Testing stress birefringence of an optical window

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ABSTRACT

This paper describes a method to measure the birefringence of an optical window. The transmitting wavefront includes the contributions from the two surfaces, the material inhomogeneity, and the birefringence. Because of the birefringence, the transmitting wavefront has different profiles for different orientations of polarization of linearly polarized beams. From this difference, the amount of phase difference for the fast and slow axes is obtained. Thus, the birefringence is calculated. With this method, the contributions from the two surfaces and the material inhomogeneity are removed. A laser rod was measured with different methods. The theoretical derivation, comparison of different methods, and experimental results are presented.

1. INTRODUCTION

A transparent isotropic solid becomes birefringent when subjected to mechanical force.[1] If the solid acts like a uniaxial crystal, the two privileged directions of the crystal at any point of the solid are parallel to the directions of the principal stresses in the plane of the wavefront at that point. In general, the direction of the two principal stresses will change from point to point in the crystal. Linearly polarized incident light, either parallel or perpendicular to those directions, emerges unchanged. For other directions of linearly polarized light or for other polarization states, the polarization state of the transmitted light changes and becomes elliptically polarized, in general. By comparing the polarization states of the incident light and the transmitted light, the birefringent property of the material can be deduced.

The most common method is simply to place both the sample under test and a quarter waveplate between crossed linear polarizers[2,3] (one polarizer and one analyzer), and observe the change of the transmitted light while rotating the analyzer. Another method is to put a Kerr-cell[4] or Babinet compensator between the sample and the analyzer to cancel the phase retardation produced by the birefringence. Thus, the emergent light from the Kerr-cell or Babinet compensator has the same polarization state as that of the light incident upon the sample.

In this paper, we use a plane wave either linearly or circularly polarized to measure the wavefront change through a laser rod of 40 cm in length and 6 cm in diameter, by means of an interferometer. The transmitted wavefront can change due to the index inhomogeneity, the birefringence, and the surface figures of the rod. By taking measurements with different configurations, and manipulating the data, the birefringence contribution of the material is deduced and the birefringence coefficient is derived. Because of the structure of the rod, the stress has a radial symmetry, and varies with the radius. The directions of the principal stress are parallel or orthogonal to the radius. Thus, for the ray along the axis of the rod, the fast axis is parallel to the radius and the slow axis is perpendicular to the radius, or vice versa. First we briefly review Jones vectors and matrices, then drive the formula of the transmitted wavefront. From the difference between the transmitted wavefront and the reference wavefront, the birefringence coefficient is derived. Comparison of different methods and the experimental results are presented, and the accuracy of this method is discussed.

2. CHARACTERIZATION OF THE TRANSMITTED LIGHT

Since the light source used is polarized, first we briefly review the Jones vector and matrix,[5] then drive the formula of the transmitted light. We explain the interference between two waves of different polarization states and how the phase shifting technique works. Polarized light written in the form of a Jones vector is

$$E = \begin{bmatrix} E_{ox}\, e^{i\varphi_x} \\ E_{oy}\, e^{i\varphi_y} \end{bmatrix}, \tag{1}$$

where $i\varphi_x$ and $i\varphi_y$ are the appropriate phases. If $i\varphi_x = i\varphi_y$, the light is linearly polarized. A retarder with a fast axis in the vertical direction is written in the form of a Jones matrix as

$$M_o = \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\phi} \end{bmatrix}, \tag{2}$$

where ϕ is the phase lead in y-axis. When $-\phi < 0$, the fast axis is in y-axis; when $-\phi > 0$, the fast axis is in x-axis. For a general case where the fast axis is rotated an angle θ with respect to horizontal, the Jones matrix can be written as

$$M_\theta = R_{-\theta} \cdot M_o \cdot R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{-i\phi} \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}. \tag{3}$$

Thus, from Eqs. (1) and (3), the transmitted light can be expressed as

$$E' = R_{-\theta} \cdot M_o \cdot R_\theta \cdot E. \tag{4}$$

By adding the two vectors E and E', we can calculate the resulting electric field of the interference between the reference (i.e. incident) and the transmitted waves. Any two complex vectors A and B are orthogonal if $A \cdot B^* = 0$. Similarly, two electric fields are orthogonal, when $E \cdot E'^* = 0$, and the contrast of the interference fringe is equal to zero. It can be proved that the absolute value of $(E \cdot E'^*)/(|E| \cdot |E'|)$ is equal to the contrast of the interference fringe.

When the incident wave is left hand circularly polarized, $[1/\sqrt{2},\ i/\sqrt{2}]$, the phase change due to birefringence is the same for the same stress difference in the two principal stress directions, regardless of the orientation of the two directions, as shown below. From Eq. (4), for an orientation, θ, the transmitted wave can be written as

$$E'(x, y) = 1/\sqrt{2} \begin{bmatrix} (c^2+s^2\cos\phi-cs\sin\phi)+i(cs-cs\cos\phi-s^2\sin\phi) \\ (cs-cs\cos\phi+c^2\sin\phi)+i(s^2+c^2\cos\phi+cs\sin\phi) \end{bmatrix}, \tag{5}$$

where $c = \cos\theta$, $s = \sin\theta$, and $\phi(x, y)$ is defined in Eq. (2). This transmitted wave can be expressed by a linear combination of left hand and right hand circular polarization states. Thus,

$$E'(x, y) = 1/2\{(1+\cos\phi)-i\sin\phi\} \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix} + 1/2\{[(c^2-s^2)(1-\cos\phi)-2cs\sin\phi] + i[(c^2-s^2)(\sin\phi)+2cs(1-\cos\phi)]\} \begin{bmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{bmatrix}. \tag{6}$$

Because only the electric fields with the same polarization state interfere, only the first term in the right hand side interferes with the incident wave. Therefore, the phase change for the return beam is determined by the ratio of the complex factor of the first term, $1/2\{(1+\cos\phi) - i\sin\phi\}$, to the complex factor of the incident wave, unity. Thus, the transmitted wavefront deformation due to birefringence is given as follows:

$$\phi_t(x, y) = \tan^{-1} \{-\sin\phi(x, y)/(1+\cos\phi(x, y))\} = - \phi(x, y)/2. \quad (7)$$

It is clear that this equation is not a function of θ, and $\phi_t(x, y)$ is equal to one half of the phase retardation between the fast and the slow axes, regardless of the orientation of the two axes. The minus sign indicates the phase lead. It can be shown that Eq. (7) is still true, if the incident wave is right hand circularly polarized. Thus, the birefringence contribution is the same for both right hand circular polarization and left hand circular polarization.

It should be noted that in general the polarization state of the transmitted beam is different from that of the incident beam, i.e., reference beam. Because the interference is the addition of the two electric fields, i.e., vector sum, the difference in the polarization state affects the contrast of the interference fringes, as explained below. Any polarization state can be described by the linear combination of two orthonormal polarization states. Thus, the phase difference between the incident and the return wave can be derived from the phase of the vector component, which is parallel to the incident wave polarization state, of the return wave polarization state. Because the other component is orthogonal to the incident wave polarization state, it only affects the contrast of the fringes, not the phase. Thus, the information about the phase can be derived from the intensity of the interference by shifting the phase of the reference beam as used in the common phase shifting technique.

3. THEORY

In this section, we explain how the wavefront changes due to birefringence, and how to retrieve the birefringence from the measurements of different configurations or polarization states. Then, we derive the formula of the transmitted wavefront for a laser rod with stresses of rotational symmetry tested using collimated and polarized wave. Figure 1 shows the schematic diagram of the experiment setup. The material under test has two flat surfaces at both ends, and is measured with a collimated polarized light. If the end surfaces of the material are not polished, the material can be immersed in an index matching liquid. An interferometer measures the return wavefront of the light after it is transmitted through the material, is reflected by a flat, and is returned. The return wavefront is the sum of the contributions of the inhomogeneity of the material, the birefringence of the material, and the figures of the end surfaces S1 and S2, the return flat R, and the transmission flat T. The return wavefront can be written as

$$W(x, y) = M + B + S1 + S2 + R + T, \quad (8)$$

where S1, S2, R and T are the surface contributions. M is the material contribution due to the inhomogeneity not including the birefringence effect, i.e. refractive index variation over the pupil for unpolarized light. B is the contribution due to the birefringence. In Eq. (8), all terms in the right hand side except for the birefringence contribution B are invariant with the polarization state of the incident beam. Therefore, by changing the polarization of the incident beam and subtracting two measurements with different polarization states, the contributions from M, S1, S2, R, and T are cancelled. From the difference, the birefringence coefficient of the material can be derived.

In the following, we derive a formula of the phase change of polarized light through a laser rod with stress. Because of the structure of the rod, the stress has a radial symmetry and varies with the radius. The directions of the principal stress are parallel or orthogonal to the radius as shown in Fig. 2A. The difference in velocity of the ordinary and the extraordinary rays at a point is proportional to the difference of the two principal stresses. Thus, for the ray along the axis of the rod, the fast axis is parallel to the radius, and the slow axis is perpendicular to the radius, or vice versa, depending on the material. Because the stress is rotational symmetric, the phase retardation between the fast and the slow axes is also rotational symmetric, as is the phase of the transmitted wave, $\phi_t(r)$. Moreover, if the stress in the rod increases with the distance from the center, then $\phi_t(r)$ increases with the distance from the center and has a rotational symmetry, similar to a defocus term.

For the same rod, tested with a wave linearly polarized in the horizontal direction, i.e., the electric filed = [1, 0], the light emerging at four orientations on the rod, V_1, V_2, H_1, and H_2, remains linearly polarized in the horizontal, as shown in Fig. 2B. The phase differences between them are given below. Because the birefringence of the rod is rotational symmetric, at any point in the rod, (r, θ), the fast axis is rotated by an angle θ. Thus, using Eq. (5), the electric field of the transmitted wave is equal to

$$E'(r,\theta) = \begin{bmatrix} (c^2+s^2\cos\phi)+i(-s^2\sin\phi) \\ (cs-cs\cos\phi)+i(cs\sin\phi) \end{bmatrix}, \tag{9}$$

where $c = \cos\theta$, $s = \sin\theta$, and $\phi(r)$ is the phase lead for the fast axis, i. e. perpendicular to the radius. In the orientation H_1, i.e. $\theta = 0^\circ$, the electric field equals [1, 0]. Both the polarization state and the phase of the transmitted wave remains unchanged. In the orientation V_1, i.e. $\theta = 90^\circ$, the electric field equals $[\cos\phi - i\sin\phi, 0]$. Thus, the polarization state remains unchanged, but the phase difference between the transmitted and the reference waves is $-\phi$. It means that the phase of the transmitted wave leads the phase of the reference wave by ϕ in the orientation V_1 and V_2. Thus, the birefringence causes the light to propagate fast in the orientations of V_1 and V_2.

It can be shown that for small stress, i.e., the phase retardation is less than a quarter wave, the deformation at (r, θ) introduced by birefringence for a wave linearly polarized in the horizontal, is approximately equal to

$$\text{Deformation}(r, \theta) = (n_e(r) - n_o(r)) \cdot \tau \cdot \sin^2\theta / \lambda, \tag{10}$$

where τ is the length of the rod and λ is the wavelength. This deformation has the form of astigmatic profiles. Because the polarization states in the orientations V_1, V_2, H_1, and H_2, remain unchanged, the fringe contrast in those orientations is equal to unity. In other orientations the contrast decreases; at the four 45 degree locations the contrast is the worst as shown in Fig. 3. If the contributions of M, S1, S2, R, and T are smaller than that of the birefringence, then from the amount of the astigmatism, we can determine the birefringence. Moreover, while changing the polarization state of the incident wave to linearly polarized in the horizontal, the astigmatic profile will flip the orientation 90 degrees. Therefore, the birefringence coefficient can be derived with the procedure below.

1. Obtain the wavefront $W_1(x, y)$,with a linear polarization.
2. Rotate the polarization state by 90 degrees, and obtain $W_2(x, y)$.
3. Subtract the two measurement results.

As we pointed out in Eq. (8), because the contributions of M, S1, S2, R, and T are invariant for different polarization states, their contributions are cancelled after the subtraction of the two measurement results. Thus, the difference is equal to the difference of the birefringence contributions from the different polarization states. If a material has a birefringence of rotational symmetry, this difference is equal to twice the birefringence contribution. Therefore, from Eq. (10), if the phase retardation is less than a quarter wave for such a material, the birefringence is approximately equal to

$$n_e(r) - n_o(r) = (W_2(r) - W_1(r))\,\lambda/2\tau. \tag{11}$$

If the contributions of M, S1, S2, R, and T are rotational symmetric or smaller than that of the birefringence, in Step 2 we can rotate the rod by -90 degrees and then mathematically rotate the measurement result back. Thus, the birefringence can also be obtained using Eq. (11). Moreover, from Eq. (8), because S1, S2, R, and T can be measured separately, the material contribution, M, can be obtained as long as $(n_e(r)-n_o(r))$ is obtained.[6]

4. EXPERIMENT

A rod was placed between two cross polarizers, and a dark cross pattern was observed, regardless of the rod orientation. The fast axis (or the slow axis) of birefringence was along the rod radius, and the other axis was perpendicular to the rod radius. We performed two experiments, since we had only two types of polarization states of the light source at the moment of the experiment: one is with a circular polarization and the other is with a linear polarization in the vertical. For each polarization state, we rotated the rod 90 degrees each time, and the contribution from the transmission flat and the return flat, T and R, were subtracted. While tested with a circularly polarized beam, the transmitted wavefront, W_C, included the combination effect of the two surfaces and the material inhomogeneity plus the birefringence. The four measurement results are W_{C0}, W_{C90}, W_{C180}, and W_{C270}, in which the rod was oriented at 0°, 90°, 180°, and 270°, as shown in Fig. 4.

While tested with a collimated beam linearly polarized in the vertical, the contribution of the birefringence has a form of astigmatism, as we pointed out in the previous section. The four measurement results are W_{L0}, W_{L90}, W_{L180}, and W_{L270}, in which the rod was oriented at 0°, 90°, 180°, and 270°, as shown in Fig. 5.

5. DISCUSSION AND CONCLUSION

Mathematically rotating the measurement results obtained using a circularly polarized beam, W_{C0}, W_{C90}, W_{C180}, and W_{C270} back to 0° and comparing, it can be seen that those four results are astigmatic and very similar. This shows that the birefringence of the rod is rotational symmetric. This was also observed using two cross polarizers. Since the surface figures were measured separately and their contributions were negligible, we concluded that this astigmatism is due to index inhomogeneity.

The four measurement results obtained with a linearly polarized beam are W_{L0}, W_{L90}, W_{L180}, and W_{L270}. For each of them, the contributions from R and T were subtracted. From Eq. (8),

$$W_{L0} = M_0 + S1_0 + S2_0 + B_{V0}, \tag{12}$$

$$W_{L90} = M_{90} + S1_{90} + S2_{90} + B_{V90}, \tag{13}$$

$$W_{L180} = M_{180} + S1_{180} + S2_{180} + B_{V180}, \text{ and} \tag{14}$$

$$W_{L270} = M_{270} + S1_{270} + S2_{270} + B_{V270}. \tag{15}$$

Because the birefringence of the rod is rotational symmetric, the birefringence contribution is the same for all orientations of the rod. Thus, $B_{V0} = B_{V90} = B_{V180} = B_{V270} = B_V$. Manipulating the data,

$$W^R_{L90} = \mathbf{R}_{-90}(M_{90}+S1_{90}+S2_{90}+B_V) = M_0 + S1_0 + S2_0 - B_V + \text{focus}, \tag{13a}$$

$$W^R_{L180} = \mathbf{R}_{-180}(M_{180}+S1_{180}+S2_{180}+B_V) = M_0 + S1_0 + S2_0 + B_V, \tag{14a}$$

$$W^R_{L270} = \mathbf{R}_{-270}(M_{270}+S1_{270}+S2_{270}+B_V) = M_0 + S1_0 + S2_0 - B_V + \text{focus}, \tag{15a}$$

where **R** is the rotation operation rotating the data back to 0°. Because the birefringence contribution is astigmatic, as given in Eq. (10), rotating the astigmatism 90° is equivalent to changing the sign of astigmatism plus a focus term. Therefore, $\mathbf{R}_{-90}(B_V) = -B_v$ + focus. Similarly, $\mathbf{R}_{-270}(B_V) = -B_v$ + focus, and $\mathbf{R}_{-180}(B_V) = B_v$. Thus, $W_{L0} - W^R_{L90} = 2B_V$ + focus. Hence,

$$\begin{aligned} B_V &= \text{astigmatism of } (W_{L0} - W^R_{L90})/2 \\ &= \text{astigmatism of } (W_{L0} - W^R_{L90} + W^R_{L180} - W^R_{L270})/4. \end{aligned} \tag{16}$$

From the experimental results, the astigmatism is about 0.36 wave. Using Eq. (11), the birefringence is about 2.85 nm/cm. Moreover, it can be shown that the average of W_{L0}, W^R_{L90}, W^R_{L180}, and W^R_{L270} is equivalent to the average of W_{C0}, W^R_{C90}, W^R_{C180}, and W^R_{C270}.

From Eq. (11), the accuracy of $(n_e(r) - n_o(r))$ is limited by the accuracy of the wavefront measurement and the length of the material under test. The longer the material, the more accurate the birefringence measurement. Since we can easily obtain repeatability of 0.02 wave p-v using a phase shifting interferometer, the accuracy of $(W_{L0} - W^R_{L90})$ is about 0.02 wave, p-v. Thus, for the 40 cm long rod, the accuracy of $(n_e(r) - n_o(r)) = 0.02*633$ nm / 2(40 cm) = 0.16 nm/cm p-v. Therefore, this method is more accurate than the polarizer analyzer method.

In summary, the transmitting wavefront is different for a circularly polarized beam and for a linearly polarized beam. If the rod is used in a linearly polarized beam system, it is better to test the rod with a linearly polarized beam. Because of the birefringence, the measurement results might be different for different polarization states. If we know the two principal axes of birefringence, the amount of birefringence of the material can be derived from the difference of the measurements. For example, if the birefringence of the material is rotational symmetric, birefringence can be derived by using a linearly polarized beam and rotating the orientation of the polarization or the rod. By mathematically manipulating the data, the entire birefringence distribution over the pupil was obtained using a common phase shifting technique. The advantage of this method is that the system error is removed during the data subtraction. Therefore, the two surfaces can be within several waves, and need not to be absolutely flat. However, one should note that the result of subtraction is equal to the difference of the birefringence contributions for different polarization states, but not directly equal to the difference of the refractive index of the extraordinary and ordinary waves, i.e., birefringence.

6. ACKNOWLEDGMENTS

The authors will like to thank Robert E. Parks for providing the ASTM references.

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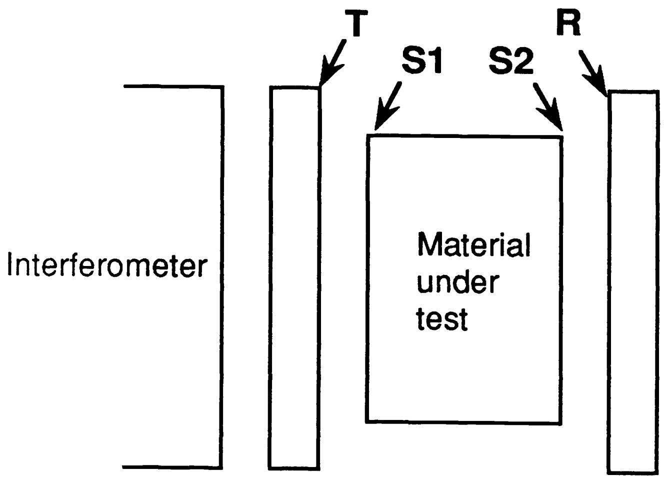


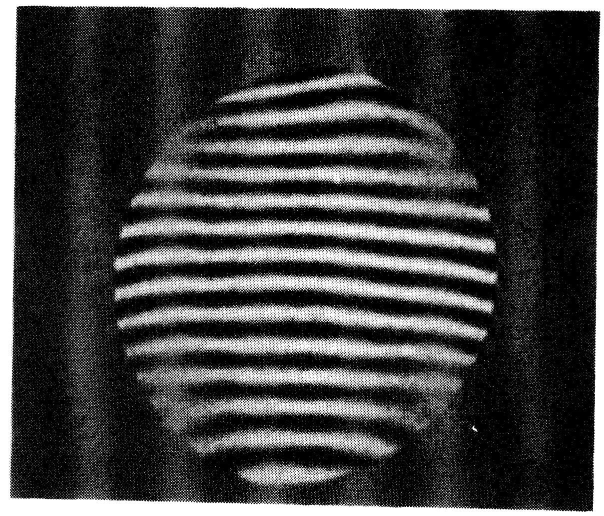
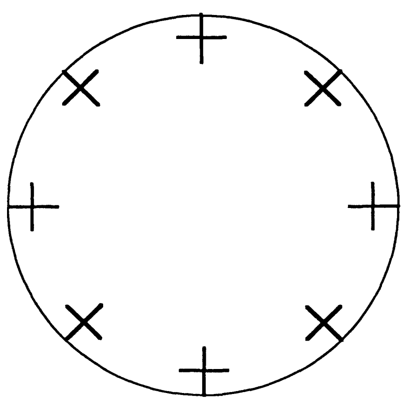


Fig. 1. The schematic diagram of the experimental setup. T is the transmission flat and R is the return flat.

Fig. 3. The interference fringes have low contrast at the four 45 degree corners, obtained with a collimated beam linearly polarized in the vertical direction.



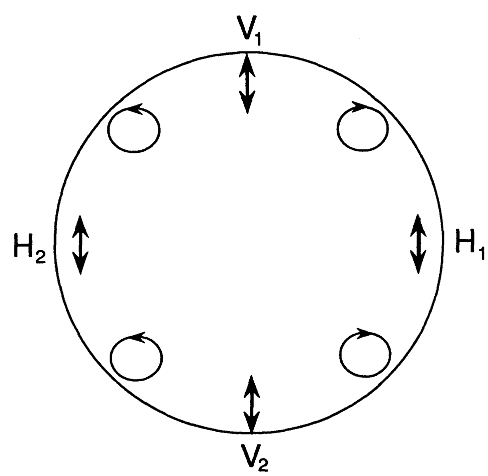


Fig. 2. (A) The principal directions of the stress in the cross section of the rod. They are either parallel or orthogonal to the radius. (B) For incident light linearly polarized in the vertical, the polarization remains unchanged in four orientations, V_1, V_2, H_1, and H_2.

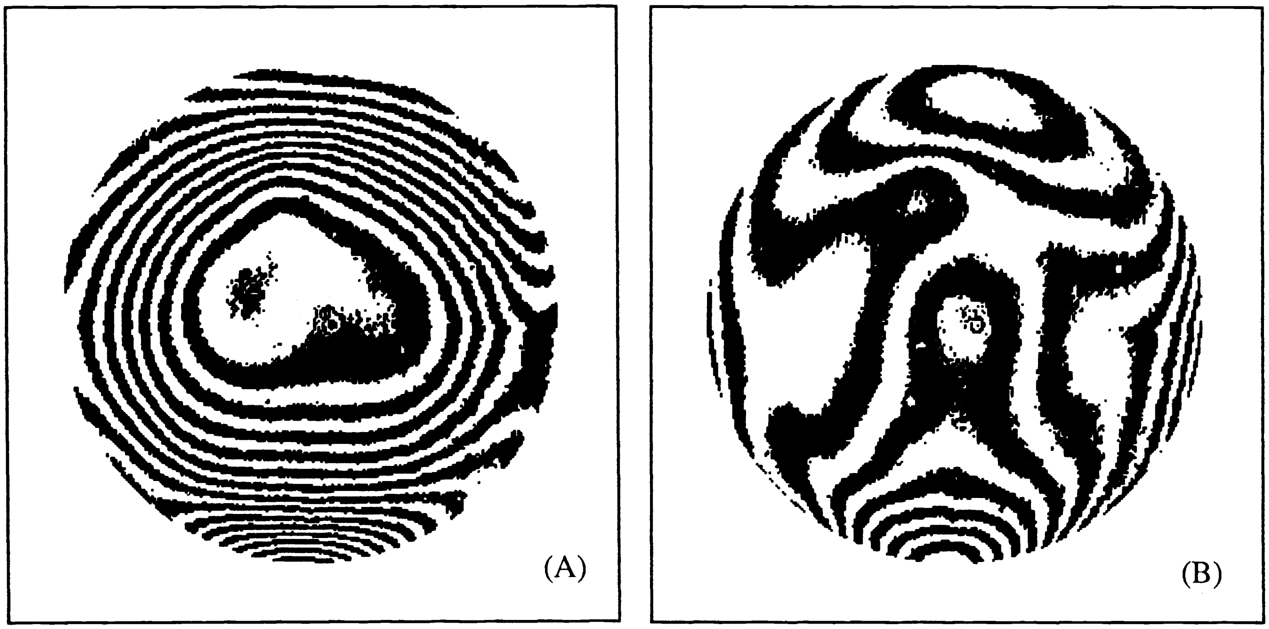


Fig. 4. (A) The measurement result obtained with circularly polarized light, W_{C0}, in which the rod was oriented at 0°. There is a strong defocus contribution. (B) The same result with the defocus contribution removed. All W_{C0}, W_{C90}, W_{C180}, and W_{C270} are very similar and have the same defocus contributions. This shows the rod has a rotational symmetry in birefringence. The interval of the isometric contour in Figs. 4 and 5 is 0.05 waves.

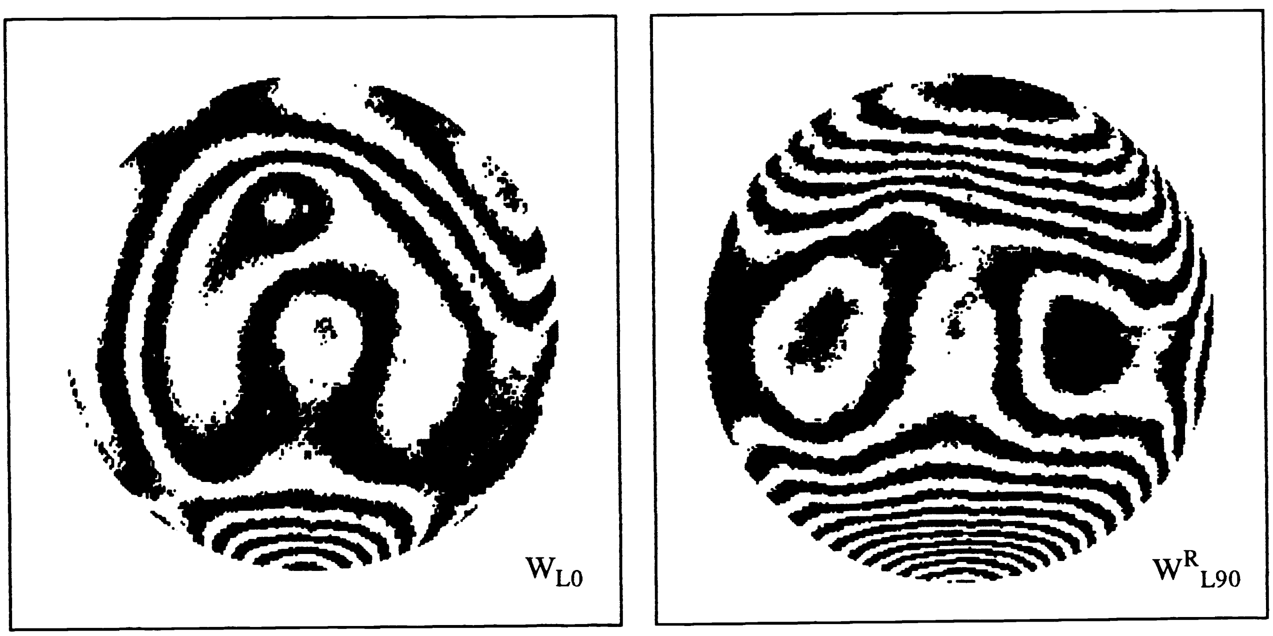


Fig. 5. The two measurement results, obtained with light linearly polarized in the vertical, in which the rod was oriented at 0° and 90°, respectively. W^{R}_{L90} is mathematically rotated back to the orientation 0°. This difference between them shows that the astigmatism is different for the two polarization states.