/2)\delta(f - f_o) + J_{-1}(m/2)\delta(f + f_o) \tag{7}$$

Finally, under propagation in the Fresnel regime we get the following distribution in our final observation plane:

$$\begin{aligned} U_f(f) &= \exp(jkz)\exp(-j\pi\lambda z f^2) * T_i(f) \\ U_f(f) &= \exp(jkz)\begin{bmatrix} J_o(m/2)\delta(f) + J_1(m/2)\exp(-j\pi\lambda z f_o^2)\delta(f - f_o) \\ + J_{-1}(m/2)\exp(-j\pi\lambda z f_o^2)\delta(f + f_o) \end{bmatrix} \end{aligned} \tag{8}$$

which gives:

$$u_f(x) = \exp(jkz) * \begin{bmatrix} J_o(m/2) + \exp(-j\pi\lambda z f_o^2)[J_1(m/2)\exp(j2\pi f_o x) + \\ J_{-1}(m/2)\exp(-j2\pi f_o x)] \end{bmatrix} \tag{9}$$

This distribution is the same as $u_i(x)$ except for the frequency dependent phase terms multiplying the last two terms. This phase term causes the expression for $u_f(x)$ to represent a sinusoidal phase grating only when that term is real. When the frequency dependent phase term is strictly imaginary, the distribution is that of a sinusoidal amplitude grating, and a combination of the two at other points.

The distance the wavefront travels between the test piece and return flat can therefore greatly affect the phase measured by the instrument. In reflection, certain frequencies will be poorly measured or not measured at all due to this effect. In transmission, where the beam passes through the test piece twice, different positions of the return flat will cause different effects. The final beam entering the interferometer may have twice the peak to valley phase extent, as expected or no apparent phase modulation at all. Figure 4 illustrates the theoretical peak to valley phase variation of a sinusoidal grating under different amounts of propagation. For a round trip distance of 80 cm, the 1 line/mm grating will have almost no phase modulation upon returning to the test object. The interferometer would see phase variation only from the second pass through the grating. The measurement would be in error by a factor of two if this were not taken into account.

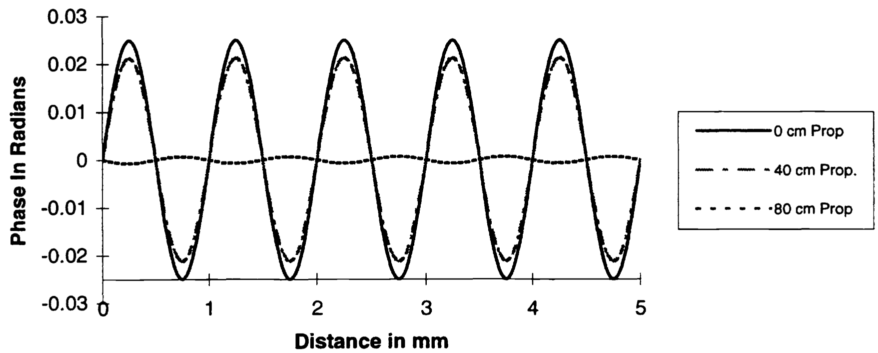

Figure 4: Phase modulation of 1mm period phase grating after propagating various distance. By 80cm of propagation, almost no phase variation is present. The wavelength used in the propagation calculations was 632.8nm.

More complex phase objects will suffer similar problems upon propagation of the beam over large distances, since they are merely superpositions of various sinusoidal components. To study defocus and propagation effects in detail, the fused silica step was measured in both transmission and reflection. For reflection measurements, the part was moved from the in-focus position of the interferometer and the transfer function calculated at various defocus locations. For measurements of the step in transmission, the system was focused on a return flat located far from the interferometer. The step was then measured at various locations between the reference flat and return flat. The transfer function calculations will show what frequencies are attenuated by propagation and how significant the attenuation actually is.

Figure 5 below shows how the profile of a step measured in reflection changes with defocus of the step. The in-focus measurement shows a very sharp edge with no apparent ringing. With the step defocused by 30cm, however, the step is less steep and ringing near the edges is apparent due to loss of some of the higher spatial frequencies.

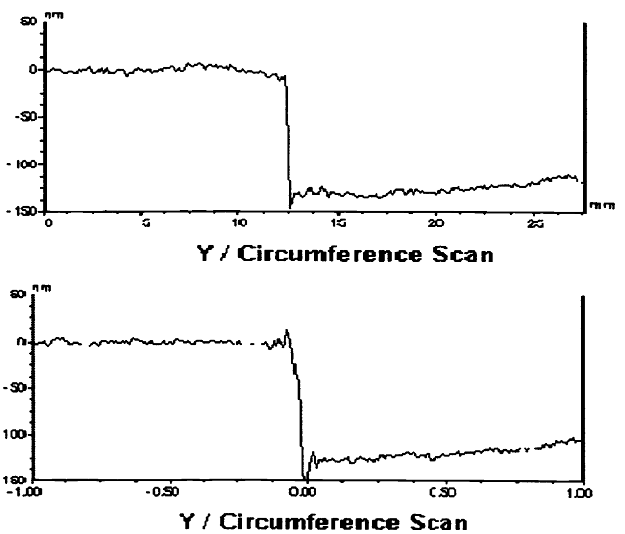

Figure 5: Step profile for an when step system is well focused on step (top) and when step is 30cm from focus (bottom). The profile of the defocused step exhibits ringing and step is not as steep due to loss of some of the higher spatial frequency content.

Figures 6 and 7 give the calculated system transfer function from the step measured in transmission and reflection. As the step moves out of focus or the distance to the return flat increases, the system transfer function begins to oscillate. Certain frequencies are highly attenuated while others are hardly attenuated at all, as one would expect from equation (9) above. As the distance from the in-focus position increases, the lowest attenuated frequency shifts inwards, as expected. In order to measure high spatial frequencies with good fidelity, the part must be as in-focus and close to the return flat as possible.

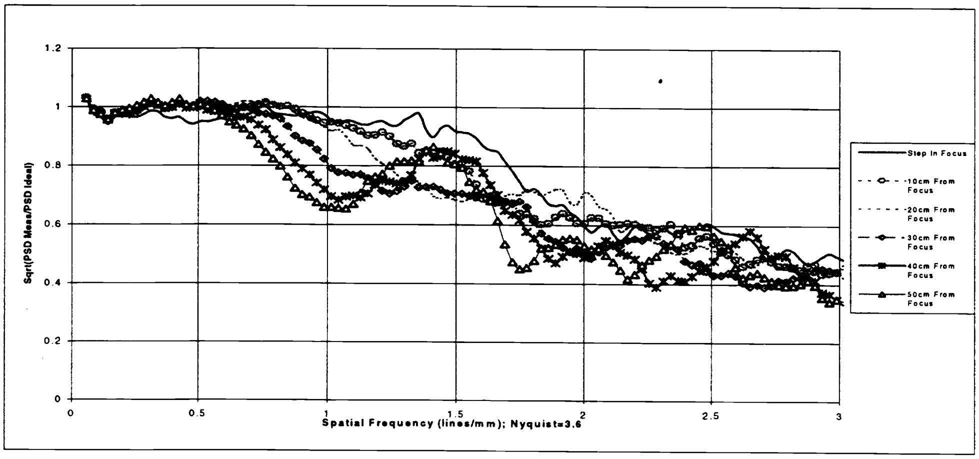

Figure 6: System transfer function from the step measured in reflection. Strictly coherent imaging was used. As the defocus distance increases, the first minimum of the system transfer function moves to lower frequencies.

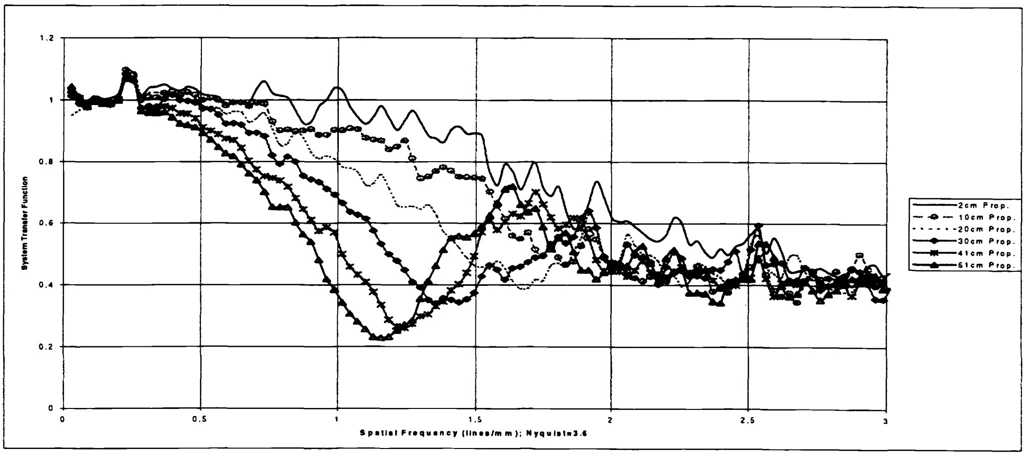

Figure 7: System transfer function from the step measured in transmission. Again strictly coherent imaging was used and the first minimum of the STF moves to lower frequencies with increasing distance from the return flat.

The calculated transfer functions from the step measurements have their first minima at the spatial frequencies predicted by equation (9). Neither of the transfer functions fall to zero at those frequencies, however, as theory would predict. The reflection measurements, in fact, show only about a 30% dip in the transfer function at the minima locations. The above plots show that the calculated system transfer function falls off much more significantly for the transmission measurements than for the reflection measurements. At this time, no satisfactory answer has been found as to why the two cases differ so significantly. Edge effects from the step as well as imperfections in the step may play a role in preserving spatial frequency content in the final beam. Also, the height of the step, nearly ¼ wave, may be sufficiently large to reduce the validity of the approximations made by assuming a small phase grating amplitude. Further study will be performed in an attempt to accurately model the two cases.

To ensure accurate measurement of the NIF optics, it was determined that the maximum distance between the test object and return flat should be no more than 20cm round trip when measuring over the 100mm square sub-aperture. Some of the large optics to be evaluated, however, will be tested at Brewster's angle since this is the angle of use in the NIF project. Since these optics may be as large as 400 by 900mm in size, there would in general be a large distance between portions of the test piece and return flat. To decrease the propagation distance for sub-aperture measurements, where the highest resolution is required, several changes were made to the system. First, instead of maintaining a fixed reference flat location, it was determined that the reference flat should be able to slide parallel to the test piece. Second, the return flat was truncated in the vertical direction. Truncating the edge of the circular flat allows the flat to be the same minimal distance from the test piece for any vertical subregion. As shown in Figure 8, such changes to the system along with appropriate changes to the mounts, allow the return flat and to approach within 5cm of the tilted test piece. Less than a 20cm round trip path difference is then maintained over a 100mm subregion of the test piece. The system should then be able to measure all spatial frequencies of interest with the required transfer function.

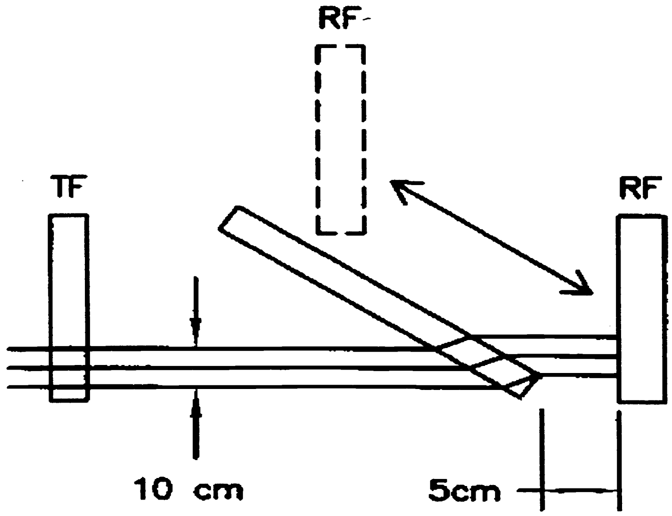

Figure 8: Measurement setup for test piece tested at Brewster's angle. Return flat is translated parallel to the test piece to minimize propagation distances.

4. CONCLUSIONS

In order to meet the measurement requirements for the NIF project, the system transfer function must be maintained at a high level out to half the Nyquist frequency of the instrument. It was seen that the rotating ground glass screen and commercial zoom lens used with most commercial interferometers must be removed from the system to achieve these results. Measurement of the system transfer function through use of simulated sinusoidal phase gratings and a high quality phase step supports the need for strictly coherent imaging. Using coherent imaging throughout the system has the disadvantage of requiring that the system be kept totally free from dust and stray reflections that may cause artifacts in the final measurement. However, the drastic increase in spatial frequency resolution this allows more than offsets these problems.

The parts to be tested for the NIF project are quite large and some are to be tested at Brewster's angle, the angle of use in the NIF project. Thus the beam which passes through the part will travel significant distances before striking the return flat and passing through the part a second time. With propagation, an additional phase term multiplies each spatial frequency and causes phase variations to change amplitude. Thus, phase terms from the outward pass through the test piece may cancel those obtained during the second pass, causing inaccurate measurement of frequency content. To eliminate such effects, it was determined that the maximum round trip propagation distance from the test part to return flat should be no more than 20cm. To achieve this specification, the return flat was truncated in the vertical direction and allowed to move parallel to the test piece. With these precautions, the measurement requirements of the system may be met.

5. REFERENCES

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