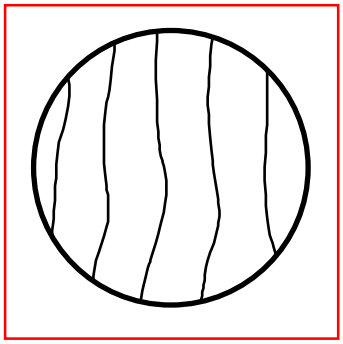
Part 9
Phase Shifting Interferometry

- Classical Interferogram Analysis
- Phase Shifting Advantages
- Phase Shifters
- Algorithms
- Removing Phase Ambiguities

Classical Analysis of Interferograms

Surface Error =
$(\lambda/2)(\Delta/S)$



- **Classical Analysis**
- **Measure positions of fringe centers.**
- **Deviations from straightness and equal spacing gives aberration.**

Computer Analysis of Interferograms

Largest Problem

Getting interferogram data into computer

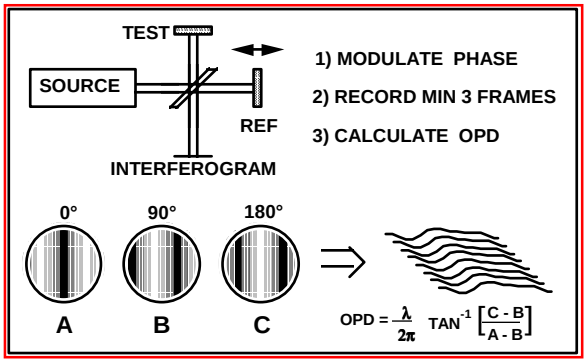
Solutions

- Graphics Tablet
- Scanner
- CCD Camera
- Phase-Shifting Interferometry

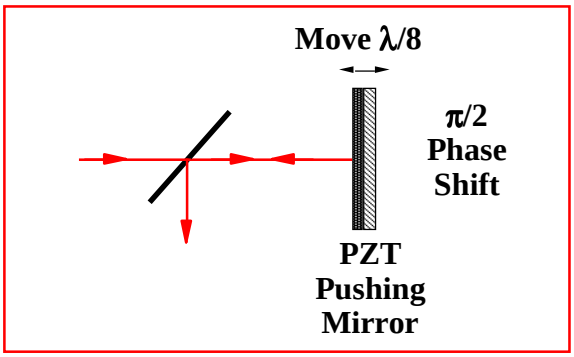
Advantages of Phase-Shifting Interferometry

- High measurement accuracy (>1/1000 fringe, fringe following only 1/10 fringe)
- Rapid measurement
- Good results with low contrast fringes
- Results independent of intensity variations across pupil
- Phase obtained at fixed grid of points
- Easy to use with large solid-state detector arrays

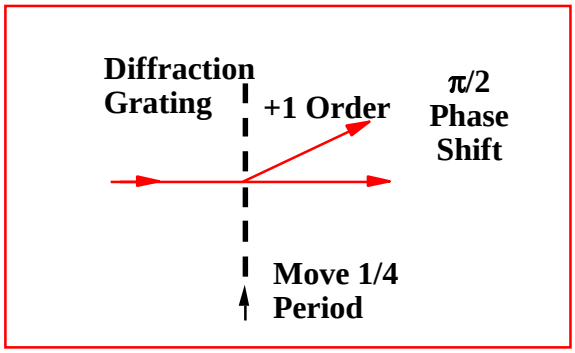
Phase-Shifting Interferometry



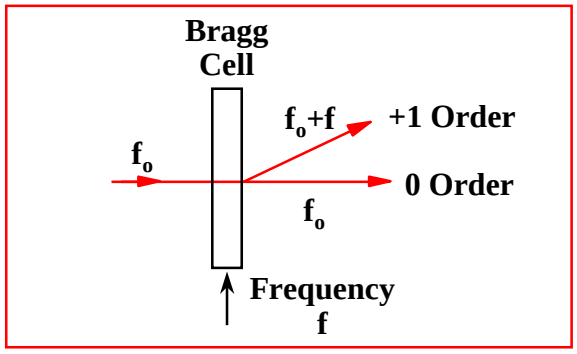
Phase Shifting - Moving Mirror



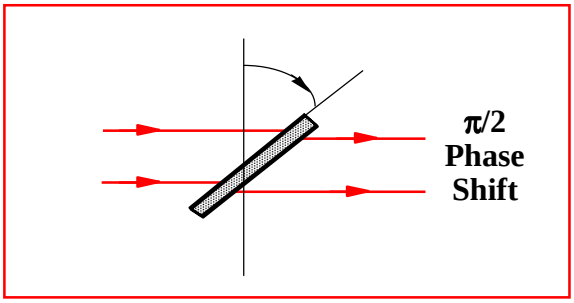
Phase Shifting - Diffraction Grating



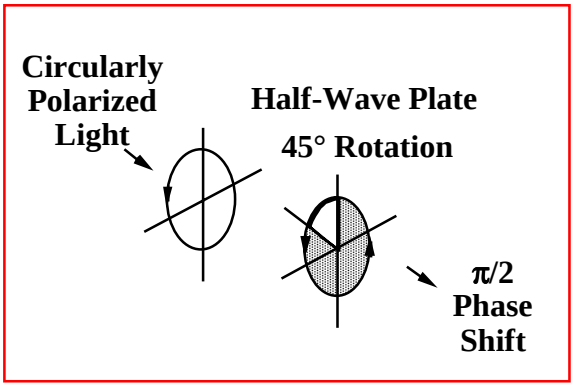
Phase Shifting - Bragg Cell



Phase Shifting - Tilted Glass Plate



Phase Shifting - Rotating Half-Wave Plate



Four Step Method

$$I(x,y) = I_0 + I' \cos[\phi(x,y) + \phi(t)]$$

$$I_1(x,y) = I_0 + I' \cos[\phi(x,y)] \quad \phi(t) = 0$$

$$I_2(x,y) = I_0 - I' \sin[\phi(x,y)] \quad \phi(t) = \pi/2$$

$$I_3(x,y) = I_0 - I' \cos[\phi(x,y)] \quad \phi(t) = \pi$$

$$I_4(x,y) = I_0 + I' \sin[\phi(x,y)] \quad \phi(t) = 3\pi/2$$

$$\text{Tan}[\phi(x,y)] = \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}$$

Relationship between Phase and Height

$$\phi(x,y) = Tan^{-1}\left[\frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}\right]$$

$$\text{Height Error}(x,y) = \frac{\lambda}{4\pi}\phi(x,y)$$

Phase-Measurement Algorithms

Three Measurements $\phi = \tan^{-1}\left[\frac{I_3 - I_2}{I_1 - I_2}\right]$

Four Measurements $\phi = \tan^{-1}\left[\frac{I_4 - I_2}{I_1 - I_3}\right]$

Hariharan Five Measurements $\phi = \tan^{-1}\left[\frac{2(I_2 - I_4)}{2I_3 - I_5 - I_1}\right]$

Carré Equation

$$\phi = \tan^{-1}\left[\frac{\sqrt{[3(I_2 - I_3) - (I_1 - I_4)][(I_2 - I_3) - (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)}\right]$$

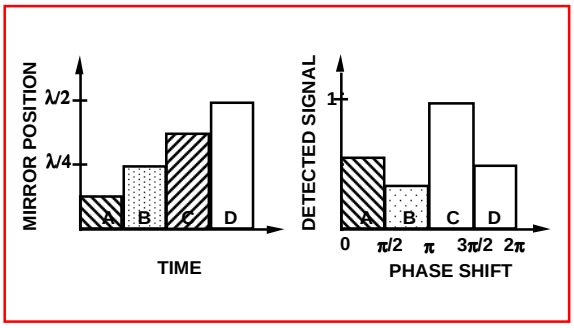
Phase-Measurement Algorithm for N Intensity Measurements

N Measurements $\phi = -\tan^{-1}\left[\frac{\sum_{i=1}^{N} I_i \sin \alpha_i}{\sum_{i=1}^{N} I_i \cos \alpha_i}\right]$

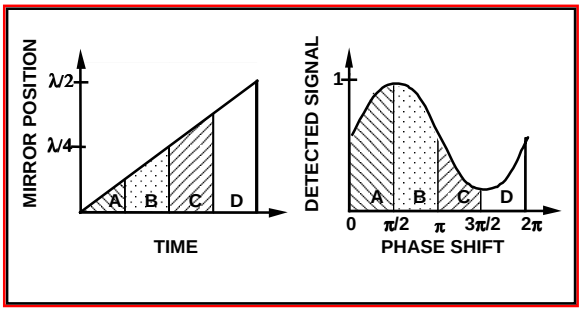
$$\alpha_i = \frac{2\pi i}{N} \quad \text{for } i = 1, \ldots, N$$

Technique is also known as synchronous detection

Phase-Stepping Phase Measurement



Integrated-Bucket Phase Measurement



Integrating-Bucket and Phase-Stepping Interferometry

Measured irradiance given by

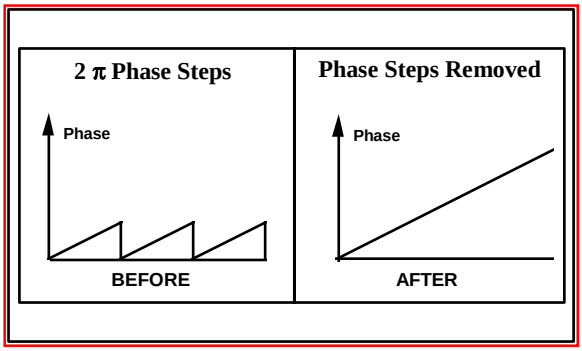
$$I_i = \frac{1}{\Delta}\int_{\alpha_i-\Delta/2}^{\alpha_i+\Delta/2} I_o\{1+\gamma_o\cos[\phi+\alpha_i(t)]\}d\alpha(t)$$

$$= I_o\left\{1+\gamma_o \mathrm{sinc}\left[\frac{\Delta}{2}\right]\cos[\phi+\alpha_i]\right\}$$

Integrating-Bucket $\Delta=\alpha$

Phase-Stepping $\Delta=0$

Phase Ambiguities



Removing Phase Ambiguities

- Arctan Mod 2π (Mod 1 wave)
- Require adjacent pixels less than π difference

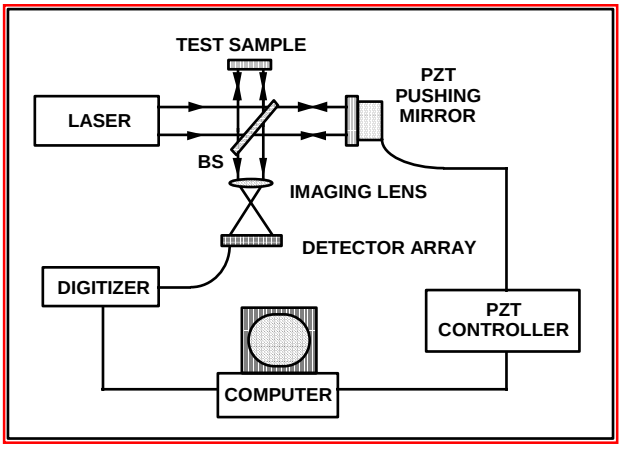
  (1/2 wave OPD)

- Trace path
- When phase jumps by > π

  Add or subtract N2π

  Adjust so < π

Phase-Shifting Interferometer



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