Part 3
Coherence and Source Requirements

- Coherence Time
- Coherence Length
- Partial Coherence
- Temporal Coherence
- Spatial Coherence

Coherence

- Temporal Coherence
  - A source is never strictly monochromatic
- Spatial Coherence
  - A source is never truly a point source

Coherence Time

Maximum transit time difference for good contrast fringes

$$E = A\cos(kx - 2\pi\nu t)$$

$\Delta\nu$ = Frequency Bandwidth

For good contrast fringes

$$\left|2\pi\bar{\nu}\Delta t - 2\pi(\bar{\nu} + \Delta\nu)\Delta t\right| < 2\pi$$

$$\Delta t < \frac{1}{\Delta\nu}$$

Coherence time

$$\Delta t = \tau_c = \frac{1}{\Delta\nu}$$

Coherence Length

Distance light travels during coherence time

Coherence length

$$l_c = c\tau_c = \frac{c}{\Delta\nu} = \frac{\bar{\lambda}^2}{\Delta\lambda}$$

Partial Coherence

$$I = \left\langle (E_1 + E_2)(E_1 + E_2)^* \right\rangle$$
$$= \left\langle |E_1|^2 + |E_2|^2 + E_1 E_2^* + E_1^* E_2 \right\rangle$$
$$= I_1 + I_2 + 2\,\mathrm{Re}\left\langle E_1 E_2^* \right\rangle$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}[\gamma_{12}(\tau)]$$

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)| e^{i\phi_{12}(\tau)}$$

Let

$$\phi_{12}(\tau) = \alpha_{12}(\tau) - \overline{\omega}\tau$$

Then

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma_{12}(\tau)| \cos(\alpha_{12}(\tau) - \overline{\omega}\tau)$$

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Degree of Coherence

$|\gamma_{12}(\tau)|$ is called degree of coherence

Effect of $|\gamma_{12}(\tau)|$ is to reduce fringe contrast

$|\gamma_{12}(\tau)| = 1,$ complete coherence
$0 < |\gamma_{12}(\tau)| < 1,$ partial coherence
$|\gamma_{12}(\tau)| = 0,$ complete incoherence

Effect of $\alpha_{12}(\tau)$ is to shift the fringes

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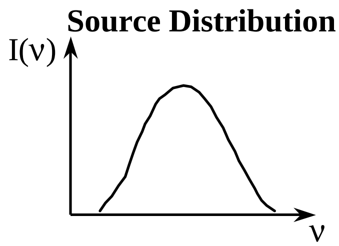
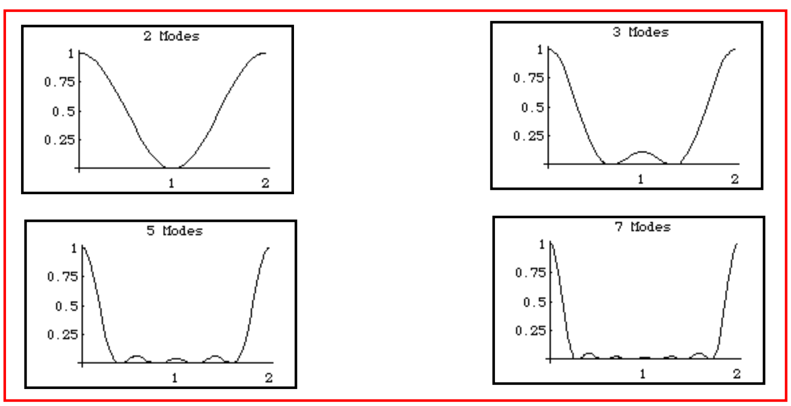
Temporal Coherence



$$\mathrm{Re}[\gamma(\tau)] = \frac{\int_0^\infty I(\nu)\cos(2\pi\nu\tau)\,d\nu}{\int_0^\infty I(\nu)\,d\nu}$$

Fringe Visibility as Function of Path Difference for Laser Having N Longitudinal Modes

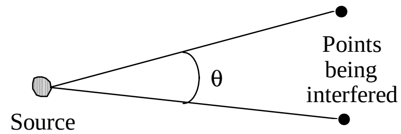


x axis is path difference in units of the laser cavity length

Spatial Coherence

$I(\xi, \eta)$ is source distribution

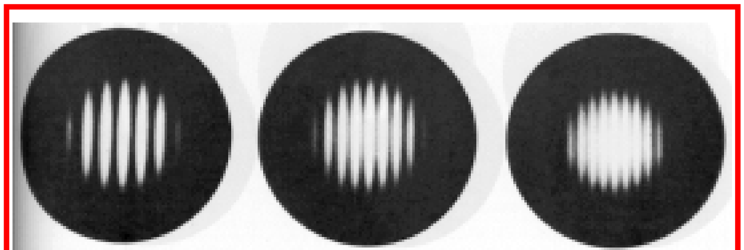
θ_x and θ_y are angular distance between points being interfered as measured from the plane of the source



Fringe Visibility

$$V = |\gamma_{12}| = \left| \frac{\iint I(\xi, \eta) e^{ik(\xi\theta_x + \eta\theta_y)} d\xi d\eta}{\iint I(\xi, \eta) d\xi d\eta} \right|$$

Interference Fringes obtained Using Partially Coherent Light



V=0.593 **V=0.361** **V=0.146**

Source size fixed, distance between points interfered varied.

From B.J. Thompson and E. Wolf, *J. Opt. Soc. Am.* **47**, 895 (1957)