

Part 2

Classical Interferometry

- **Basic Interference**
 - Two plane waves
 - Two spherical waves
 - Plane wave and spherical wave
- **Classes of Interferometers**
 - Division of wavefront
 - Division of amplitude

Optical Detectors Respond to Square of Electric Field

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = E_1 \hat{a}_1 e^{i(\omega_1 t + \alpha_1)} + E_2 \hat{a}_2 e^{i(\omega_2 t + \alpha_2)}$$

$$\begin{aligned} \bar{I} &= \text{Constant} |\vec{E}_1 + \vec{E}_2|^2 \\ &= \text{Constant} [E_1^2 + E_2^2 + 2E_1 E_2 (\hat{a}_1 \cdot \hat{a}_2) \cos[(\omega_1 - \omega_2)t + \alpha_1 - \alpha_2]] \end{aligned}$$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1 \bar{I}_2} (\hat{a}_1 \cdot \hat{a}_2) \cos[(\omega_1 - \omega_2)t + \alpha_1 - \alpha_2]$$

Irradiance at each point varies cosinusoidally with time at the difference frequency

Interference Fringes

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1\bar{I}_2}(\hat{a}_1 \cdot \hat{a}_2) \cos[(\omega_1 - \omega_2)t + \alpha_1 - \alpha_2]$$

Let $\omega_1 = \omega_2$

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1\bar{I}_2}(\hat{a}_1 \cdot \hat{a}_2) \cos(\alpha_1 - \alpha_2)$$

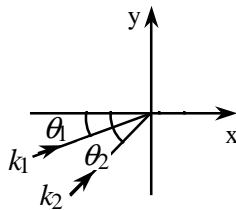
Bright interference fringe

$$\alpha_1 - \alpha_2 = 2\pi m$$

Dark interference fringe

$$\alpha_1 - \alpha_2 = 2\pi \left(m + \frac{1}{2} \right)$$

Interference of Two Plane Waves



$$\vec{E}_1 = E_1 e^{i(\vec{k}_1 \cdot \vec{r} - \omega t + \phi_1)} \hat{a}_1$$

$$\vec{E}_2 = E_2 e^{i(\vec{k}_2 \cdot \vec{r} - \omega t + \phi_2)} \hat{a}_2$$

$$\vec{k}_1 = k(\cos \theta_1 \hat{i} + \sin \theta_1 \hat{j})$$

$$\vec{k}_2 = k(\cos \theta_2 \hat{i} + \sin \theta_2 \hat{j})$$

$$\vec{r} = x \hat{i} + y \hat{j}, \quad k = \frac{2\pi}{\lambda}$$

$$\alpha = \alpha_1 - \alpha_2 = \vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r} + \phi_1 - \phi_2$$

Let $\phi_1 = \phi_2$

Bright fringe

$$\alpha = 2\pi m$$

$$= k[x(\cos \theta_1 - \cos \theta_2) + y(\sin \theta_1 - \sin \theta_2)]$$

Dark fringe

$$\alpha = 2\pi \left(m + \frac{1}{2} \right)$$

Fringe Spacing

$$\bar{I} = \bar{I}_1 + \bar{I}_2 + 2\sqrt{\bar{I}_1\bar{I}_2}(\hat{a}_1 \cdot \hat{a}_2)\cos(\alpha_1 - \alpha_2)$$

Bright fringe

$$\alpha = \alpha_1 - \alpha_2 = 2\pi m = k[x(\cos \theta_1 - \cos \theta_2) + y(\sin \theta_1 - \sin \theta_2)]$$

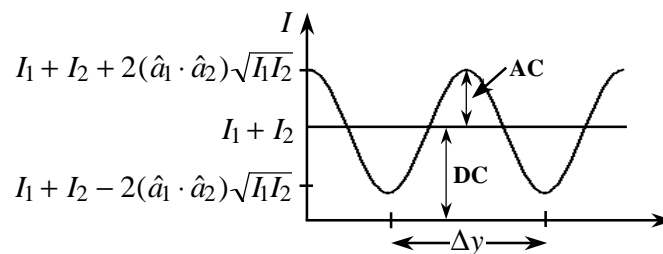
Straight equi-spaced fringes

Look in $x=0$ plane

Fringe spacing

$$\Delta y = \frac{\lambda}{\sin \theta_1 - \sin \theta_2}$$

Fringe Visibility

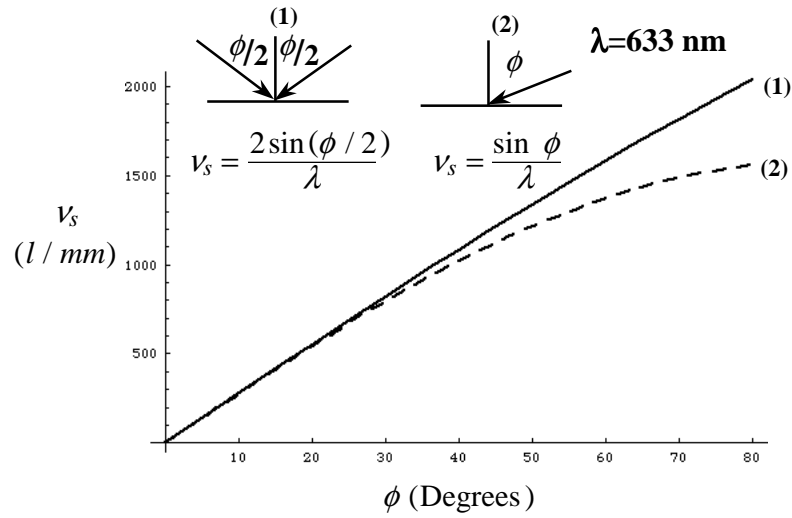


$$\text{Fringe Visibility} = V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$\text{if } \hat{a}_1 \cdot \hat{a}_2 = 1$$

$$V = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} = \frac{AC}{DC}$$

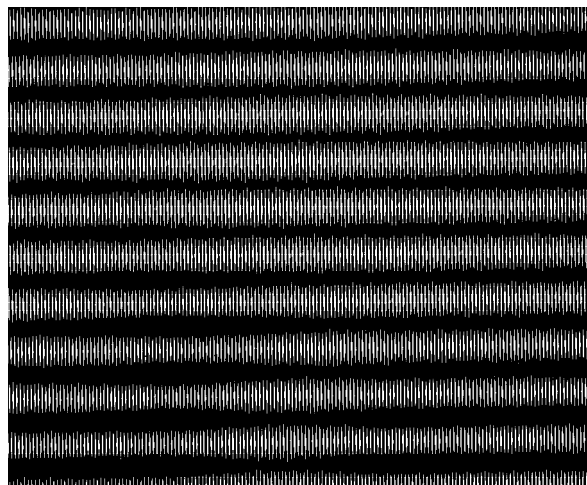
Fringe Spatial Frequency



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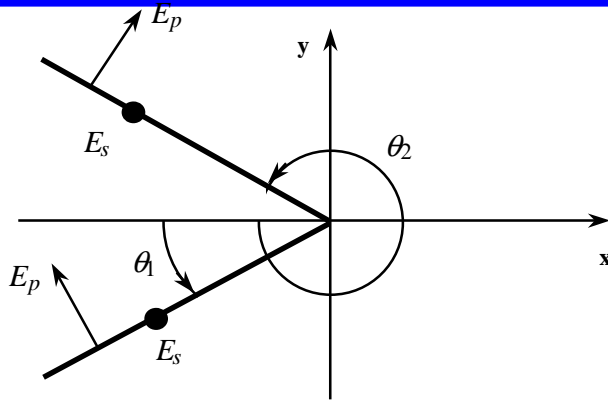
Moiré Pattern - Two Plane Waves



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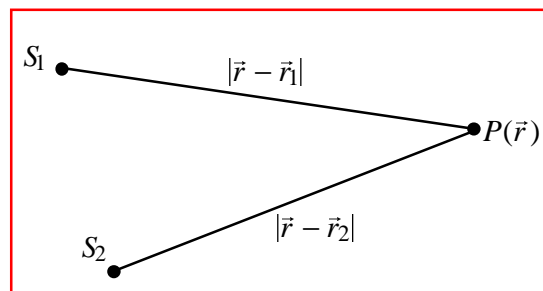
Effect of Polarization Direction



Dependence of $\hat{a}_1 \cdot \hat{a}_2$ on angle for s and p polarization

s polarization: $\hat{a}_1 \cdot \hat{a}_2 = 1$ for all angles
 p polarization: $\hat{a}_1 \cdot \hat{a}_2$ depends upon angle

Interference of Two Spherical Waves



$$\vec{E}_1 = \hat{a}_1 \frac{B_1}{|\vec{r} - \vec{r}_1|} e^{i[k|\vec{r} - \vec{r}_1| - \omega t + \phi_1]}$$

$$\vec{E}_2 = \hat{a}_2 \frac{B_2}{|\vec{r} - \vec{r}_2|} e^{i[k|\vec{r} - \vec{r}_2| - \omega t + \phi_2]}$$

Two Spherical Waves - Fringe Shape

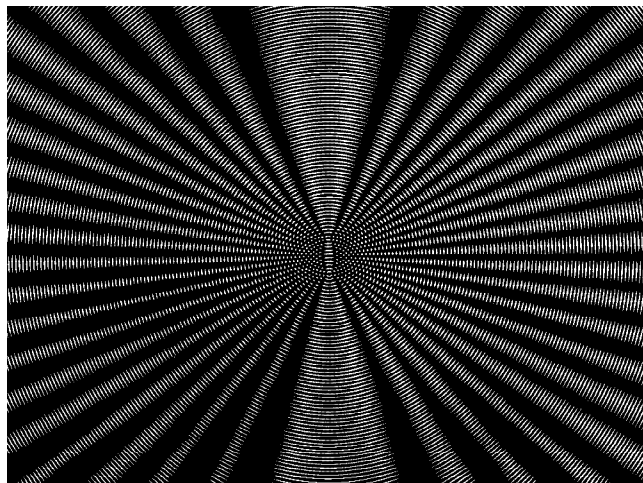
$$\vec{E}_1 = \hat{a}_1 \frac{B_1}{|\vec{r} - \vec{r}_1|} e^{i[k|\vec{r} - \vec{r}_1| - \omega t + \phi_1]} \quad \sqrt{I_1} = \frac{B_1}{|\vec{r} - \vec{r}_1|} \approx \text{Constant}$$
$$\vec{E}_2 = \hat{a}_2 \frac{B_2}{|\vec{r} - \vec{r}_2|} e^{i[k|\vec{r} - \vec{r}_2| - \omega t + \phi_2]} \quad \sqrt{I_2} = \frac{B_2}{|\vec{r} - \vec{r}_2|} \approx \text{Constant}$$

$$\alpha = \frac{2\pi}{\lambda} \{|\vec{r} - \vec{r}_1| - |\vec{r} - \vec{r}_2|\} + \phi_1 - \phi_2$$

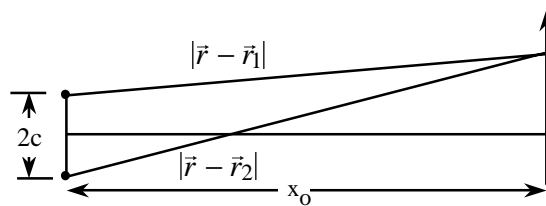
= Constant for given fringe

Hyperbolic Fringes

Moiré Pattern - Spherical Waves



Spherical Waves - Special Case #1

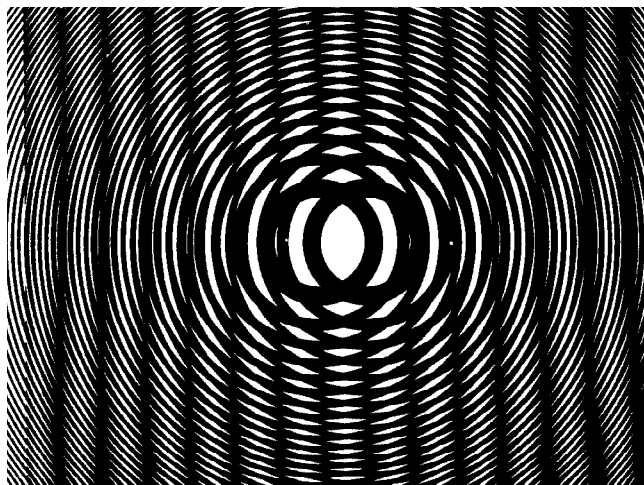


if $x_0 \gg 2c$ then

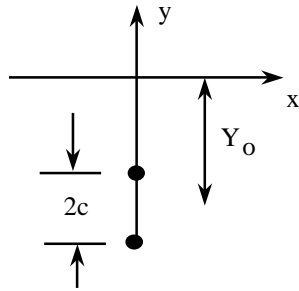
$$m\lambda = \frac{2cy}{x_0}$$

Same result as for two plane waves

Moiré Pattern - Straight Line Fringes



Spherical Waves - Special Case #2



For bright fringe

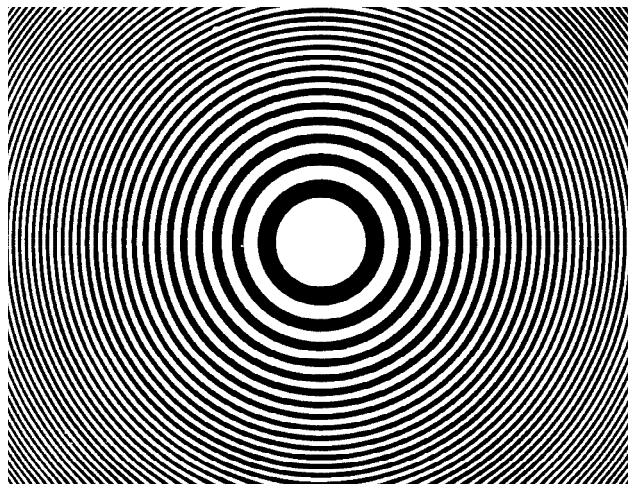
$$m\lambda = |\bar{r} - \bar{r}_1| - |\bar{r} - \bar{r}_2|$$

$$= 2c - \frac{(x^2 + z^2)}{Y_0^2} c$$

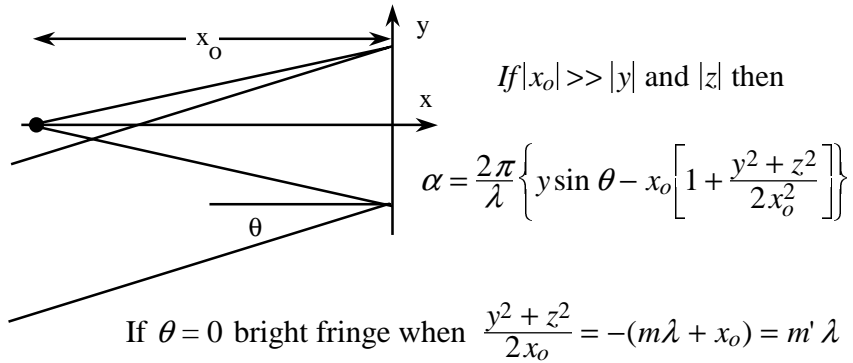
Fringes are concentric circles

$$r = \sqrt{\frac{2c - m\lambda}{c}} Y_0 \quad \text{spatial frequency} = \frac{1}{\Delta r} = \frac{2c}{y_0^2 \lambda} r$$

Concentric Circular Fringes



Interference of Plane Wave and Spherical Wave

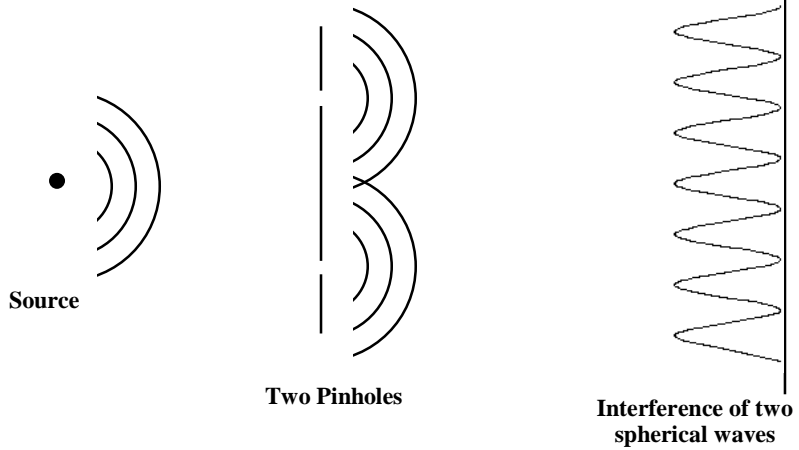


Circular fringes of radius $r = \sqrt{2x_o m' \lambda}$

Two Basic Classes of Interferometers

- **Division of Wavefront**
 - **Division of Amplitude**

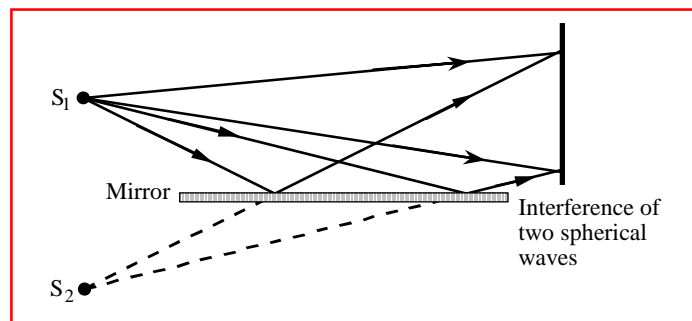
Division of Wavefront (Young's Two Pinholes)



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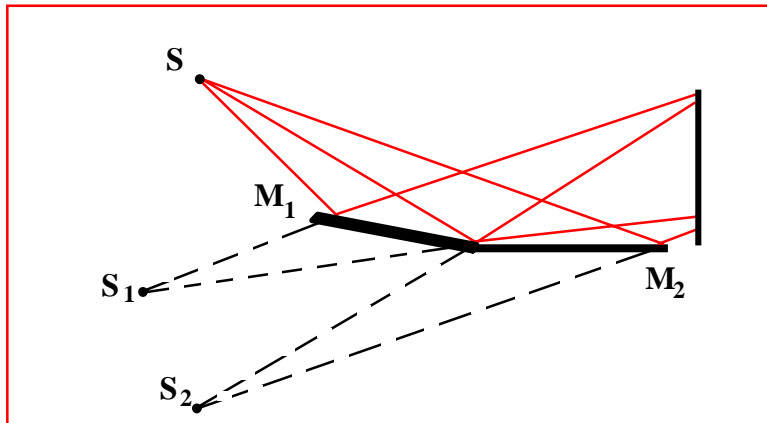
Division of Wavefront (Lloyd's Mirror)



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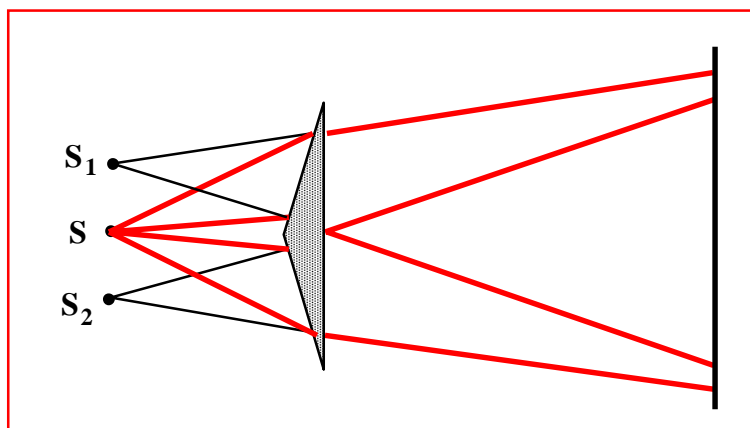
Division of Wavefront (Fresnel's Mirrors)



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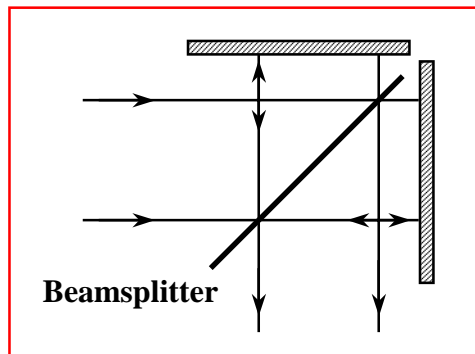
Division of Wavefront (Fresnel's Biprism)



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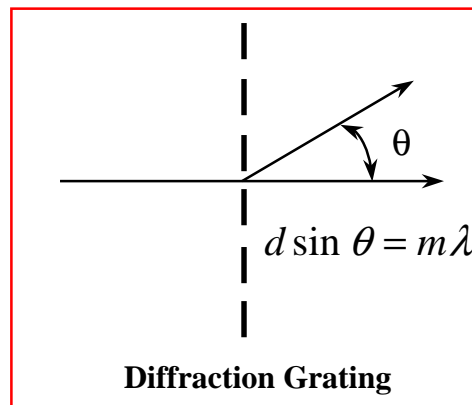
Division of Amplitude (Beamsplitter)



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Division of Amplitude (Diffraction)

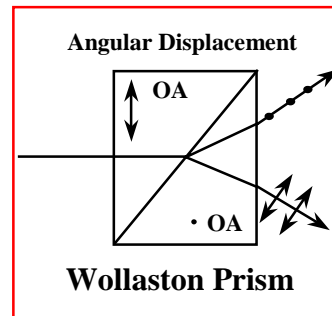
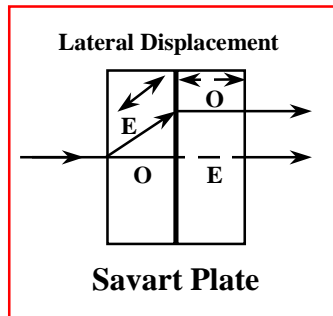


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Division of Amplitude and Division of Wavefront

Polarization

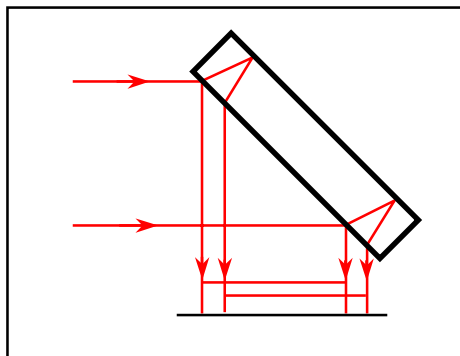


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Division of Amplitude and Division of Wavefront

Plane Parallel Plate



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